PHENOMENOLOGICAL AND COSMOLOGICAL ASPECTS OF A MINIMAL GUT SCENARIO

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Several phenomenological and cosmological aspects of a minimal extension of the Georgi-Glashow model, where the Higgs sector is composed by $\mathbf{5}_H$, $\mathbf{15}_H$, and $\mathbf{24}_H$, are studied. It is shown that the constraints coming from the unification of gauge interactions up to two-loop level predict light scalar leptoquarks. In this GUT scenario, the upper bound on the total proton decay lifetime is $\tau_p \leq 1.4 \times 10^{36}$ years. The possibility to explain the matter-antimatter asymmetry in the universe through the decays of $SU(2)_L$ scalar triplets is also studied. We find that a successful triplet seesaw leptogenesis implies an upper bound on the scalar leptoquark mass, $M_{\Phi_b} \lesssim 10^{6-7}$ GeV. We conclude that this GUT scenario can be tested at the next generation of proton decay experiments and future colliders through the production of scalar leptoquarks.

I. INTRODUCTION

Grand unified theories (GUTs) based on the $SO(10)$ gauge symmetry [1, 2] are usually considered as the most appealing candidates for the unification of electroweak and strong interactions. They offer a number of advantages over $SU(5)$ theories [3]: (i) They provide a natural explanation of the smallness of neutrino masses through the seesaw mechanism [4]; (ii) they accommodate all fermions of one generation into one representation; (iii) they represent, in their minimal form, the most promising theory of fermion masses (For realistic grand unified theories based on the $SO(10)$ gauge symmetry see e.g. Refs. [5, 6, 7].).

Nevertheless, it is well known that the only promising way to test the idea of grand unification is through nucleon decay. Therefore, it is very important to investigate the simplest realistic grand unified theory where proton decay can be well predicted. This crucial issue brings us back to

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non-supersymmetric GUT scenarios, since the unification scale is rather low ($M_{GUT} \approx 10^{14}$ GeV). In particular, we focus on the simplest realistic $SU(5)$ theories, where the unification scale can be accurately predicted. Even though $SU(5)$ possesses uncorrelated regions in the Yukawa sector, the simplicity of the Higgs sector in the non-supersymmetric case offers a hope that the theory can be verified in near future.

In a recent work [8], some of us argued that the simplest realistic extension of the Georgi-Glashow (GG) model is the one containing the $\mathbf{5}_H$, $\mathbf{15}_H$ and $\mathbf{24}_H$ representations in the Higgs sector. The purpose of this paper is to demonstrate that the next generation of collider and proton decay experiments will refute or verify this minimal $SU(5)$ scenario. A first attempt was made in Ref. [8] in this direction. Here we offer the full two-loop treatment of the gauge coupling unification and discuss the constraints on the Higgs sector. To show the testability of the model, we include all the presently available experimental limits in our discussion. The upper bound on the total proton decay lifetime in our GUT model is corrected, and we investigate the possibility to explain the baryon asymmetry observed in the universe through the decays of $SU(2)_L$ scalar triplets living in $\mathbf{15}_H$, showing how important the constraints coming from leptogenesis turn out to be. The latter, when combined with the unification constraints, lead us to conclude that the present GUT scenario could be tested at the next generation of collider experiments through the production of light leptoquarks.

II. A MINIMAL $SU(5)$ SCENARIO

Ever since its inception in 1974, the $SU(5)$ model of Georgi and Glashow [3] has been considered as the minimal grand unified theory. It offers partial matter unification of one standard model (SM) family a ($a = 1, 2, 3$) in the anti-fundamental $\overline{\mathbf{5}}_a$ and antisymmetric $\mathbf{10}_a$ representations. The GUT symmetry is broken down to the standard model by the vacuum expectation value (VEV) of the Higgs field in the $\mathbf{24}_H$, while the SM Higgs resides in the $\mathbf{5}_H$. The beauty of the model is undeniable, but the model itself is not realistic. Indeed, there are several problems some of which are correlated:

- Gauge coupling unification.

  The most dramatic problem of the naive $SU(5)$ is the lack of unification. Namely, using the whole freedom of the model, one can compute the maximum value of the ratio between the parameters $B_{23} = b_2 - b_3$ and $B_{12} = b_1 - b_2$, where b_i ($i = 1, 2, 3$) are the beta functions of the particle content of the theory for $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$, respectively. One gets $B_{23}^{SU(5)}/B_{12}^{SU(5)} \leq 0.60$, while the present experimental data requires $B_{23}/B_{12} = 0.719 \pm 0.005$.

- Relation between Yukawa couplings of quarks and leptons.

  A second major problem is related to the predicted relation $Y_D = Y_E^T$ between the down quark and charged lepton Yukawa coupling matrices. This prediction is in strong disagreement with the experiment, especially in the case of the first and second generations of fermions. There are two solutions to this issue: one can add higher-dimensional operators [9] or introduce the $\mathbf{45_H}$ representation [10].

- Neutrino masses.

  In the Georgi-Glashow model, neutrinos are massless. However, today we know that they do have a tiny mass. Therefore, the model has to be extended in order to account for non-vanishing neutrino masses. There are two possible solutions: one can introduce at least two right-handed neutrinos and use the so-called Type I seesaw mechanism [4], or one can add the representation $\mathbf{15}_H$ in order to generate neutrino masses through the Type II seesaw mechanism [11].

