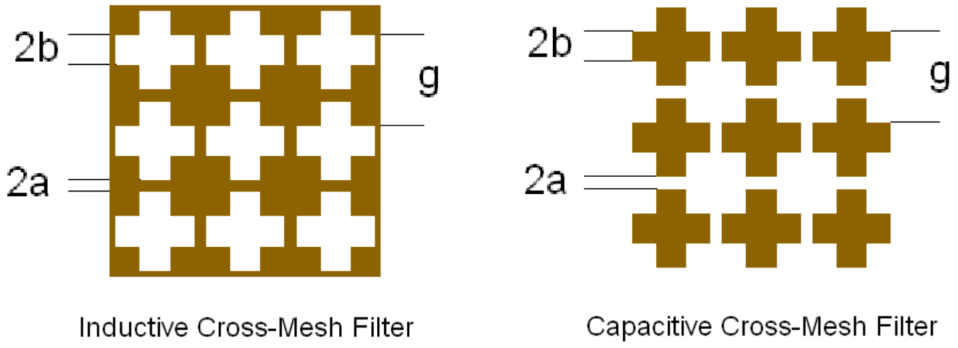
Cross Filters

Khodnevych Vitalii
Nicolas Delerue

1 Introduction



Inductive Cross-Mesh Filter

Capacitive Cross-Mesh Filter

Figure 1: Cross-mesh filters layout (from [1])

Cross filters layout is presented on figure 1. Characteristic sizes (g,a,b) determine filtering frequency and width of transmissivity function. On resonant wavelength

$$\lambda_r = 2.27g - 4a - 2b$$

transmission is zero for capacitive filter and one for inductive cross-mesh filter.

We will be interested only by inductive filters. Characteristic impedance in this case is:

$$Y(f) = \frac{1}{a_1 \pm i\frac{gA_1}{\lambda_r \Omega(f)}}$$

where $a_1 = 0.0001$ is metal loss parameter and $A_1 = 0.53$ is bandwidth parameter. Normalized frequency is

$$\Omega(f) = \frac{\lambda_r f}{c} - \frac{c}{\lambda_r f}$$

2 Matrix methods

To determine the transmittance and reflectance of cross filters matrix method is used. Systems with several filters will be reduced simple matrix multiplication. Filter scattering matrix depends only on the impedance of equivalent circuit (see fig. 2).

Incident wave a_1 and reflected wave b_1 on the l.h.s. relate to the wave amplitudes a_2, b_2 on r.h.s. by following matrix:

$$\begin{pmatrix} b_1 \\ a_1 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} b_2 \\ a_2 \end{pmatrix} \tag{1}$$

If $a_2 = 0$, transmission coefficient

$$t = \frac{b_2}{a_1} = \frac{1}{S_{22}}$$

. By applying Kirchoff's laws we could find S_{ij} elements:

$$S = \begin{pmatrix} -\frac{Y(f)}{2} + 1 & -\frac{Y(f)}{2} \\ \frac{Y(f)}{2} & \frac{Y(f)}{2} + 1 \end{pmatrix} \tag{2}$$

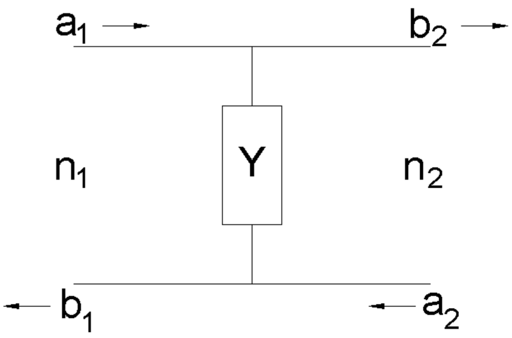


Figure 2: Equivalent circuit of cross filter (from [1])

As Y is frequency-dependent, so transmittance also will be frequency dependent.

