

# COMPARISON OF COHERENT SMITH-PURCELL RADIATION AND COHERENT TRANSITION RADIATION\*

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## Abstract

Smith-Purcell radiation and Transition Radiation are two radiative phenomenon that occur in charged particles accelerators. For both the emission can be significantly enhanced with sufficiently short pulses and both can be used to measure the form factor of the pulse. We compare the yield of these phenomenon in different configurations and look at their application as bunch length monitors, including background filtering and rejection. We apply these calculations to the specific case of the CLIO Free Electron laser

## INTRODUCTION

### Coherent Transition Radiation (CTR)

When a relativistic charged particle crosses the interface between two media of different dielectric properties, transition radiation (TR) is emitted. This process was calculated analytically by Ginzburg and Frank [1].

$$\frac{d^2 I_{GF}}{d\omega d\Theta} = \frac{q_0^2}{4\pi^3 \epsilon_0 c} \frac{\beta^2 \sin^2 \Theta}{(1 - \beta^2 \cos^2 \Theta)^2}, \quad (1)$$

where  $q_0$  is electron charge,  $\epsilon_0$  – vacuum permittivity,  $c$  is the speed of light,  $\beta$  is relativistic velocity and  $\Theta$  is the observation angle.

We use their formula with virtual-quanta method to compute the backward TR from a finite screen [2]

$$\frac{d^2 I_{disk}}{d\omega d\Theta} = \frac{d^2 I_{GF}}{d\omega d\Theta} [1 - T(\gamma, \omega a, \Theta)]^2 \quad (2)$$

where  $T$  is correction term to finite size of the screen. This gives the single electron yield (SEY). From the SEY the whole spectrum can be derived using the following formula:

$$\frac{d^2 I}{d\omega d\Theta} = \frac{d^2 I_1}{d\omega d\Theta} [N + N(N - 1)F(\omega)] \quad (3)$$

Where  $N$  is the number of electrons in the bunch and  $F(\omega)$  is the form factor of the time profile of the bunch. Using phase recovery methods, such as Kramers-Kronig or Hilbert [3], it is possible to recover the phase and then the time profile of the bunch.

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### Coherent Smith-Purcell Radiation (CSPR)

The same formula, but with a different SEY is used for calculation of the Smith-Purcell radiation (SP) spectrum. SP radiation occurs when a charged particle move above a metallic periodic structure. Unlike TR, SP has the advantage that the emitted radiation is not concentrated in a small observation angle ( $\Theta_{max} \simeq 1/\gamma$ ), but spread in angle. The wavelength of the radiation for SP depends on the observing angle according to the following:

$$\lambda = \frac{l}{n} \left( \frac{1}{\beta} - \cos \Theta \right) \quad (4)$$

where  $l$  is the grating period,  $n$  is the order of radiation,  $\Theta$  is the observation angle and  $\beta$  is the relativistic velocity.

To calculate the SEY and the total spectrum for SP effect, the gfw code was used [4]. The calculation is based on the surface current model. Taking into account the fact that the grating have a finite width, the energy per solid angle for a single electron can be written as:

$$\frac{dI}{d\Omega} = 2\pi e^2 \frac{Z}{l^2} \frac{n^2 \beta^3}{(1 - \beta \cos \Theta)^3} R^2 \quad (5)$$

where  $Z$  is the grating length,  $e$  is the electron charge and  $R^2$  is the grating efficiency factor.

## COMPARISON

### Single Electron Yield

Calculation of the SEY for SP and TR are presented on figure 1. The screen used in this case for TR is a metallic disk with radius 20 mm and the grating for SP has a length of 180 mm and a width of 40 mm with a pitch of 8 mm and a 30 degrees blaze angle. The beam-grating distance (from center of the beam to the top of the teeth) is 3 mm. The lorentz factor  $\gamma$  for both cases is 118 (60.3 MeV). The other parameters were taken from the CLIO accelerator simulation in ASTRA reported in [5].

The SEY spatial energy distribution for SP and TR are also significantly different. The spatial distribution of the energy per solid angle and per grating length is presented on figure 2.

### Coherent Radiation

The calculation of coherent radiation was done with the same parameters than for the SEY. For others grating this distribution would be different, but this gives us an approximate space distribution of the CSPR. From these simulations,

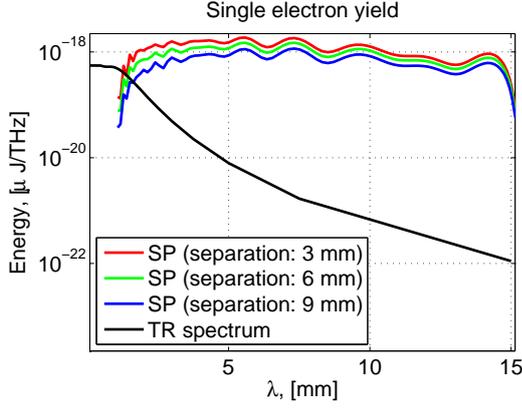


Figure 1: Single electron yield for TR and SP. The screen diameter for TR is 40mm. SP SEY is presented for different beam-grating separation (3,6,9 mm). The grating used here is  $40 \times 180 \text{ mm}^2$  with 8 mm pitch and  $30^\circ$  blaze angle.

