Proton decay and flavour structure

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OUTLINE

1. Introduction: proton decay via

$\longrightarrow$ dimension 6 operators

$\longrightarrow$ supersymmetry and dimension 5 operators

2. 4-dimensional SU(5) GUT

- effects of the flavour structure: excluded or not ?

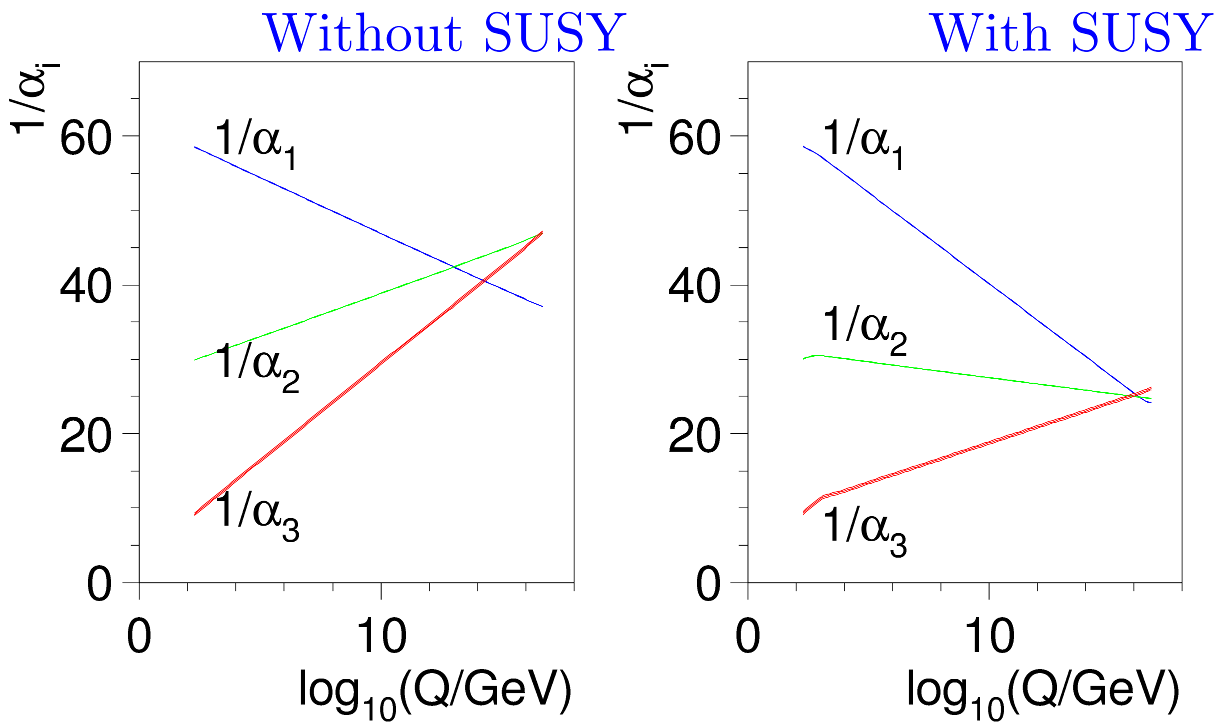
3. A higher dimensional case: $SO(10)$ orbifold GUT

4. Conclusions and Outlook

The Standard Model describes successfully the present data, but leaves many questions open, like the fractional charges, the SM representations, the absence of anomalies, neutrino masses, etc..

Can we do better ? YES !

Key observation: the SM gauge couplings meet at a high scale M_G (with supersymmetry...) !



[W. de Boer & C. Sander '04]

$\Rightarrow$ Simple Unified Gauge Group at M_G !

BUT quarks and leptons are in the same multiplets

$\Downarrow$

The proton is not stable !

