COMPARISON OF COHERENT SMITH-PURCELL RADIATION AND COHERENT TRANSITION RADIATION*

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Abstract

Smith-Purcell radiation and Transition Radiation are two radiative phenomenon that occur in charged particles accelerators. For both the emission can be significantly enhanced with sufficiently short pulses and both can be used to measure the form factor of the pulse. We compare the yield of these phenomenon in different configurations and look at their application as bunch length monitors, including background filtering and rejection. We apply these calculations to the specific case of the CLIO Free Electron laser

INTRODUCTION

Coherent Transition Radiation (CTR)

When a relativistic charged particle crosses the interface between two media of different dielectric properties, transition radiation (TR) is emitted. This process was calculated analytically by Ginzburg and Frank [1].

$$\frac{d^2 I_{GF}}{d\omega d\Theta} = \frac{q_0^2}{4\pi^3 \epsilon_0 c} \frac{\beta^2 sin^2\Theta}{(1 - \beta^2 cos^2\Theta)^2}, \qquad (1)$$

where q_0 is electron charge, c is the speed of light, β is relativistic velocity and Θ is the observation angle.

We use their formula with virtual-quanta method to compute the backward TR from a finite screen [2]

$$\frac{d^2 I_{disk}}{d\omega d\Theta} = \frac{d^2 I_{GF}}{d\omega d\Theta}[1 - T(\gamma, \omega a, \Theta)]^2 \qquad (2)$$

where T is correction tern to finite size of the screen. This gives the single electron yield (SEY). From the SEY the whole spectrum can be derived using the following formula:

$$\frac{d^2 I}{d\omega d\Theta} = \frac{d^2 I_1}{d\omega d\Theta}[N + N(N-1)F(\omega)] \qquad (3)$$

Where N is the number of electrons in the bunch and $F(\omega)$ is the form factor of the time profile of the bunch. Using phase recovery methods, such as Kramers-Kronig or Hilbert [3], it is possible to recover the phase and then the time profile of the bunch.

Coherent Smith-Purcell Radiation (CSPR)

The same formula, but with a different SEY is used for calculation of the Smith-Purcell radiation (SP) spectrum. SP radiation occurs when a charged particle move above a metallic periodic structure. Unlike TR, SP has the advantage that the emmited radiation is not concentrated in a small observation angle ($\Theta_{max} \simeq 1/\gamma$), but spread in angle. The wavelength of the radiation for SP depends on the observing angle according to the following:

$$\lambda = \frac{l}{n}(\frac{1}{\beta} - cos\Theta) \qquad (4)$$

where l is the grating period, n is the order of radiation, Θ is the observation angle and β is the relativistic velocity.

To calculate the SEY and the total spectrum for SP effect, the gfw code was used [4]. The calculation is based on the surface current model. Taking into account the fact that the grating have a finite width, the energy per solid angle for a single electron can be written as:

$$\frac{dI}{d\Omega} = 2\pi e^2 \frac{Z}{l^2} \frac{n^2 \beta^3}{(1 - \beta cos\Theta)^3} R^2 \qquad (5)$$

where Z is the grating length, e is the electron charge and R^2 is the grating efficiency factor.

COMPARISON

Single Electron Yield

Calculation of the SEY for SP and TR are presented on figure 1. The screen used in this case for TR is a metalic disk with radius 20 mm and the grating for SP has a length of 180 mm and a width of 40 mm with a pitch of 8 mm and a 30 degrees blaze angle. The beam-grating distance (from center of the beam to the top of the teeth) is 3 mm. The lorentz factor γ for both cases is 118 (60.3 MeV). The other parameters were taken from the CLIO accelerator simulation in ASTRA reported in [5].

The SEY spatial energy distribution for SP and TR are also significantly different. The spatial distribution of the energy per solid angle and per grating length is presented on fig. 2.

Coherent Radiation

This calculation was done with same parameters as for SEY. For others grating this distribution will be different, but it give us approximate space distribution of SP radiation. At this point, we can conclude, that most of radiation is confined in approx. $\pm 6°$ in azimuthal (ϕ) angle. So a standard 50 mm parabolic mirror at a distance of 300 mm from the grating will collect most of the radiation.