- Doublet-triplet (DT) splitting problem.

  Another problem in the naive $SU(5)$ is that it cannot explain why the Higgs doublet living in $\mathbf{5}_H$ is light. Although there are no solutions to this issue in the context of a non-supersymmetric scenario, different mechanisms are conceivable in SUSY $SU(5)$ to achieve the splitting between the triplet and the doublet. (See for example Ref. [12] for a review.)

The simplest way [8] to address the first three problems listed above in a non-supersymmetric framework consists of extending the minimal GG model with the $\mathbf{15}_H$ and allowing for higher-dimensional operators. More precisely, the Higgs sector is $\mathbf{24}_H = \Sigma = (\Sigma_8, \Sigma_3, \Sigma_{(3,2)}, \Sigma_{(\bar{3},2)}, \Sigma_{24}) = (\mathbf{8},\mathbf{1},0) + (\mathbf{1},\mathbf{3},0) + (\mathbf{3},\mathbf{2},-5/6) + (\overline{\mathbf{3}},\mathbf{2},5/6) + (\mathbf{1},\mathbf{1},0)$, $\mathbf{15}_H = \Phi = (\Phi_a, \Phi_b, \Phi_c) = (\mathbf{1},\mathbf{3},1) + (\mathbf{3},\mathbf{2},1/6) + (\mathbf{6},\mathbf{1},-2/3)$, $\mathbf{5}_H = \Psi = (\Psi_D, \Psi_T) = (\mathbf{1},\mathbf{2},1/2) + (\mathbf{3},\mathbf{1},-1/3)$, where $\Sigma_{(3,2)}$ and $\Sigma_{(\bar{3},2)}$ are fields eaten by the superheavy gauge fields V. In what follows we define the GUT scale through their mass: $M_{GUT} = M_V$. As emphasized in [8], in this non-supersymmetric grand unified model, the GUT scale is low and can be predicted with great precision. This gives us the possibility to test the grand unification idea at future proton decay experiments. In fact, in our view, grand unified theories are the theories for the decay of the proton, since they provide us with the necessary input to compute the corresponding partial lifetimes. Of course, one can also think of other minimal extensions of grand unified theories based on higher groups. However, since in those models it is

very difficult to predict the GUT scale and the masses of the superheavy gauge bosons mediating nucleon decay, the predictions for the lifetime of the proton are far from accurate.

In the GUT scenario proposed in [8] the scalar potential reads as:

$$\begin{aligned} V &= V_{SU(5)}^{\rm naive} + \overline{5}_{ai}^{T}\, C\, h^{ab}\, \overline{5}_{bj}\, 15_H^{ij} - \frac{\mu_\Phi^2}{2}{\rm Tr}\, 15_H^\dagger 15_H + \frac{a_\Phi}{4}({\rm Tr}\, 15_H^\dagger 15_H)^2 \\ &+ \frac{b_\Phi}{2}\,{\rm Tr}\,(15_H^\dagger 15_H 15_H^\dagger 15_H) + c_2\,{\rm Tr}\,(15_H^\dagger 24_H 15_H) + c_3\, 5_H^\dagger\, 15_H\, 5_H^* \\ &+ c_3^*\, 5_H^T\, 15_H^\dagger 5_H + b_1\,{\rm Tr}\,(15_H^\dagger 15_H){\rm Tr}\, 24_H^2 + b_3\, 5_H^\dagger 5_H\,{\rm Tr}\,(15_H^\dagger 15_H) \\ &+ b_5\, 5_H^\dagger 15_H 15_H^\dagger 5_H + b_6\,{\rm Tr}\,(15_H 15_H^* 24_H^2) + b_7\,{\rm Tr}\,(15_H^* 24_H 15_H 24_H^*) \\ &+ b_8\, 5_H^\dagger 24_H 15_H 5_H^* + b_8^*\, 5_H^T 24_H 15_H^\dagger 5_H + \text{higher-dimensional terms}, \end{aligned} \tag{1}$$

where $V_{SU(5)}^{\rm naive}$ is the scalar potential of the Georgi-Glashow model.

III. UNIFICATION

In this section we present the constraints that exact gauge coupling unification places on the masses of the scalars of the theory. We first present the one-loop level analysis to outline the basic features of the scalar mass spectrum and only then we resort to the more accurate two-loop analysis.

A. One-Loop Analysis

The one-loop level relations between the gauge couplings at M_Z and the unifying gauge coupling $\alpha_{GUT} = g_{GUT}^2/(4\pi)$ at M_{GUT} are

$$\alpha_i^{-1}\big|_{M_Z} = \alpha_{GUT}^{-1} + \frac{b_i}{2\pi}\ln\frac{M_{GUT}}{M_Z}, \tag{2}$$

where $i = 1, 2, 3$ for $U(1)$, $SU(2)$, and $SU(3)$, respectively, and b_i are the familiar one-loop β function coefficients. The SM particle content with n light Higgs doublet fields yields $b_1 = 40/10 + n/10$, $b_2 = -20/6 + n/6$ and $b_3 = -7$.