Simplest GUT: $SU(5)$

unifies quarks and leptons in the same multiplets:

$$\bar{\mathbf{5}} \equiv (\mathbf{d^c}, \ell) \qquad \mathbf{10} \equiv (\mathbf{u^c}, \mathbf{q}, \mathbf{e^c})$$

$$\rightarrow m^d \simeq m^e \qquad \text{bottom-}\tau \text{ unification}$$

and also needs many Higgs's multiplets, two containing the SM Higgs doublet and its conjugate

$$\mathbf{5} \equiv (\mathbf{h_c}, \mathbf{h}) \qquad \bar{\mathbf{5}} \equiv (\bar{\mathbf{h}}_\mathbf{c}, \bar{\mathbf{h}})$$

Doublet-triplet splitting problem: $h, \bar{h}$ belong to the same multiplets as $h_c, \bar{h}_c$, but must have a mass at the EW, not the GUT scale.

At least one Higgs large representation, e.g. **24** is needed to break $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$.

There are also 12 additional gauge bosons, $\mathcal{X} = (\bar{\mathbf{3}}, \mathbf{2}, \mathbf{5}/\mathbf{6})$, such that in 5×5 notation

$$\mathbf{24} \equiv \begin{pmatrix} G_{ij} - \sqrt{\frac{2}{15}}\, B & \bar{\mathcal{X}} \\ \mathcal{X} & W_{ab} + \sqrt{\frac{3}{10}}\, B \end{pmatrix}$$

where G denotes gluons, W, B are the EW bosons.

Note that $\mathcal{X}$ can turn quarks into leptons and also the Higgs colored triplets $h_c, \bar{h}_c$ Yukawa couplings mix quarks and leptons ! $\Rightarrow$ PROTON DECAY !

Long history of predictions for proton decay in GUT models starting from the '70's...

e.g. PROTON DECAY in $SU(5)$: integrating away the heavy states generates 4 fermion (dimension 6) operators, which allow for baryon decay into lepton and meson, e.g. $p \to \pi^0 \ell^+, \pi^+ \bar{\nu}, K^0 \ell^+, K^+ \bar{\nu}$

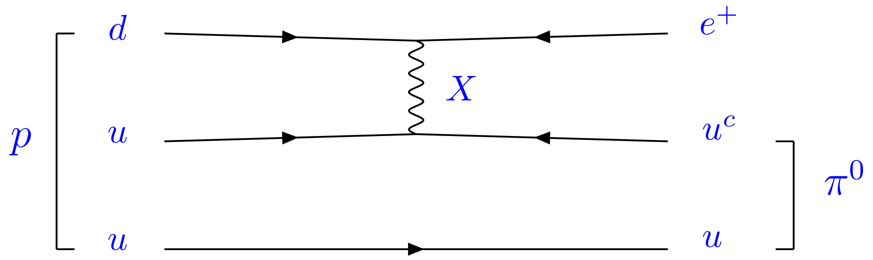
$$\frac{1}{M_{h_c}^2} \epsilon_{\alpha\beta\gamma} \left(Y_{qq} Y_{q\ell} \, q_\alpha q_\beta q_\gamma \ell + Y_{ue} Y_{ud} \, u_\alpha^c e^c u_\beta^c d_\gamma^c \right)$$

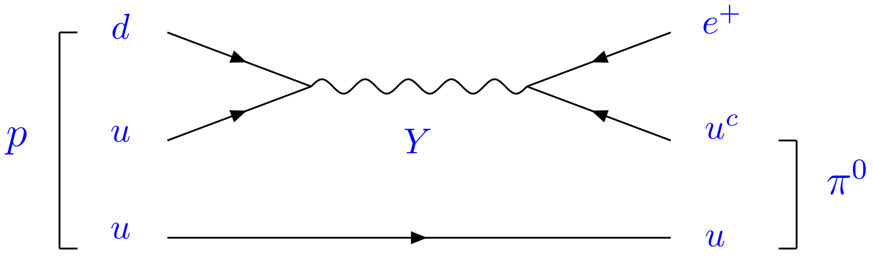
Higgs mediated

$$\frac{g_5^2}{M_{\mathcal{X}}^2} \epsilon_{\alpha\beta\gamma} \left(\bar{d}_{\alpha,k}^c \bar{u}_{\beta,j}^c q_{\gamma,j} \ell_k - \bar{e}_k^c \bar{u}_{\alpha,j}^c q_{\beta,j} q_{\gamma,k} \right)$$

gauge mediated

The dominant channel is $p \to \pi^0 e^+$: [Ellis et al, '79 ...]