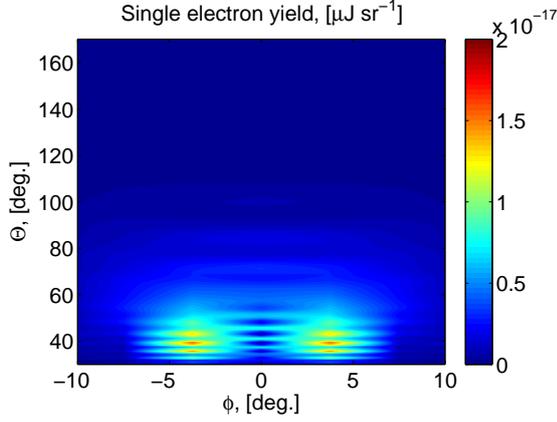


Figure 2: SEY for SP effect. Grating  $40 \times 180 \text{ mm}^2$  with 8 mm pitch and  $30^\circ$  blaze angle.

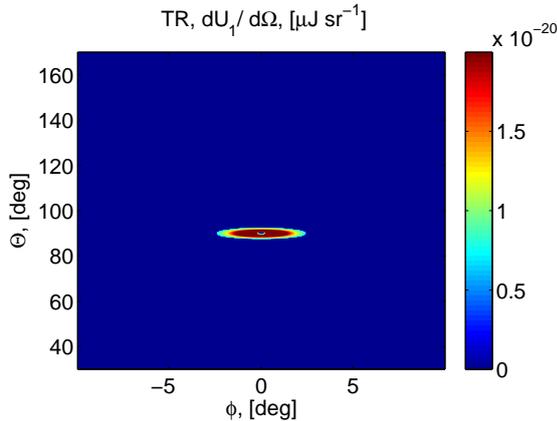


Figure 3: SEY for TR effect. Screen is turned at  $45^\circ$  to beam propagation direction and have diameter 40 mm.

we can conclude that most of the radiation is confined in approximately  $\pm 6^\circ$  in azimuthal ( $\phi$ ) angle. So a standard 50 mm parabolic mirror at a distance of 300 mm from the grating will collect most of the radiation.

To choose the most appropriate grating pitch, one should use Smith-Purcell condition given in equation 4. For maximum emission at  $90^\circ$  deg. formula is applicable.

$$l = \frac{2\pi c}{2\sqrt{2}ln(2)} p_t \approx 8 \times 10^8 p_t \quad (6)$$

where  $l$  is grating pitch in meters and  $p_t$  – bunch fwhm in seconds.

The relation between the the grating pitch, the pulse length and the angle of maximum emission is given on figure 4. We can see that the green band ( $90^\circ$  emission) follows the rule given in equation 6. We can also look at the total energy emitted by the grating as a function of the pulse length and the grating energy. This is shown on figure 5. On this figure the transition between the coherent and incoherent regime can clearly be seen.

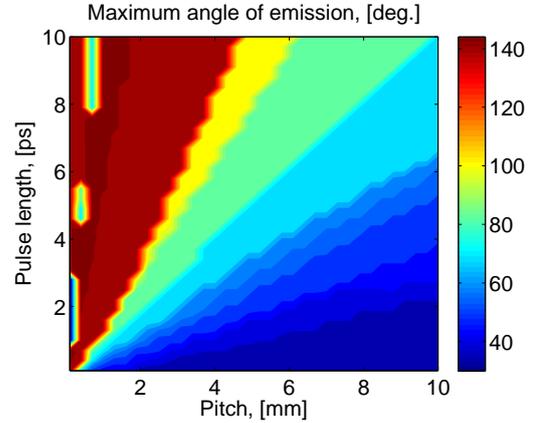


Figure 4: Maximum angle of emission for SP effect as function of pulsewidth and grating pitch. Grating  $40 \times 180 \text{ mm}^2$  with  $30^\circ$  blaze angle. The beam-grating separation is 3 mm.

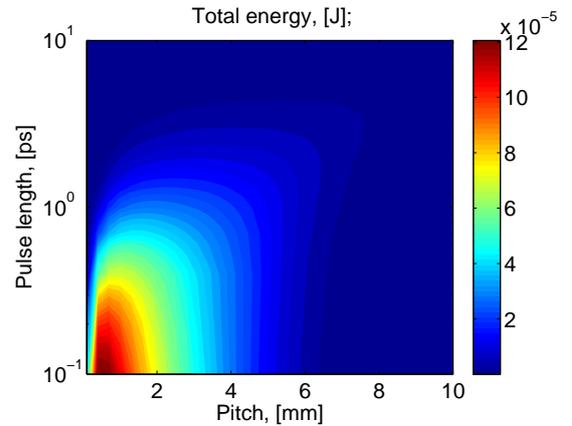


Figure 5: Total energy for SP effect presented as function of pulsewidth and grating pitch. Grating is  $40 \times 180 \text{ mm}^2$  with  $30^\circ$  blaze angle.