Eqs. (2) hold under the assumption that there is a particle "desert" between M_Z and M_{GUT}. However, there is no particular reason that this should be the case. If there are I particles with intermediate masses M_I ($M_Z \leq M_I \leq M_{GUT}$), these equations remain unaltered except for the substitutions $b_i \to B_i$, where $B_i = b_i + \sum_I b_{iI} r_I$ are the so-called effective coefficients. Here b_{iI} are the appropriate one-loop coefficients of the particle I and $r_I = (\ln M_{GUT}/M_I)/(\ln M_{GUT}/M_Z)$ ($0 \leq r_I \leq 1$) is its "running weight".

The elimination of α_{GUT} from Eqs. (2) leaves the following two equations that connect the effective coefficients $B_{ij} = B_i - B_j$ with the low-energy observables [13]:

$$\frac{B_{23}}{B_{12}} = \frac{5}{8}\frac{\sin^2\theta_W - \alpha_{em}/\alpha_s}{3/8 - \sin^2\theta_W}\,, \qquad \ln\frac{M_{GUT}}{M_Z} = \frac{16\pi}{5}\frac{3/8 - \sin^2\theta_W}{\alpha_{em}B_{12}}\,. \tag{3}$$

Adopting the following experimental values at M_Z in the $\overline{MS}$ scheme [14]: $\sin^2\theta_W = 0.23120 \pm 0.00015$, $\alpha_{em}^{-1} = 127.906 \pm 0.019$ and $\alpha_s = 0.1187 \pm 0.002$, these read

$$\frac{B_{23}}{B_{12}} \;=\; 0.719 \pm 0.005\,, \tag{4a}$$

$$\ln\frac{M_{GUT}}{M_Z} \;=\; \frac{184.9 \pm 0.2}{B_{12}}\,. \tag{4b}$$

Eq. (4a) is sometimes referred to as the B-test. It basically shows whether unification takes place or not. Eq. (4b), on the other hand, could be referred to as the GUT scale relation since it yields the GUT scale value once Eq. (4a) is satisfied.

The B-test fails badly in the SM case ($B_{23}/B_{12} = 0.53$), and hence the need for extra light particles with suitable B_{ij} coefficients to bring the value of the B_{23}/B_{12} ratio in agreement with its experimental value. In our case the presence of the $\mathbf{15}_H$ is essential. The B_{ij} coefficients for all the particles in our scenario are presented in Table I. Clearly, Σ_3, Φ_a and Φ_b improve unification with respect to the SM case, while Σ_8, Ψ_T and Φ_c act in the opposite manner. We recall that we set $M_V = M_{GUT}$, where M_V is the mass of the superheavy gauge bosons. We thus take Σ_8, Ψ_T and Φ_c to reside at or above the GUT scale in our numerical analysis. We relax this assumption later to discuss its impact on our findings.

TABLE I: B_{ij} coefficients.

	Higgsless SM	Ψ_D	Ψ_T	V	Σ_8	Σ_3	Φ_a	Φ_b	Φ_c
B_{23}	$\frac{11}{3}$	$\frac{1}{6}$	$-\frac{1}{6}r_{\Psi_T}$	$-\frac{7}{2}r_V$	$-\frac{1}{2}r_{\Sigma_8}$	$\frac{1}{3}r_{\Sigma_3}$	$\frac{2}{3}r_{\Phi_a}$	$\frac{1}{6}r_{\Phi_b}$	$-\frac{5}{6}r_{\Phi_c}$
B_{12}	$\frac{22}{3}$	$-\frac{1}{15}$	$\frac{1}{15}r_{\Psi_T}$	$-7r_V$	0	$-\frac{1}{3}r_{\Sigma_3}$	$-\frac{1}{15}r_{\Phi_a}$	$-\frac{7}{15}r_{\Phi_b}$	$\frac{8}{15}r_{\Phi_c}$

The value of B_{12} determines the scale of unification through the GUT scale relation (4b). Therefore, a lower bound on B_{12} translates into an absolute upper bound on the GUT scale. If we naively set $M_{\Sigma_3} = M_{\Phi_a} = M_{\Phi_b} = M_Z$ ($r_{\Sigma_3} = r_{\Phi_a} = r_{\Phi_b} = 1$) we obtain $B_{12} > 6.4$ and accordingly $M_{GUT} < 3.2 \times 10^{14}\,\text{GeV}$. This constraint, when combined with the relation $M_{GUT} = M_V$, determines an upper bound on the proton lifetime, if the corresponding value of α_{GUT} is known. To find the latter we resort to the numerical analysis. However, the non-supersymmetric nature of our scenario suggests this value to be around 1/39.

In the one-loop analysis we treat M_{GUT}, M_{Σ_3}, M_{Φ_a} and M_{Φ_b} as free parameters and investigate the possibility to find a consistent scenario with *exact* gauge coupling unification. Since we have four free parameters and two equations—Eqs. (4a) and (4b)—we opt to present the M_{Σ_3} and M_{Φ_a} contours in the M_{GUT}–M_{Φ_b} plane in Fig. 1. The line of constant α_{GUT}^{-1} is also shown.