$$\tau_{exp} \geq 6.9 \times 10^{33} \text{ years} \Rightarrow M_{\mathcal{X}} \geq 7.3 \times 10^{15} \text{GeV}$$

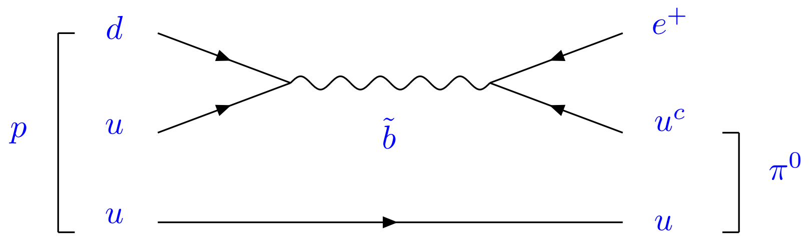
NOTE: the Higgs mediated contribution strongly depends on the Yukawas ! The gauge part not, but still one has to rotate current to mass eigenstates...

In SUSY models more dangerous terms arise:

• there are renormalizable couplings that violate explicitly B and L number:

$$W = \lambda LLE^c + \lambda' LQD^c + \lambda'' U^c D^c D^c + \mu_i L_i H_2$$

$\Rightarrow$ Dimension 4 proton decay operators

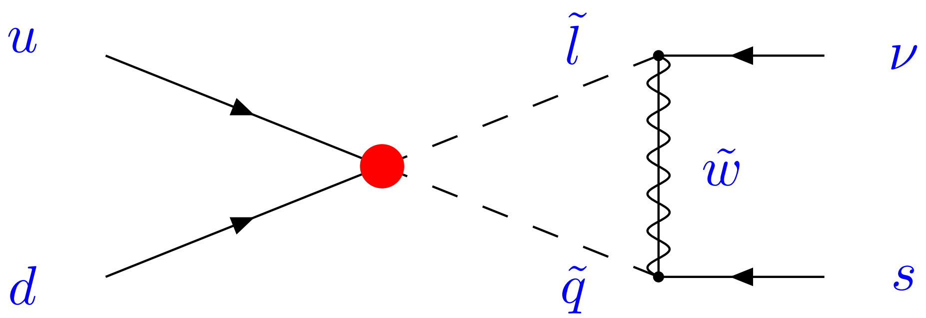


$\propto \frac{1}{m_{\tilde{b}}^2}$

To avoid fast proton decay, impose a discrete symmetry called R-parity, which forbids these terms

$\Rightarrow$ No dimension 4 proton decay (and LSP is stable)!

• there is a contribution to the 4-fermion operators from a superparticle loop, called "dimension 5":



$\Rightarrow$ enhanced by $\frac{M_{h_c}}{m_{SUSY}}$ compared to the non-SUSY contribution and therefore most dangerous !

Dimension five operators

$$\frac{1}{M_{H_c}}\left[\underbrace{\frac{1}{2}Y_{qq}^{ij}Y_{ql}^{km}\left(Q_iQ_jQ_kL_m\right)}_{LLLL\text{ operator}}+\underbrace{Y_{ue}^{ij}Y_{ud}^{km}\left(u_i^{\mathrm{C}}e_j^{\mathrm{C}}u_k^{\mathrm{C}}d_m^{\mathrm{C}}\right)}_{RRRR\text{ operator}}\right]$$

$$\epsilon_{\alpha\beta\gamma} \qquad\qquad \epsilon_{\alpha\beta\gamma}$$

$$\left(u_i^\alpha d_j^\beta - d_i^\alpha u_j^\beta\right)\left(u_k^\gamma l_m - d_k^\gamma \nu_m\right) \qquad \left(u_i^{\mathrm{C}\alpha}e_j^{\mathrm{C}}\right)\left(u_k^{\mathrm{C}\beta}d_m^{\mathrm{C}\gamma}\right)$$

Colour antisymmetry requires decay to be flavour non-diagonal $\Rightarrow$ dominant decay channel: $p \to K^+\bar{\nu}$.

