Study of Phase Reconstruction Techniques applied to Smith-Purcell Radiation Measurements

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Coherent Radiation as a bunch profile monitor

- Coherent emission encodes the Fourier transform of the bunch longitudinal profile:

$$I(\lambda) = I_1(\lambda)(N + |F(\lambda)|^2 N^2)$$

☞ Can be used as a diagnostic to measure the longitudinal profile of an electron bunch.
☞ Example: Coherent Smith-Purcell radiation produced when a bunch of charged particles passes above a grating.

Simulations

- Simulate multi-gaussian profiles

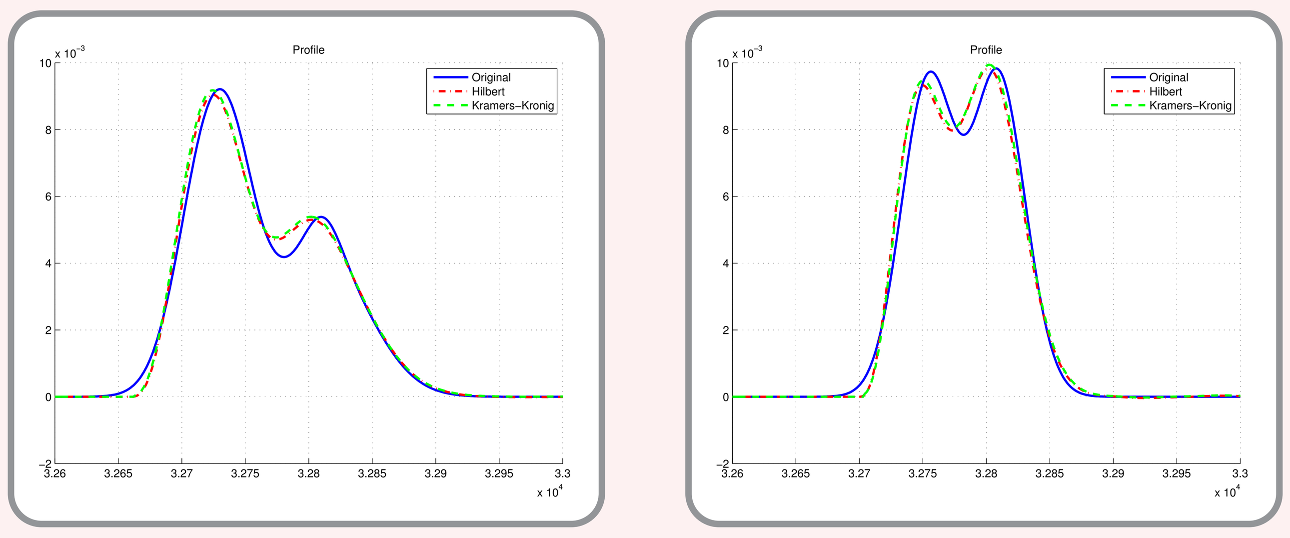
$$\mathcal{G}(x) = \sum_{i=1}^{5} A_i \exp \frac{-(\frac{x}{mX} - \mu_i)^2}{2\sigma_i^2}$$

☞ A_i, μ_i and σ_i are random numbers with $x \in [1; mX]$, $A_i \in [0; 1]$, $\mu_i \in 0.5 + [-7.5; +7.5] \times 10^{-4}/mX$ and $\sigma_i \in [3; 9] \times 10^{-9}$; $mX = 65536$