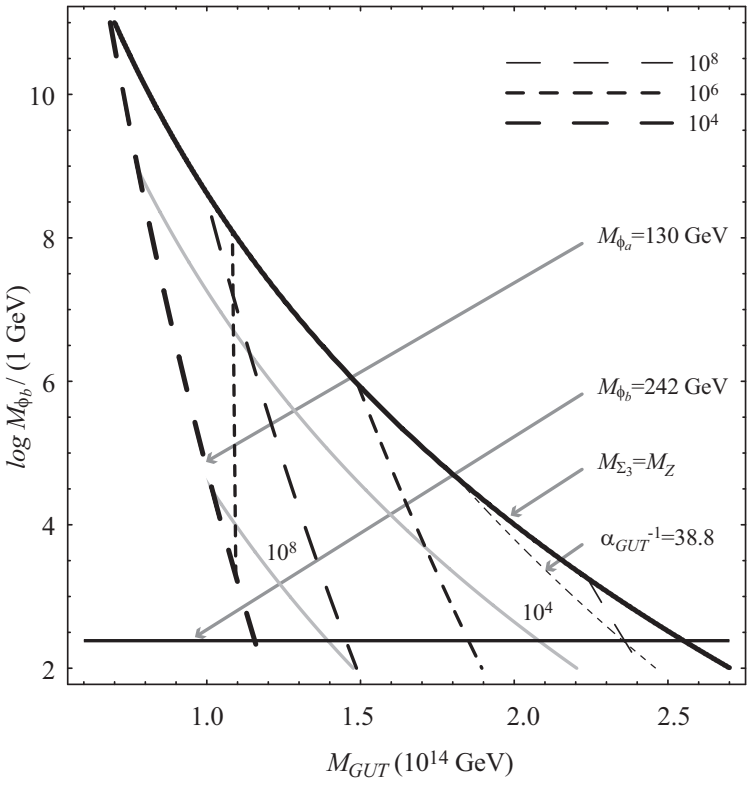

FIG. 1: Plot of lines of constant M_{Σ_3} and M_{Φ_a} in the M_{GUT}–$log\,(M_{\Phi_b}/1\,\mathrm{GeV})$ plane, assuming exact one-loop unification. We use the central values for the gauge couplings as given in the text. All the masses are given in GeV units. The triangular region is bounded from the left (below) by the experimental limit on M_{Φ_a} (M_{Φ_b}). The right bound is $M_{\Sigma_3} \geq M_Z$. The two grey solid (thick dashed) lines are the lines of constant M_{Σ_3} (M_{Φ_a}). The line of constant α_{GUT}^{-1} is also shown. The region to the left of the vertical dashed line is excluded by the proton decay experiments if $\alpha = 0.015\,\mathrm{GeV}^3$.

The triangular region in Fig. 1 represents the available parameter space under the assumption that Ψ_T, Σ_8 and Φ_c reside at or above the GUT scale. The region is bounded from the left and below by the experimental limits on M_{Φ_a} and M_{Φ_b}, respectively. (For the discussion on the origin of the experimental limits see [8] and references therein.) The right bound stems from the requirement that $M_{\Sigma_3} \geq M_Z$. It is expected that the Large Hadron Collider (LHC) will place a more stringent lower limit on the mass of the scalar leptoquark Φ_b at around 1 TeV (For experimental bounds on leptoquark masses see Refs. [15].).

Fig. 1 reveals that the masses of the three scalar fields that improve unification, namely, Σ_3, Φ_a and Φ_b, have to be below the GUT scale. This, however, does not hold at two-loop level. The GUT scale is rather low and, for a given value of α_{GUT}, the predicted value for the proton decay lifetime is within the reach of the present and future proton decay experiments. More precisely, if the nucleon matrix element is taken to be $\alpha = 0.015\,\mathrm{GeV}^3$, the region to the left of the vertical thick dashed line in Fig. 1 is already excluded by the present limits on the proton decay lifetime. In order to generate this bound we assume maximal flavor suppression of the gauge $d = 6$ proton decay operators as explained in the next section in more details. Clearly, due to the simplicity of our scenario, experimental limits place firm upper bounds on M_{Σ_3}, M_{Φ_a} and M_{Φ_b}.

What happens if we relax the $M_{\Psi_T}, M_{\Sigma_8}, M_{\Phi_c} \geq M_{GUT}$ assumption? If either M_{Ψ_T} or M_{Φ_c} are below the GUT scale, then they both decrease M_{GUT} due to an increase of the B_{12} coefficient. They also change the B_{23}/B_{12} ratio in the wrong direction, which has to be compensated by appropriate changes in the Σ_3, Φ_a and Φ_b contributions. Pictorially, as one lowers M_{Ψ_T} and M_{Φ_c} the $M_{\Phi_a} = 130\,\mathrm{GeV}$ line in Fig. 1 moves very slowly to the left while, at the same time, $M_{\Sigma_3} = M_Z$ moves very rapidly in the same direction until the triangular region shrinks to a point. In other words, any scenario in which M_{Ψ_T} or M_{Φ_c} or both are below the GUT scale would be more significantly exposed to the tests through the proton decay lifetime measurements and accelerator searches than the scenario shown in Fig. 1.