Calculation of the decay width of $p \to K^+\bar{\nu}$

$$\Gamma = \frac{(m_p^2-m_K^2)^2}{32\pi m_p^3 f_\pi^2}\sum_{i=e,\mu,\tau}\left|\mathcal{K}^{usd\nu}C_{\mathrm{LL/RL}}^{usd\nu}+\mathcal{K}^{uds\nu}C_{\mathrm{LL/RL}}^{uds\nu}+\mathcal{K}^{dsu\nu}C_{\mathrm{LL/RL}}^{dsu\nu}\right|^2$$

where $$C_{\mathrm{LL}} = \beta\; A_l\; f_{\mathrm{L}}(M;m_1,m_2)\,\frac{1}{M_{H_{\mathrm{C}}}}\left(Y_{qq}Y_{ql}\right)\; A_s$$

using chiral Langrangian method

- Wilson coefficients $C_{5L} = Y_{qq}Y_{ql}$ and $C_{5R} = Y_{ue}Y_{ud}$ evaluated at M_G, evolved down to $M_{\mathrm{SUSY}} \to \left(C_{5L/5R}\; A_s\right)$
- Triangle diagram factor $f_{\mathrm{L/R}}(M;m_1,m_2)$ in the decoupling scenario $m \gg M:\; f_{\mathrm{L/R}} \to M/m^2$
- hadron matrix elements:

$$\alpha u_{\mathrm{L}}(\mathbf{k}) = \epsilon_{\alpha\beta\gamma}\,\langle 0|(d^\alpha u_{\mathrm{R}}^\beta)u_{\mathrm{L}}^\gamma|\,p\,(\mathbf{k})\rangle$$
$$\beta u_{\mathrm{L}}(\mathbf{k}) = \epsilon_{\alpha\beta\gamma}\,\langle 0|(d^\alpha u_{\mathrm{L}}^\beta)u_{\mathrm{L}}^\gamma|\,p\,(\mathbf{k})\rangle$$