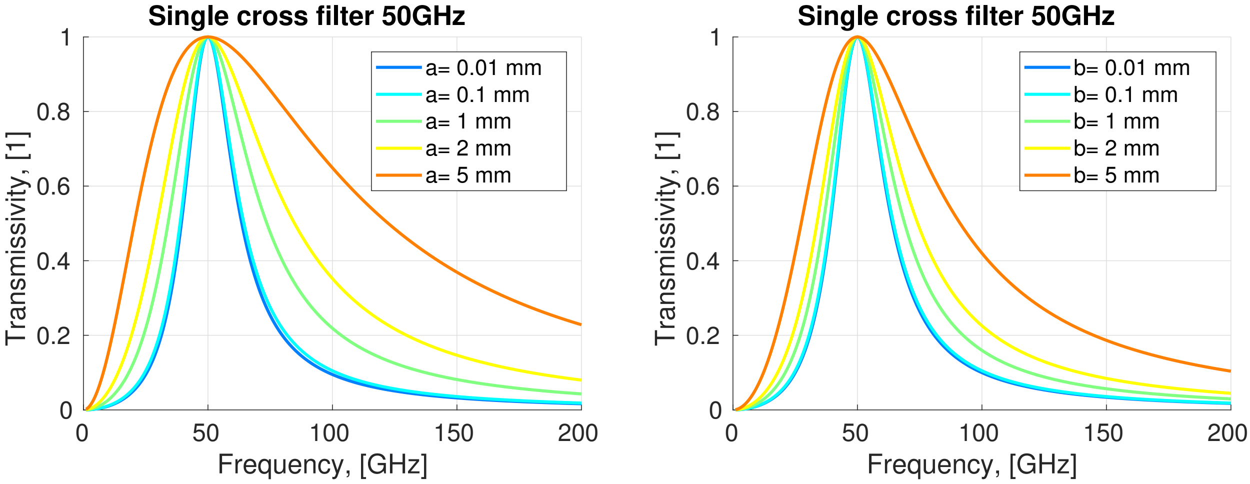
In case of series of elements, we multiply scattering matrix of each element $S_{total} = ...S_3S_2S_1$. Transport matrix (in medium between filters):

$$T = \begin{pmatrix} e^{-\frac{2\pi i f d}{c}} & 0 \\ 0 & e^{\frac{2\pi i f d}{c}} \end{pmatrix} \tag{3}$$

where d is distance between filters. In case of repetitive system of several filters $S_{total} = ...S_3T_{32}S_2T_{21}S_1$. Transmission coefficient in this case calculated from S_{total} matrix.

In calculation code also taken into account change of refractive index from medium to medium (in case filter is grown on substrate), angle of incident wave fall, loss of propagation between filters, polarization angle of incident wave.

Change of transmissivity as function of geometrical parameters a and b are presented on figures 3a and 3b.



(a) Change of transmissivity as function of geometrical parameter a

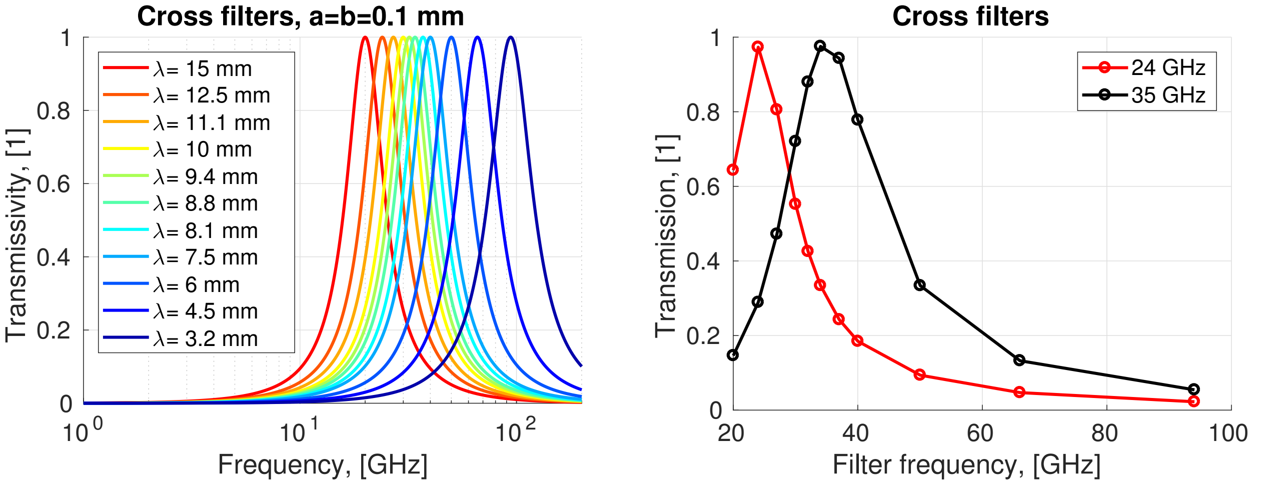
(b) Change of transmissivity as function of geometrical parameter b