If, on the other hand, one lowers the mass of Σ_8, the $M_{\Phi_a} = 130$ GeV line moves to the right more rapidly than the $M_{\Sigma_3} = M_Z$ line until the triangular region becomes a point when M_{Σ_8} reaches $10^5\,\mathrm{GeV}$. At that point M_{GUT} reaches the upper bound[1] of 3.2×10^{14} GeV for $M_{\Sigma_8} = 10^5\,\mathrm{GeV}$, $M_{\Sigma_3} = M_Z$, $M_{\Phi_a} = 130\,\mathrm{GeV}$, $M_{\Phi_b} = 242\,\mathrm{GeV}$ and $\alpha_{GUT}^{-1} = 37.3$. Again, the upper bound on the masses of Σ_3, Φ_b and Φ_a would be significantly lower as long as $M_{\Sigma_8} < M_{GUT}$.

B. Two-Loop Analysis

The simplicity of the Higgs sector allows us to repeat the same analysis at the two-loop level. We require exact unification and present the available parameter space in Fig. 2. In the two-loop analysis we must also take into account the one-loop running of the Yukawa couplings. The relevant input parameters at the M_Z scale, such as the fermion masses and CKM angles that are used in the running are specified in Table I of Ref. [16]. The stars in Fig. 2 represent points that correspond

[1] We will confront this bound with the outcome of the two-loop analysis and use it to evaluate the corresponding upper bound on the total proton decay lifetime.

to exact unification.

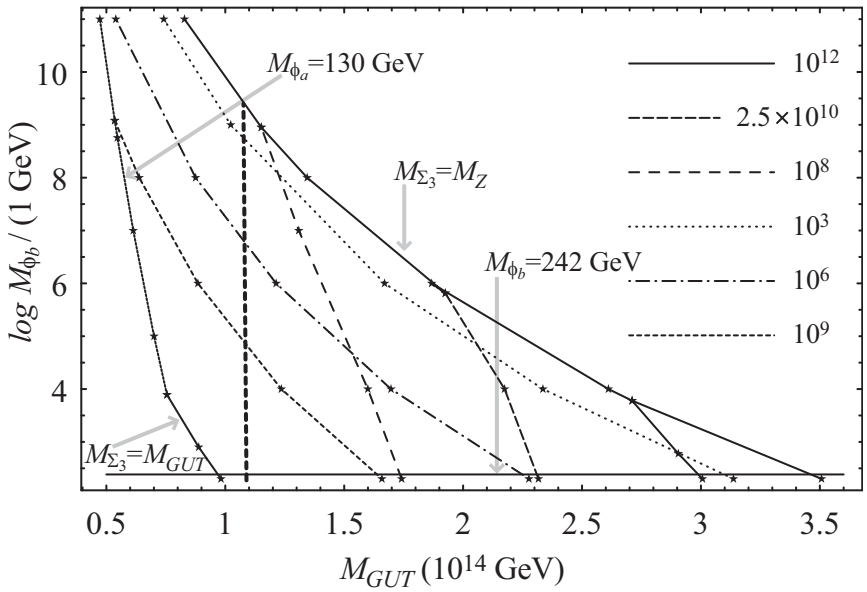

FIG. 2: Unification of the gauge couplings at the two-loop level. Stars correspond to exact numerical two-loop unification solutions. There are two sets of lines of constant value. The steeper set is associated with M_{Φ_a} and the other one represents the lines of constant M_{Σ_3}. All the masses are in GeV units. The region to the left of the vertical dashed line is excluded by the proton decay experiments if $\alpha = 0.015\,\mathrm{GeV}^3$.

The upper bound on the GUT scale is shifted to a higher value (about a factor of $\sqrt{2}$) with respect to the one-loop case. There also appears a line along which M_{Σ_3} is not required to be below M_{GUT} to accomplish exact unification. Moreover, the allowed values of M_{Φ_a} are significantly larger than in the one-loop analysis. As before, by relaxing the assumption $M_{\Psi_T}, M_{\Phi_c} \geq M_{GUT}$ ($M_{\Sigma_3} \geq M_{GUT}$), the allowed region moves to the left (right) and shrinks as we lower the relevant masses.

At two-loop level, the corrected upper bound on M_{GUT} is $4.6 \times 10^{14}\,\mathrm{GeV}$ for $M_{\Sigma_8} = M_Z$, $M_{\Sigma_3} = M_Z$, $M_{\Phi_a} = 6.4 \times 10^3\,\mathrm{GeV}$, $M_{\Phi_b} = 242\,\mathrm{GeV}$ and $\alpha_{GUT}^{-1} = 37.06$. We use these values in the next section to derive an accurate upper bound on the proton decay lifetime. If $\alpha = 0.015\,\mathrm{GeV}^3$ the proton decay lifetime measurements already exclude the part to the left of the thick dashed line ($M_{GUT} \gtrsim 1.1 \times 10^{14}\,\mathrm{GeV}$) and establish a triangular region with the maximal value for M_{Φ_b} around $10^9\,\mathrm{GeV}$.

With the two-loop analysis at hand, we can finally answer the following question. How much improvement in the lifetime limits do proton decay experiments need in order to completely exclude our scenario? In the "worst" case scenario the two-loop GUT scale is approximately by a factor of

four larger than the current proton decay bound presented in Fig. 2. Hence, an improvement in the measurements of proton lifetime by a factor of 4^4 is called for to completely rule out this GUT scenario. The situation is actually much better than that, since even the slightest improvement in the proton lifetime bounds (by a factor of fifteen) will make our scenario incompatible with exact unification unless either Φ_b or Σ_3 resides below $10^3\,\mathrm{GeV}$, thus making them accessible in accelerator experiments.