In the case of CSPR as the pulse length change the angular distribution of the energy will also change. This is shown on figure 6. Same dependence for CTR is shown on figure 7.

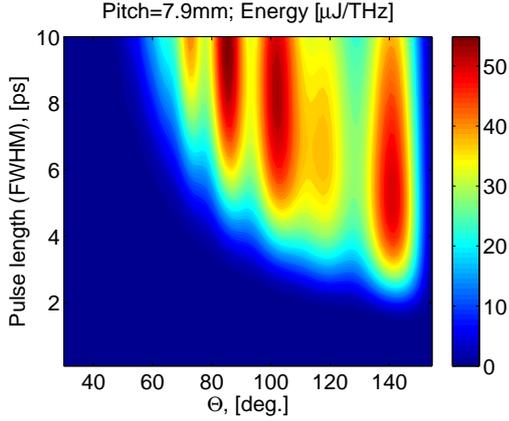


Figure 6: Evolution of the C SPR spectrum with a constant grating pitch and a wavelength changing with the angle. The grating dimensions are  $40 \times 180 \text{ mm}^2$  with 7.9 mm pitch and  $30^\circ$  blaze angle.

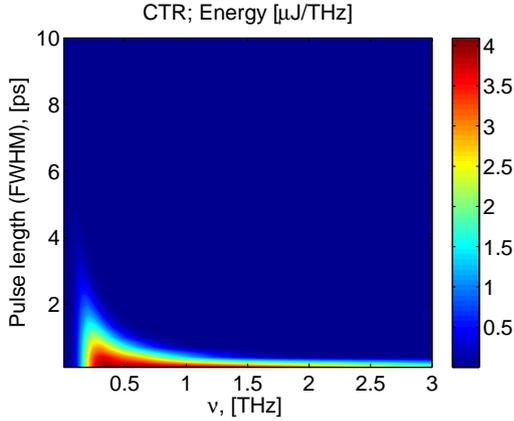


Figure 7: Evolution of CTR spectrum with changing wavelength. The screen has a diameter of 40 mm.

Using the bunch profile predicted for the CLIO Free Electron Laser [5], as shown on figure 8 we can predict the spectrum for both C SPR and CTR as shown on figure 9. We can see that the intensity of the CTR signal is lower, but it is concentrated in a small solid angle. For C SPR the signal intensity depends on the beam-grating separation.

## CONCLUSION

We have studied both C SPR and CTR and studied how to optimize the experimental parameters. Using the CLIO parameters we expect a signal (in the range 0.03-3 THz [ 0.1 - 10 mm]) of  $8.37 \times 10^{-7} \text{ J}$  for C SPR and  $7.35 \times 10^{-8} \text{ J}$  for CTR.

## REFERENCES

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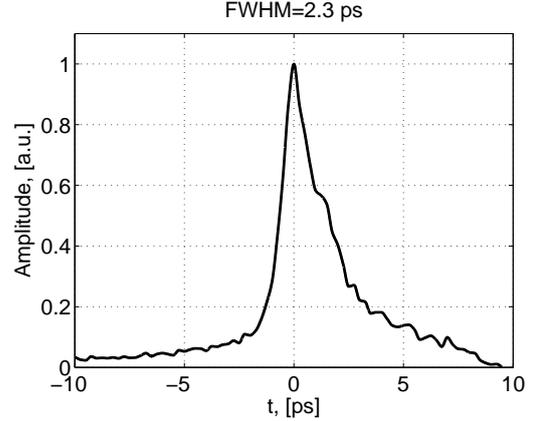


Figure 8: Profile of the bunch at the exit of the CLIO accelerating section (see [5]).

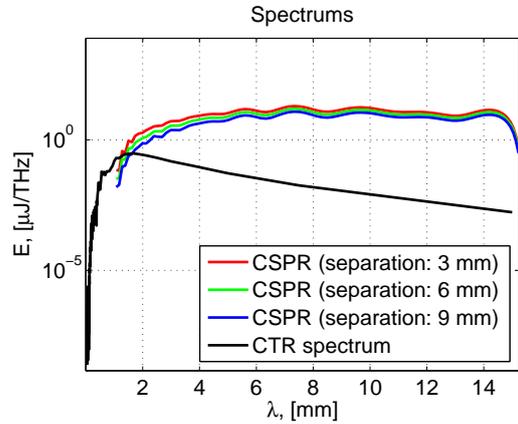


Figure 9: C SPR and CTR energy density as function of wavelength. The C SPR spectrum is presented for beam-grating separations of 3 mm, 6 mm and 9 mm. The grating dimensions are  $40 \times 180 \text{ mm}^2$  with 8 mm pitch and  $30^\circ$  blaze angle. The screen diameter for TR is 40 mm. The signal is measured as integrated with a 50 mm diameter parabolic mirror located 300 mm from the beam axis.

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