Figure 3

3 Proposed experimental check of filter properties

ETALON project at LAL have two RF sources: on 24 and 35 GHz. They will be used to check properties of cross filters. For this purpose we design 16 filters. Experiment will consist from two parts:

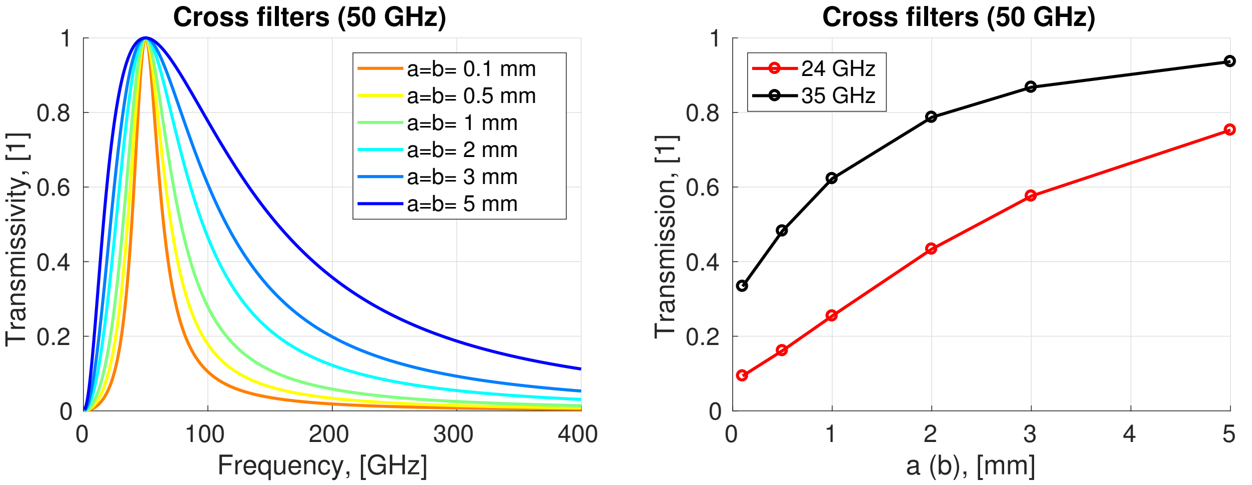
- Check transmissivity of filters as function of filter frequency (see 4a and 4b)
- Check transmissivity of filters as function of a,b parameters for fixed filter frequency (see 5a and 5b)



(a) Transmissivity functions for different test wavelengths

(b) Transmission for different filter with present sources

Figure 4: First part of experiment: check transmission



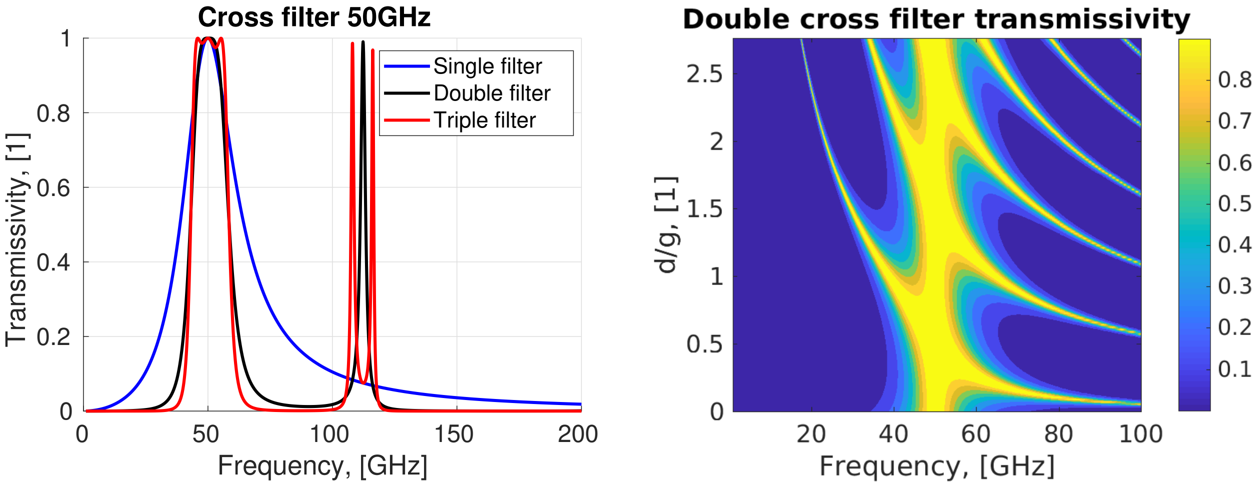
(a) Transmissivity functions for different filter parameters