C. $\Sigma \to -\Sigma$ Invariance

Fig. 2 shows that Σ_3 and Σ_8 are highly non-degenerate in some parts of the allowed parameter space. This, however, seems in conflict with the tree-level relation $M_{\Sigma_3} = 4M_{\Sigma_8}$ of the Σ potential, which is invariant under the $\Sigma \to -\Sigma$ transformation [17]. Since we commit to the scenario that includes all possible terms allowed by the gauge symmetry, we are forced to depart from this commonly used invariance. This has three important consequences: (i) the cubic term ($\mathrm{Tr}\,\Sigma^3$) in the potential violates the validity of the $M_{\Sigma_3} = 4M_{\Sigma_8}$ relation [18]; (ii) the higher dimensional terms linear in Σ/Λ in the Yukawa part of the Lagrangian allow for masses of quarks and leptons that are in agreement with the experimentally observed values [2]; (iii) a term linear in both Σ and $\mathbf{15}_H$ appears in the scalar part of the potential that is relevant for both the proton decay and neutrino masses if $\mathbf{15}_H$ couples to matter fields.

To demonstrate how constraining the demand for $\Sigma \to -\Sigma$ invariance is, we present one example in a somewhat simpler setting. Recall the Georgi-Glashow $SU(5)$ model with two Higgs fields—one in the adjoint and the other in the fundamental representation. After imposing the $\Sigma \to -\Sigma$ invariance there are seven terms left in the scalar potential. It is easy to show [17] that once Σ_{24} gets a VEV v_1 of the order of the GUT scale and Ψ_D gets the electroweak scale VEV v_2, the electrically neutral component of Σ_3 must get a VEV of the order of $v_3 \sim v_2^2/v_1$ (For the relevant equations and further discussion see [19]. We note that there is a term $+\lambda_4 v_2 v_3^2$ missing on the right hand side of Eq. (11) of Ref. [19].) The above statement, however, is not necessarily correct if we include two more terms in the potential that are absent under the $\Sigma \to -\Sigma$ invariance. In other words, a classical solution with appropriate VEVs for Σ_{24} and Ψ_D but with no VEV for Σ_3 is allowed if the potential is not invariant under $\Sigma \to -\Sigma$.

We conclude this section with the following observation. In our analysis we include all the

[2] Here Λ is the scale where some new physics, relevant for the ultraviolet (UV) completion of the theory, enters.

terms allowed by the underlying gauge symmetry. In this way we insure that the predictions of our scenario are independent of any particular set of assumptions. The benefit of such an approach is clear: if the predictions of our scenario are experimentally refuted, then the scenario is ruled out regardless of any particular assumptions.

IV. UPPER BOUND ON THE TOTAL PROTON DECAY LIFETIME

Proton decay is a generic prediction coming from matter unification. We thus believe this to be the most promising way to test the beautiful idea of grand unification. It is commonly thought that non-supersymmetric GUT scenarios are ruled out by the limits on nucleon decay lifetimes, since the unification scale is around 10^{14} GeV. This, however, holds only in GUT scenarios where the Yukawa sector is quite constrained. In general, this is no longer true because proton decay predictions are different for each model of fermion masses [20]. To show that our minimal non-supersymmetric GUT scenario based on $SU(5)$ gauge group is not ruled out by such limits, we look for an upper bound on the total proton decay lifetime. For new experimental lower bounds on the partial lifetime of the proton see Ref. [21].

It is well known that in any non-supersymmetric scenario the most important contributions to the decay of the proton are the so-called gauge $d = 6$ contributions. In the physical basis, these effective operators read as [22]:

$$O(\nu_l, d_\alpha, d^C_\beta) \;=\; \frac{g^2_{GUT}}{2M^2_V}\; (V_1\; V_{UD})^{1\alpha}\; (V_3\; V_{EN})^{\beta l}\; \overline{u^C}\; \gamma^\mu\; L\; d_\alpha\; \overline{d^C_\beta}\; \gamma_\mu\; L\; \nu_l\,, \tag{5}$$

$$O(e^C_\alpha, d_\beta) \;=\; \frac{g^2_{GUT}}{2M^2_V}\; \left(V_1^{11}\; V_2^{\alpha\beta} \;+\; (V_1\; V_{UD})^{1\beta}(V_2\; V^\dagger_{UD})^{\alpha 1}\right)\; \overline{u^C}\; \gamma^\mu\; L\; u\; \overline{e^C_\alpha}\; \gamma_\mu\; L\; d_\beta\,, \tag{6}$$

$$O(e_\alpha, d^C_\beta) \;=\; \frac{g^2_{GUT}}{2M^2_V}\; V_1^{11}\; V_3^{\beta\alpha}\; \overline{u^C}\; \gamma^\mu\; L\; u\; \overline{d^C_\beta}\; \gamma_\mu\; L\; e_\alpha\,. \tag{7}$$

In the above equations $V_1 = U^\dagger_C\, U$, $V_2 = E^\dagger_C\, D$, $V_3 = D^\dagger_C\, E$, $V_4 = D^\dagger_C\, D$, $V_{UD} = U^\dagger\, D$, $V_{EN} = E^\dagger\, N$ are mixing matrices; $L = (1 - \gamma_5)/2$ and $\alpha, \beta = 1, 2$; $l = 1, 2, 3$. Our convention for the diagonalization of the up, down, charged lepton and neutral lepton Yukawa matrices is specified by

$$U^T_C\; Y_U\; U = Y_U^{\rm diag}\,, \quad D^T_C\; Y_D\; D = Y_D^{\rm diag}\,, \quad E^T_C\; Y_E\; E = Y_E^{\rm diag}\,, \quad N^T\; Y_\nu\; N = Y_\nu^{\rm diag}\,. \tag{8}$$

The quark and leptonic mixing are given in our notation by $V_{UD} = U^\dagger D = K_1 V_{CKM} K_2$ and $V_{EN} = K_3 V_{PMNS}$, respectively, where K_1, K_3 and K_2 are diagonal matrices containing three and two phases, respectively.