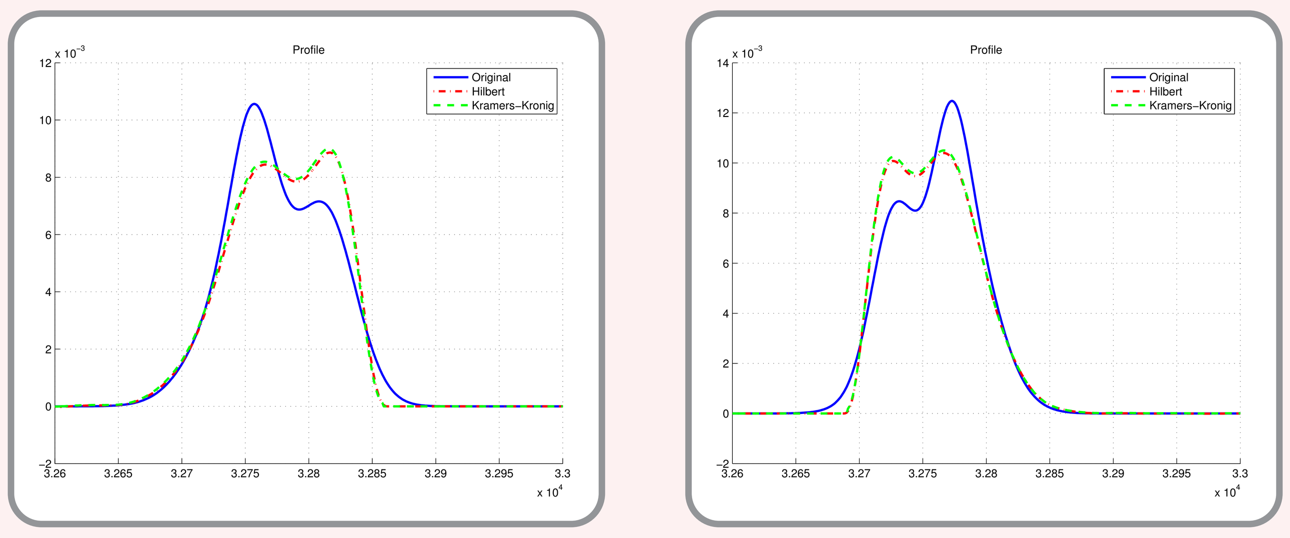
- $\mathcal{F} = \|\mathrm{FFT}(\mathcal{G})\|$
- Sampling at some limited frequencies (33) for example: $F_i = \mathcal{F}(\omega_i)$
- Several sampling models investigated (linear, log, E-203 like,...).

Reconstructed profiles

Good reconstructions



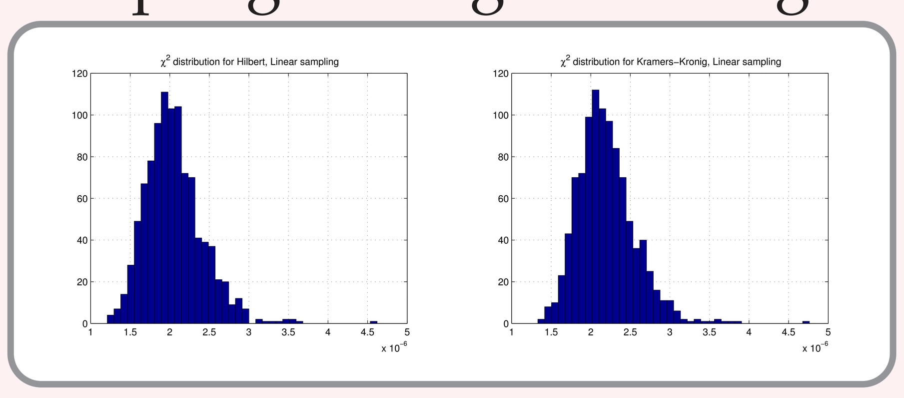
Bad reconstructions



Lorenzian profiles

- Instead of multi-guassian, use Lorenzian profiles.

Effect of sampling and sigma scaling



Phase recovery methods

- $|F(\lambda)|^2$ ☞ Phase information lost
- However in an analytical function ($\varepsilon(\omega)$) there is a relation between phase and amplitude:
  ☞ Kramers-Kronig relations
- Rewrite as $log(\varepsilon(\omega)) = log(\rho(\omega)) + i\Theta(\omega)$ with $\rho(\omega)$

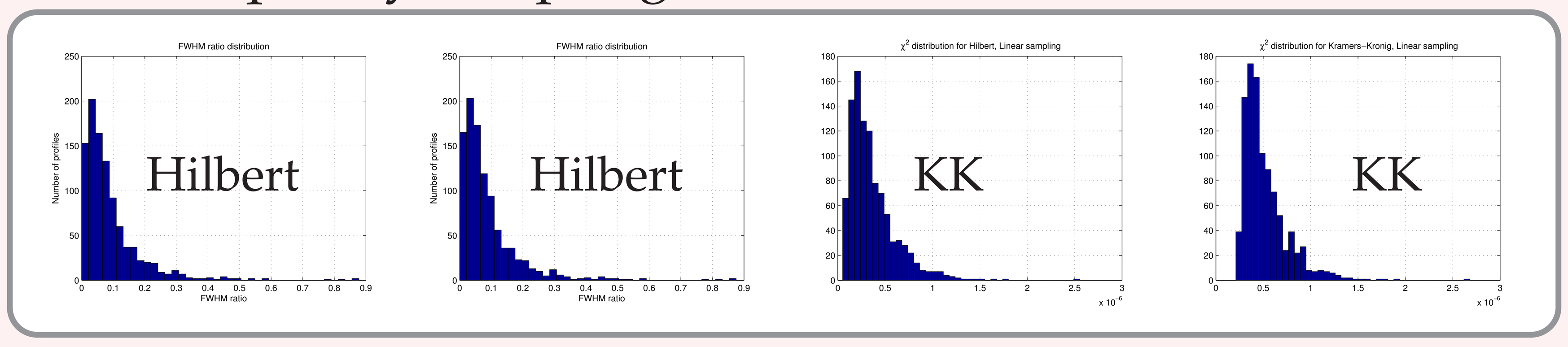
  ☞ Then: $\Theta(\omega_0) = \frac{2\omega_0}{\pi} P \int_0^{+\infty} \frac{ln(\rho(\omega))}{\omega_0^2 - \omega^2} d\omega$