To choose the most appropriate grating pitch, one should use Smith-Purcell condition given in equation 4. For maxi-
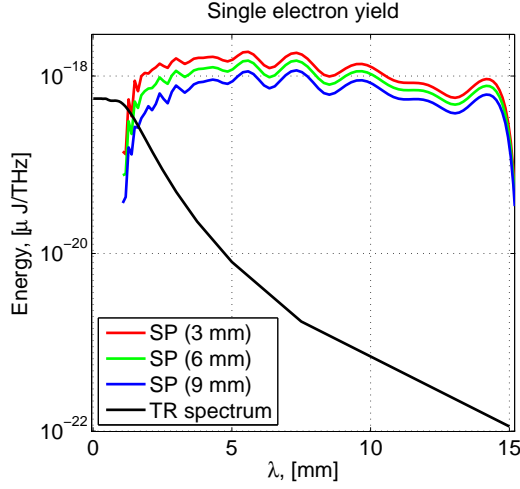
Figure 1: Single electron yield for TR and SP. The screen diameter for TR is 40mm. SP SEY is presented for different beam-grating separation (3,6,9 mm). The grating used here is 40mm × 180mm with 8 mm pitch and 30° blaze angle.
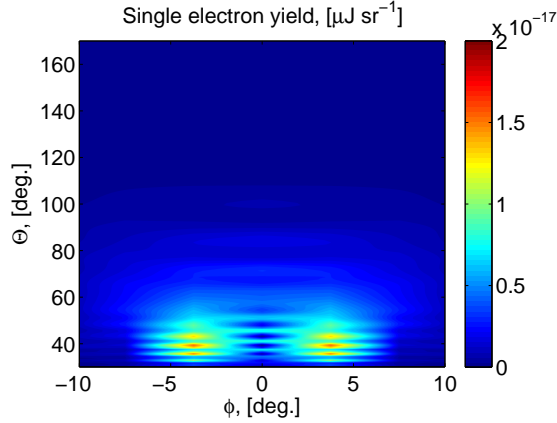

Figure 2: SEY for SP effect. Grating 40mm × 180mm with 8 mm pitch and 30° blaze angle.
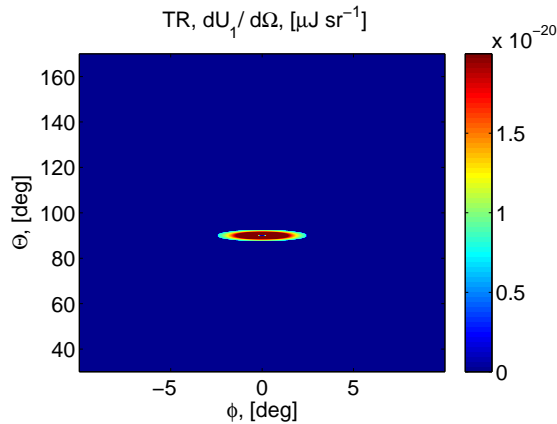

Figure 3: SEY for TR effect. Screen is turned at 45° to beam propagation direction and have diameter 40 mm.

mum emission at 90 deg. formula is applicable.

$$l = \frac{2\pi c}{2\sqrt{2ln(2)}} p_t \approx 8 \times 10^8 p_t \qquad (6)$$

where l is grating pitch in meters and p_t – bunch fwhm in seconds.

The relation between the the grating pitch, the pulse length and the angle of maximum emission is given on figure 4. We can see that the green band (90° emission) follows the rule given in equation 6. We can also look at the total energy emitted by the grating as a function of the pulse length and the grating energy. This is shown on figure 5. On this figure the transition between the coherent and incoherent regime can clearly be seen.
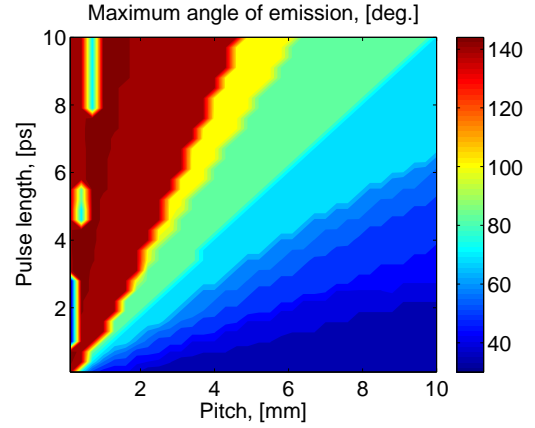

Figure 4: Maximum angle of emission for SP effect as function of pulsewidth and grating pitch. Grating 40mm × 180mm with 30° blaze angle. The beam-grating separation is 3 mm.
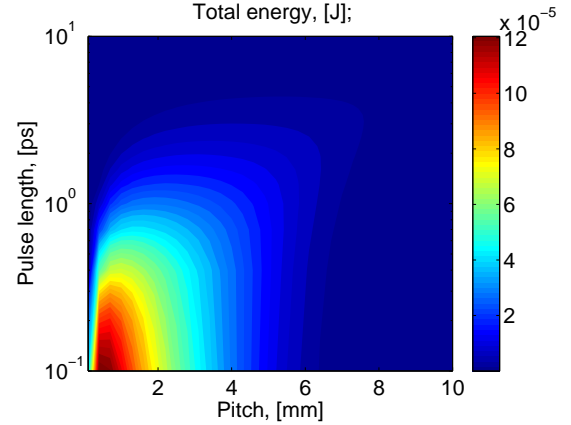

Figure 5: Total energy for SP effect presented as function of pulsewidth and grating pitch. Grating is 40 × 180mm with 30° blaze angle.

