Reflection at the Boundary of an Absorbing Medium

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Let a plane wave be incident on the boundary of a medium having a complex index of refraction

$$\mathcal{N} = n + i\kappa$$

And let's denote the angle of incidence by θ and the angle of refraction by ϕ. Now we express the law of refraction in terms of the complex index of refraction in a purely formal way as

$$\mathcal{N} = \frac{\sin\theta}{\sin\phi}$$

Here the angle ϕ is a complex number. It turns out that ϕ is very useful in simplifying the equations related to reflection and refraction by an absorbing medium. From the above definition of ϕ, we have

$$\cos\phi = \sqrt{1 - \frac{\sin^2\theta}{\mathcal{N}^2}}$$

If we derive the coefficient of reflection using the boundary conditions giving the continuity of the tangential components of the electric and magnetic fields for TE polarization[1] and TM polarization[2].

$$r_{TE} = \frac{\cos\theta - \mathcal{N}\cos\phi}{\cos\theta + \mathcal{N}\cos\phi} \qquad (TE\ polarization)$$

$$r_{TM} = \frac{-\mathcal{N}\cos\theta + \cos\phi}{\mathcal{N}\cos\theta + \cos\phi} \qquad (TM\ polarization)$$

Now we can get the reflectance for TE and TM polarization.

$$R_{TE} = |r_{TE}|^2 \qquad (TE\ polarization)$$

[1] Transverse Electric, the electric field vector of incident wave is perpendicular to the plane of incidence

[2] Transverse Magnetic, the magnetic field vector of the incident wave is perpendicular to the plane of incidence

$$R_{TM} = |r_{TM}|^2 \qquad (TM\ polarization)$$

Above two cases are only for purely TE and TM polarized optical photon [1].

In GEANT4, if we don't set polarization for optical photon, the optical photon has random polarization. In other words, an optical photon can have both a TE polarization component and a TM polarization component. So, by the definition of the reflectance[3], we can derive the reflectance for a randomly polarized optical photon. Let $\vec{E_0}$ be the electric field vector of incident wave and $\vec{E'}$ be the electric field vector of reflected wave for a randomly polarized optical photon. Also, the electric field vector of incident wave is perpendicular and parallel to the plane of incidence for TE and TM polarization, respectively. The electric field vector is $\vec{E_\perp}$ and $\vec{E_\parallel}$ for TE and TM polarization respectively. If we write $\vec{E_0}$ and $\vec{E'}$ in terms of $\vec{E_\perp}$ and $\vec{E_\parallel}$

$$\vec{E_0} = \vec{E_\perp} + \vec{E_\parallel}$$

$$\vec{E'} = |r_{TE}|\vec{E_\perp} + |r_{TM}|\vec{E_\parallel},$$

we can get the reflectance for a randomly polarized optical photon at the boundary of an absorbing medium.

$$R = |r|^2 = \frac{|\vec{E'}|^2}{|\vec{E_0}|^2} = \frac{|r_{TE}|^2 E_\perp^2 + |r_{TM}|^2 E_\parallel^2}{E_\perp^2 + E_\parallel^2}$$

References

[1] Grant R. Fowles, *Introduction to Modern Optics, Second Edition*, Holt, Rinehart and Winston, Inc., 1975.

[3] The *reflectance* is defined as the fraction of the incident light energy that is reflected