The way to find an upper bound on the total proton decay lifetime by investigating the possible freedom in the Yukawa sector of grand unified theories has been pointed out in Ref. [20]. For a given value of α_{GUT} and the super-heavy gauge boson mass, it has been shown that the upper bound in the case of Majorana neutrinos is given by [20]:

$$\tau_p \leq \frac{6}{\alpha_{GUT}^2} \times \left(\frac{M_V}{10^{16}\text{GeV}}\right)^4 \times \left(\frac{0.003\text{GeV}^3}{\alpha}\right)^2 \times 10^{39} \text{ years}\,. \tag{9}$$

Here, α is the value of the matrix element, usually taken $\alpha = 0.003\,\text{GeV}^3$ as a conservative value. However, in a recent lattice calculation, $\alpha = 0.015\,\text{GeV}^3$ has been obtained [23]. (There is also a factor 4 difference between the previous equation and Eq. (9) of Ref. [20]. This is due to an erroneous normalization in [20].)

By inspecting the full parameter space where unification can be achieved in our GUT scenario (cf. Figs. 1 and 2), it is then possible to find the upper bound on the total proton decay lifetime. Using the maximal M_{GUT} value and associated values for α_{GUT}, we can estimate the upper bound on τ_p, taking into account the one- and two-loop running of the gauge couplings, respectively. Using $\alpha = 0.015\,\text{GeV}^3$ [23], these bounds read as

$$\tau_p^{(\text{one-loop})} \leq 3.5 \times 10^{35}\,\text{years}, \tag{10}$$

$$\tau_p^{(\text{two-loop})} \leq 1.4 \times 10^{36}\,\text{years}. \tag{11}$$

There is a difference of a factor 4 for the upper bounds on the total proton lifetime between the two cases. As can be appreciated, our grand unified scenario is not ruled out by the present experimental lower bound on the proton decay lifetime (typically $\tau_{\text{exp}} \gtrsim 10^{33}$ years [21]). The regions that are ruled out are presented by the vertical dashed thick lines in Figs. 1 and 2.

In order to complete our study, let us also discuss the $d = 6$ Higgs contributions. In Ref. [8], the predictions coming from those operators were studied in detail. It was shown that besides the usual $d = 6$ Higgs terms there are also contributions due to the mixing between the colored triplet Ψ_T and the light leptoquark Φ_b. This mixing comes from the interaction term $c_3\ 5_H^\dagger\ 15_H\ 5_H^*$. However, there is an extra contribution to this mixing coming from the term $b_8\ 5_H^\dagger\ 24_H\ 15_H\ 5_H^*$ that we mentioned before. Therefore, when applying Eq. (10) of [8] to the present case, one should replace c_3 by $c_3 - \lambda b_8/\sqrt{30} \equiv \tilde{c}_3/2$. Since the Higgs contributions are very ambiguous, one can verify that the upper bound on the proton decay lifetime in our GUT scenario is indeed given by Eqs. (10). In particular, we remark that in the present scenario it is always possible to set to zero all Higgs contributions to the nucleon decay by choosing the matrices $\underline{A}_{ij} = -\underline{A}_{ji}$ and $D_{ij} = 0$, except for $i = j = 3$. (See Ref. [8] for notation and details.)

Certainly, if proton decay is not observed, the next generation of experiments will improve the lower bounds on partial lifetimes by a few orders of magnitude. For instance, the goal of Hyper-Kamiokande is to explore the proton lifetime at least up to $\tau_p/B(p \to e^+\pi^0) > 10^{35}$ years and $\tau_p/B(p \to K^+\bar{\nu}) > 10^{34}$ years in about 10 years [24]. Thus, our minimal GUT scenario will be tested or ruled out at the next generation of proton decay experiments, since the upper-bound on the total proton decay lifetime in our scenario is $\tau_p \leq 1.4 \times 10^{36}$ years.

V. CONSTRAINTS FROM TRIPLET SEESAW LEPTOGENESIS

The origin of the baryon asymmetry observed in the universe is an outstanding problem in particles physics and cosmology. The most recent Wilkinson Microwave Anisotropy Probe (WMAP) results and big bang nucleosynthesis analysis of the deuterium abundance imply [25]

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.1 \pm 0.3) \times 10^{-10}\,, \tag{12}$$

for the baryon-to-photon ratio of number densities. Among the viable mechanisms to explain this primordial matter-antimatter asymmetry, leptogenesis [26] has undoubtedly become one of the most compelling scenarios. Indeed, the evidence for non-vanishing neutrino masses and the possibility that their origin is directly linked to lepton number violation point towards leptogenesis as a natural mechanism for the generation of the cosmological baryon asymmetry. Moreover, in GUT scenarios, where the existence of heavy (boson or fermion) particles is predicted, leptogenesis can be easily realized by means of the out-of-equilibrium decays of such particles at temperatures below their mass scale. The lepton asymmetry generated in the presence of CP-violating processes is then partially converted into a baryon asymmetry by the sphalerons [27].