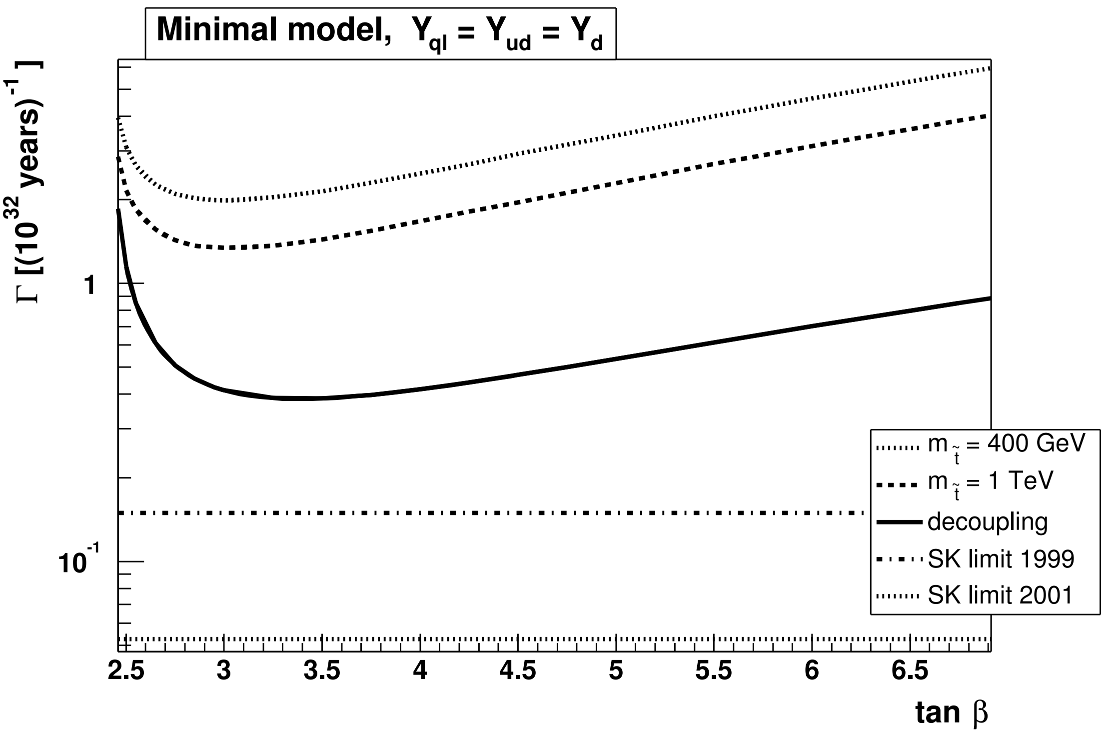
Proton decay in minimal SUSY $SU(5)$

[D. Emmanuel-Costa & S. Wiesenfeldt '03]

Minimal $SU(5)$ predicts $Y_d = Y_e = Y_{ql} = Y_{ud} = Y_2$ at M_G. But : RGE evolution to $M_G \rightarrow Y_d \neq Y_e$!

Usual procedure : Assume

$Y_{qq} = Y_{ue} = Y_u,\ Y_{ql} = Y_{ud} = Y_d$



experimental limit by SuperKamiokande experiment :

$\tau = 6.7 \times 10^{32}$ years [Hayato et al (SuperKamiokande) 1999]

$\tau = 1.9 \times 10^{33}$ years [Ganezer (SuperKamiokande) 2001]

Decay rate is always above the experimental limit.

$\rightarrow$ lead to **exclusion of minimal SUSY SU(5) ?**

[Goto, Nihei '99, Murayama, Pierce '02]