In the case of CSPR as the pulse length change the angular distribution of the energy will also change. This is shown on figure 6. Same dependence for CTR is shown on figure 7.

Using the bunch profile predicted for the CLIO Free Electron Laser [5], as shown on figure 8 we can predict the spectrum for both CSPR and CTR as shown on figure 9. We can see that the intensity of the CTR signal is lower, but it is concentrated in a small solid angle. For CSPR the signal intensity depends on the beam-grating separation.
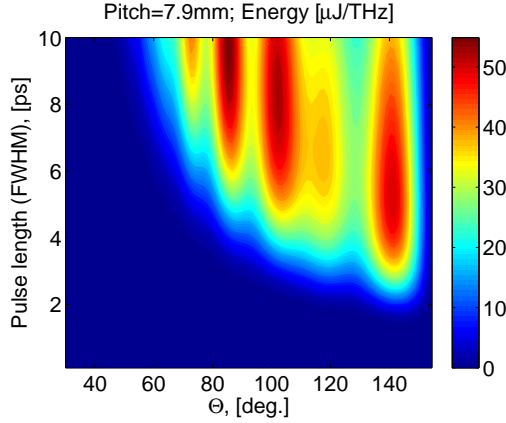
Figure 6: Evolution of SP spectrum with constant grating pitch and changing wavelength. Grating 40×180mm with 7.9 mm pitch and 30° blaze angle.
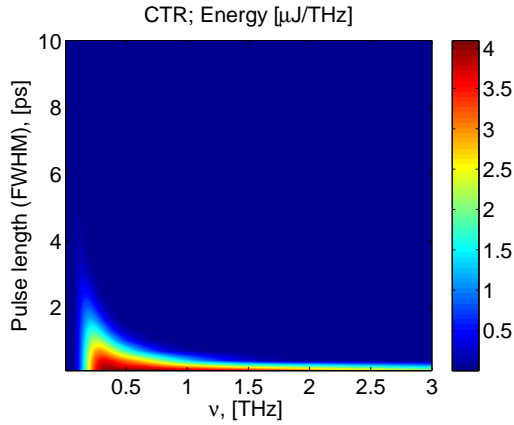

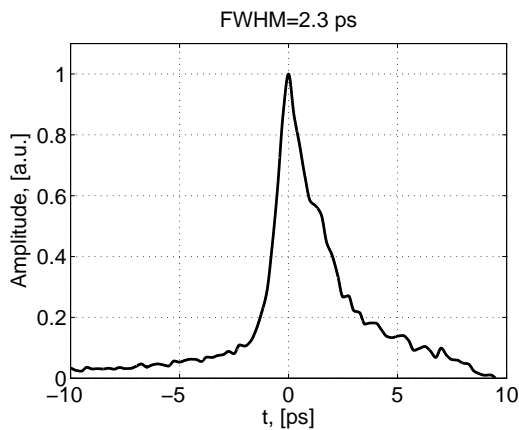Figure 7: Evolution of CTR spectrum with changing wavelength. Screen is 40mm in diameter.

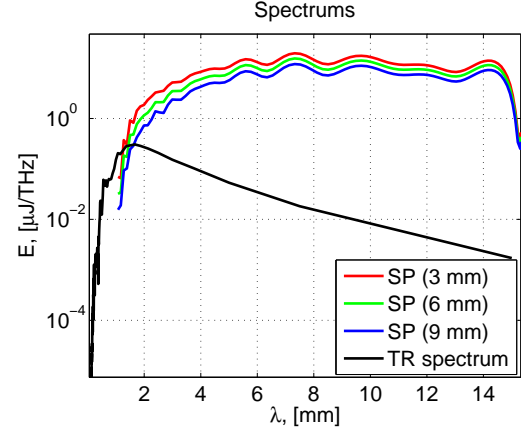Figure 8: Profile of the bunch at the exit of the CLIO accelerating section.

Figure 9: CSPR and CTR energy density as function of wavelength. SP spectrum is presented for different beam-grating separation (3,6,9 mm).Grating 40×180mm with 8 mm pitch and 30° blaze angle. Screen diameter for TR is 40mm. The signal is measured as integrated with a 50mm diameter parabolic mirror located 300 mm from the beam axis.

CONCLUSION

We have studied both CSPR and CTR and studied how to optimize the experimental parameters. Using the CLIO parameters we expect a signal (in the range 0.3-3 THz [0.1 - 1 mm]) of 5.3e-7 J for CSPR and 1.86e-06 J for CTR with 50 mm parabolic mirror at distance 300mm from grating.

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