(b) Transmission for different filter with present sources

Figure 5: Second part of experiment: check filters parameters dependence

4 Multiple filters

To decrease tails of transmissivity functions possible to use several cross filters. On figure 6a is shown transmissivity for single, double and triple filters. Double filters significantly decrease tails but produce narrow window in region far from resonant frequency. Triple filters didn't give big effect, but require more filters in realization. So we will use double cross filters.



(a) Transmissivity function of single, double and triple cross filters

(b) Change of transmissivity function for double cross filters as function of filters separation

Figure 6: Multiple filters

Question which arise is at which optimal distance put second filter with respect to first one. We find (see 6b) that when filters separation is equal half of g parameter, transmissivity function have best characteristics.

5 CLIO filters

Using matrix method for calculation cross filters we could calculate double structure filters for Smith-Purcell experiment at CLIO. Filter size is 3x3 cm. Filter separation is equal half of g parameter. Frequency of filters are calculated for current setup (7^o detector separation, 6 mm grating). Transmissivity functions are presented on figure 7.

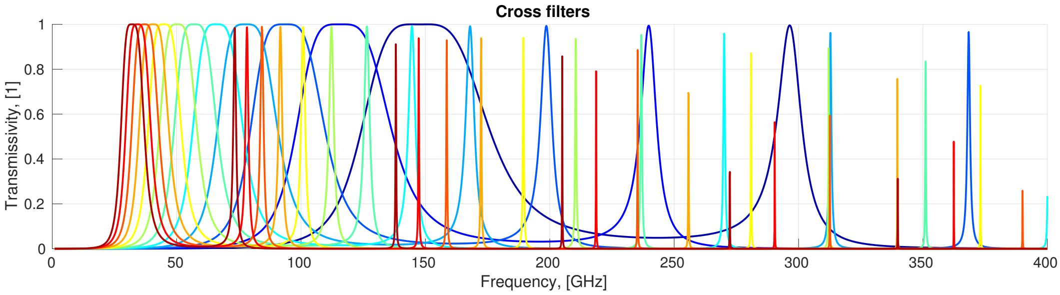


Figure 7: Transmissivity functions of cross filters for Smith-Purcell experiment at CLIO.

Usage of filters give big advantages for experiment, as it significantly cut background components of the signal. But it's also require precise alignment. Shift of detector mount with respect to grating base cause decrease of signal (as detector then aligned on other frequency and filters stay at designed). As wavelengths of Smith-Purcell radiation are distributed by cosine law, shift of setup in one or other side is not same (see fig. 8). To estimate this effect we calculate the ratio of transmitted fraction of radiation with and without filter. Integration borders are determined by acceptance of detection system (25mm OAP + pyrodetector). Result of this calculation presented on figure 8. We conclude that even with error of detector system mount, we still should detect more than 90% of emitted radiation.

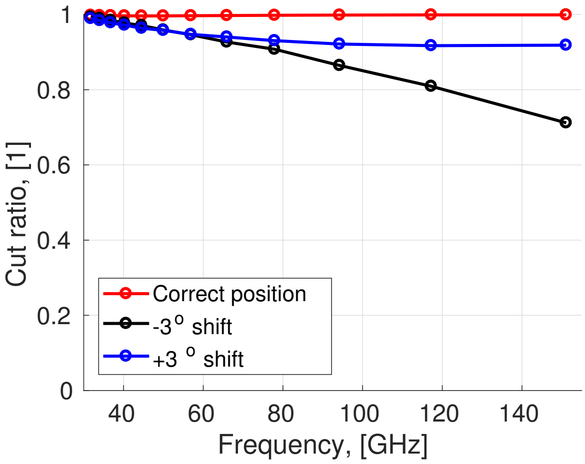


Figure 8: Impact of detectors shift on fraction of measured radiation with double cross filters

References

[1] Benjamin Hooberman *Everything You Ever Wanted to Know About Frequency-Selective Surface Filters but Were Afraid to Ask*, May 2005