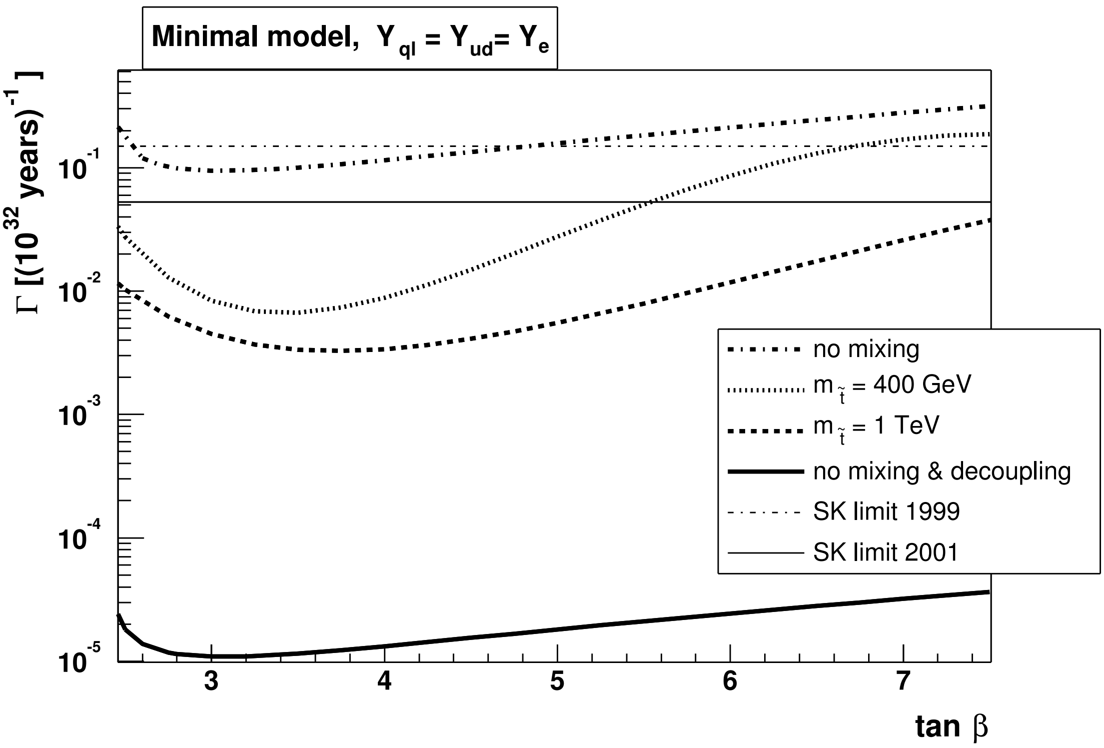
Minimal model : Flavour dependence

[D. Emmanuel-Costa & S. Wiesenfeldt '03]

Choose $Y_{qq} = Y_{ue} = Y_u$, $Y_{ql} = Y_{ud} = Y_e$

$\longrightarrow$ Mixing matrix $\mathcal{M} = U_u^\dagger U_e$ instead of $V_{\text{CKM}} = U_u^\dagger U_d$

Study first $\mathcal{M} = \mathbb{1}$; then take $\mathcal{M}$ arbitrary $\rightarrow$ minimize the decay rate.



A sufficiently low decay rate can be found !

$\longrightarrow$ illustrates the dependence on flavour mixing and

$\Rightarrow$ **uncertainty due to failure of Yukawa unification**

Motivation for $SO(10)$ in 6D:

Same as for 4D:

* Unification not only of the gauge group: SM generation + RH neutrino entirely accommodated into the **16** spinor representation
* Anomaly-free group in 4D
* Gauged $U(1)_{B-L}$ (and R-symmetry in SUSY): $B-L$ breaking gives rise automatically to a Majorana mass for the RH neutrino and the seesaw mechanism

BUT also

* Simpler breaking by orbifold/boundary conditions making the breaking pattern unique without large Higgs representations
* No dimension 5 proton decay ! The Higgs triplets in H_u and H_d do not have a common mass term, the Kaluza-Klein mass comes from the 6D kinetic term and mixes them with their N=2 SUSY partners.

$SO(10)$ GUT in 6D

[Asaka, Buchmüller & Covi '01]
[Hall, Nomura, Okul & Smith '01]

6D N=1 supersymmetric $SO(10)$ gauge theory compactified on a torus with three Z_2 parities, acting non-trivially on the $SO(10)$ indices. All parities break $N=1$ $6D$ SUSY to $N=1$ $4D$ SUSY and

$$P_{GG} : SO(10) \rightarrow SU(5) \times U(1)$$

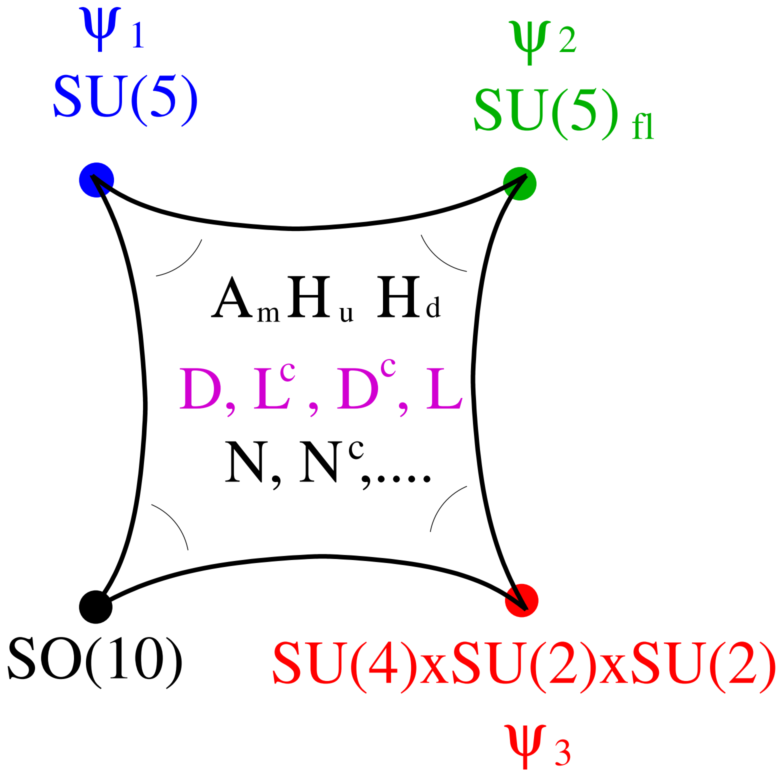
$$P_{PS} : SO(10) \rightarrow SU(4) \times SU(2) \times SU(2)$$