- In some cases this can be done using the Hilbert transform
- Hilbert transform directly implemented in Matlab (very fast)
- Wrote a Matlab implementation for the Kramers-Kronig relations.

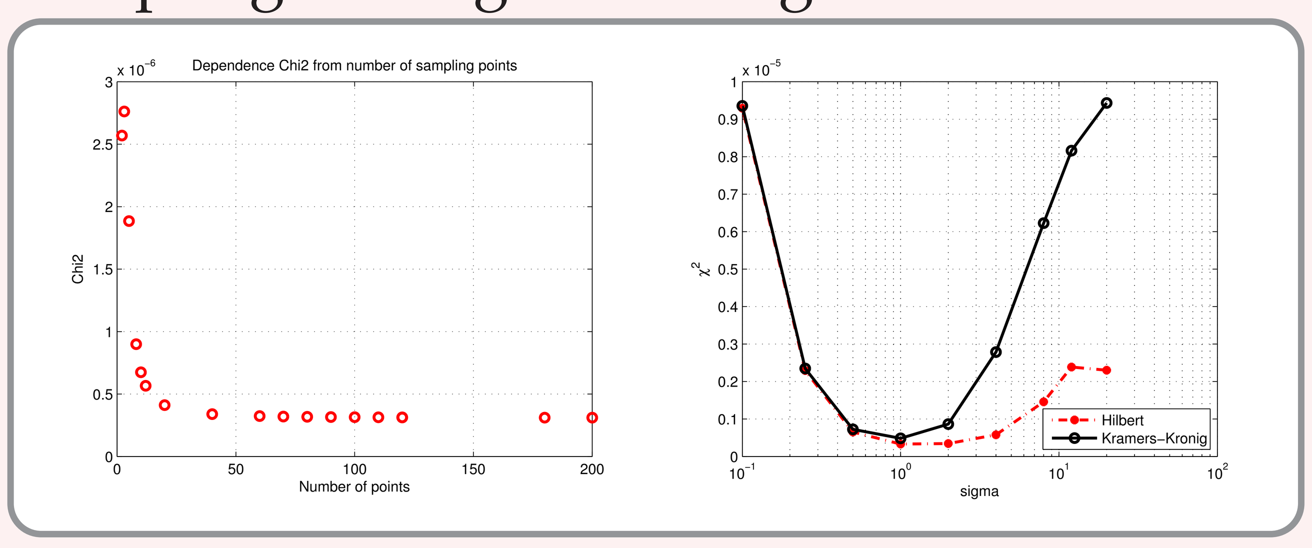
Statistics

$$\Delta_{FWXM} = \mathrm{Max}_{X \in \mathrm{rset}} \left| \frac{FWXM_{\mathrm{orig}} - FWXM_{\mathrm{reco}}}{FWXM_{\mathrm{orig}}} \right|$$

Linear frequency sampling



Effect of sampling and sigma scaling



Discussion

- Both methods give good reconstruction accuracy.
- Hilbert directly implemented in Matlab => faster
- More detailed study in progress to find the limits of validity of the methods.

References

- **G. Doucas et al.** Reconstruction of the time profile of 20.35 GeV, subpicosecond long electron bunches by means of coherent Smith-Purcell radiation. *Phys. Rev. ST Accel. Beams*, 17:052802, May 2014.
- **O. Grimm and P. Schmüser** Principles of Longitudinal Beam Diagnostics with Coherent Radiation, TESLA FEL 2006-03