The zero modes can belong only to the intersection of the two symmetric subgroups: $\Rightarrow SM \times U(1)'$

Note: a forth non-independent parity is given by

$$P_{fl} = P \cdot P_{GG} \cdot P_{PS} \; : SO(10) \rightarrow SU(5)_{fl} \times U(1)$$

$\rightarrow$ ALL breaking of $SO(10)$ are present !



Hierarchical fermion masses

Basic idea: connect the hierarchy to the location of the SM generations in the extra space-coordinates, placing ψ_1 at GG-brane, ψ_2 at $SU(5)_{fl}$-brane and ψ_3 at PS-brane

$\rightarrow$ "diagonal" mass matrices in flavour space

$\rightarrow$ the Yukawa couplings satisfy only the GUT symmetry of the local point, not full $SO(10)$

$\rightarrow$ the mixing arises from the coupling to bulk fields zero modes with the same quantum numbers (only for down quarks and leptons !) Note: bulk fields are split multiplets, so the mixing does not respect any GUT !

$\rightarrow$ lopsided structure for m^d, m^e and m^D !

$$m^i = \begin{pmatrix} \mu_1 & 0 & 0 & \tilde{\mu}_1 \\ 0 & \mu_2 & 0 & \tilde{\mu}_2 \\ 0 & 0 & \mu_3 & \tilde{\mu}_3 \\ \tilde{M}_1 & \tilde{M}_2 & \tilde{M}_3 & \tilde{M}_4 \end{pmatrix},$$

where $\mu_i, \tilde{\mu}_i = \mathcal{O}(v_{EW})$ and $\tilde{M}_i = \mathcal{O}(M_{GUT})$

[Sato & Yanagida '98]

A LARGE rotation on the RIGHT is needed to go to the mass eigenstates.

Proton decay in 6D

Dimension 5 operators do not appear since the triplets in H_1 and H_2 have no common mass term: the Kaluza-Klein mass comes from the 6D kinetic term and mixes H_i with its $N=2$ SUSY partner $\bar{H}_i$.
$\longrightarrow$ dimension 6 operators are dominant !

ψ_1 at Z_{GG}: Proton decay via $SU(5)$
$\mathcal{X} = (X, Y)$-boson exchange!

What is the difference with the $SU(5)$ $4D$ case ?

[Hall, Nomura '02; Hebecker, March-Russell '02]

[Buchmüller et al. '04]

- there are not only one $\mathcal{X}$, but a Kaluza-Klein tower with masses $M^2_{\mathcal{X}}(n,m) = \frac{(2n+1)^2}{R_5^2} + \frac{(2m)^2}{R_6^2}$
- $\mathcal{X}$ bosons do not couple to ψ_2, ψ_3 because they vanish on $Z_{\mathrm{PS}}, Z_{\mathrm{GG}'}$

non-universal coupling of $\mathcal{X}$!

- there is a subleading contribution from $\mathcal{X}'$ if brane localized derivative operators are present

Sum over the Kaluza-Klein tower: logarithmic divergent ! The model is non-renormalizable (6D !) and valid only up to the cut-off M_*, so

$$\frac{1}{M_{\mathcal{X}}^2} \Rightarrow \frac{\pi}{4} R_5 R_6 \left(\log\left(R_5 M_*\right) + \dots\right)$$

slight enhancement compared to $4D$

Different flavour structure:

$$\frac{g_5^2}{(M_{\mathcal{X}}^{\mathrm{eff}})^2} \epsilon_{\alpha\beta\gamma} \left(\bar{d}^c_{\alpha,1} \bar{u}^c_{\beta,1} q_{\gamma,1} \ell_1 - \bar{e}^c_1 \bar{u}^c_{\alpha,1} q_{\beta,1} q_{\gamma,1} \right) + h.c.$$

$$= \frac{g_5^2}{(M_{\mathcal{X}}^{\mathrm{eff}})^2} \epsilon_{\alpha\beta\gamma} \left(d'^{\,c}_{\alpha,l} \left(U_R^{d\top}\right)_{l1} u^c_{\beta,1} q'_{\gamma,m} \left(U_L^q\right)_{1m} \left(U_L^\ell\right)_{1j} \ell'_j \right.$$
$$\left. - e'^{\,c}_k \left(U_R^{e\top}\right)_{k1} u^c_{\alpha,1} q'_{\beta,m} \left(U_L^q\right)_{1m} \left(U_L^q\right)_{1l} q'_{\gamma,l} \right) + h.c.$$

$\rightarrow$ very different branching ratios for similar (phenomenologically acceptable) mixing matrices !

decay channel	Branching Ratios [%]		
	6D SO(10)		$SU(5) \times U(1)_F$
	case I	case II	models A & B
$e^+\pi^0$	75	71	54
$\mu^+\pi^0$	4	5	< 1
$\bar{\nu}\pi^+$	19	23	27
e^+K^0	1	1	< 1
μ^+K^0	< 1	< 1	18
$\bar{\nu}K^+$	< 1	< 1	< 1
$e^+\eta$	< 1	< 1	< 1
$\mu^+\eta$	< 1	< 1	< 1

So for the proton decay in this 6D model we can conclude:

- the dominant decay mode is $p \to \pi^0 e^+$ with

$$\Gamma \simeq K^{\pi^0}_{\rm had} \frac{\pi^2}{16\, M_c^4} \left(\ln\left(\frac{M_*}{M_c}\right) + 2.3 \right)^2 \left[4V_{ud}^4 + \frac{\tilde{M}_2^{d\,2}}{\tilde{M}_1^{d\,2} + \tilde{M}_2^{d\,2}} \frac{\tilde{M}_2^{e\,2}}{\tilde{M}_1^{e\,2} + \tilde{M}_2^{e\,2}} \right]$$

  where $M_c = R_{5,6}^{-1}$ and

$$K^{\pi^0}_{\rm had} = \frac{(m_p^2 - m_{\pi^0}^2)^2}{32\pi m_p^3 f_\pi^2}\, \alpha^2\, A^2 \left(\frac{1 + D + F}{\sqrt{2}} \right)^2$$

  contains the hadronic matrix element, kinematical factors and the running of the dimension 6 operator from the GUT to 1 GeV.

  For $M_* = 10^{17}\,\mathrm{GeV}$ and $\tilde{M}^{d,e}_{1,2} = \mathcal{O}(1)$, the experimental limit $\tau \geq 6.9 \times 10^{33}$ yields $M_c \geq 9.6 \times 10^{15}\,\mathrm{GeV}$, very close to the 4D GUT scale. Is proton decay around the corner ?

- the localization of the states in the extra dimensional space gives characteristic signatures in the branching ratios and in particular suppresses strongly the mode $K^0 \mu^+$.

Conclusions and Outlook

- Proton decay is (still...) the smoking gun for any GUT theory
- It is possible to suppress the decay rate, but in many cases proton decay is just around the corner: keep looking !
- Once proton decay is observed in different channels, its branching ratios could give us additional information on the flavour structure (especially the RH sector) and distinguish between models for fermion masses.