In its simplest framework, consisting on the addition of hierarchical heavy right-handed neutrinos to the standard model, successful leptogenesis implies a lower bound on the mass of the lightest heavy Majorana neutrino, $M_{N_1} \gtrsim 10^8$ (10^9) GeV, which holds assuming a thermal (zero) initial abundance of the N_1 neutrinos before decaying. On the other hand, if the same heavy neutrinos are responsible for the generation of the light neutrino masses via the well-known seesaw mechanism [4], then their natural mass scale is expected to be $M_N \sim v^2/m_\nu \sim 10^{14}$ GeV, for a light neutrino mass scale around the atmospheric neutrino scale, i.e. $m_\nu \sim m_{atm} \simeq 5 \times 10^{-2}$ eV. However, in the presence of other lepton-number violating interactions, such as the ones mediated by $SU(2)_L$ scalar triplets, the leptonic asymmetry produced by the out-of-equilibrium decay of the heavy Majorana singlets can be totally washed out, if the triplet mass scale is lower than M_N. In

the latter case, leptogenesis could proceed through the decay of the lightest triplet scalar. As we shall see below, the $\Phi_a\,(\subset \mathbf{15}_H)$ $SU(2)_L$ scalar triplet with a mass $M_{\Phi_a} \geq 10^{9-10}$ GeV constitutes a natural candidate for a successful triplet seesaw leptogenesis in our minimal $SU(5)$ scenario.

To study the viability of thermal leptogenesis, we consider the simplest extension of the minimal $SU(5)$ model proposed in [8], which consists on the addition of right-handed neutrinos. We remark that, in the present framework, the introduction of a single heavy Majorana neutrino is the minimal extra particle content required to implement the leptogenesis mechanism through the out-of-equilibrium decay of the triplet Φ_a into leptons involving the virtual exchange of the right-handed neutrino. The relevant terms of the right-handed Majorana neutrino and scalar triplet Lagrangian are

$$\mathcal{L} \ni -\frac{1}{2} M_N N^T C\, N - H^\dagger \bar{N}\, Y_N\, \ell - M^2_{\Phi_a} \mathrm{Tr}\, \Phi^\dagger_a \Phi_a - \frac{1}{2} \ell^T C i\sigma_2 \Phi_a Y_\nu\, \ell + \frac{1}{2} \tilde{c}_3 H^T i\sigma_2 \Phi_a H + \mathrm{H.c.}\,, \quad (13)$$

with $\ell = (\nu, e)^T$, $H = (H^0, H^-)^T$,

$$\Phi_a = \begin{pmatrix} \dfrac{1}{\sqrt{2}}\,\delta^+ & \delta^{++} \\ \delta^0 & -\dfrac{1}{\sqrt{2}}\,\delta^+ \end{pmatrix}\,, \quad (14)$$

M_N is the right-handed neutrino mass matrix, Y_N and Y_ν are the coupling matrices. For simplicity, flavour indices have been omitted. The triplet (type-II seesaw) contribution to the effective neutrino mass matrix $\mathcal{M}_\nu$ is given by

$$M^{II}_\nu = \tilde{c}_3 Y_\nu \frac{v^2}{M^2_{\Phi_a}}\,, \quad (15)$$

$v = \langle H_0 \rangle = 174$ GeV, while the usual right-handed neutrino (type-I seesaw) contribution is

$$M^I_\nu = -v^2 Y^T_N M^{-1}_N Y_N\,. \quad (16)$$

In the presence of CP-violating interactions, the Φ_a decay into two leptons generates a non-vanishing CP asymmetry

$$\varepsilon = 2\,\frac{\Gamma(\Phi^*_a \to \ell + \ell) - \Gamma(\Phi_a \to \bar{\ell} + \bar{\ell})}{\Gamma_\Phi + \Gamma_{\Phi^*}}\,, \quad (17)$$

where Γ_Φ denotes the total triplet decay width. Since in the present minimal $SU(5)$ framework there are only two decay modes, $\Phi_a \to \ell + \ell$ and $\Phi_a \to H + H$, one can write

$$\mathcal{B}_\ell\, \Gamma_\Phi \equiv \Gamma(\Phi_a \to \ell + \ell) = \frac{M_{\Phi_a}}{16\pi} \mathrm{Tr}\, Y^\dagger_\nu Y_\nu\,, \quad (18)$$

$$\mathcal{B}_H \Gamma_\Phi \equiv \Gamma(\Phi_a \to H + H) = \frac{1}{16\pi}\,\frac{|\tilde{c}_3|^2}{M_{\Phi_a}}\,, \quad (19)$$

where $\mathcal{B}_\ell$ and $\mathcal{B}_H$ are the corresponding tree-level branching ratios ($\mathcal{B}_\ell + \mathcal{B}_H = 1$). A nonzero ε asymmetry is then generated by the interference of the tree-level decay process with the one-loop vertex diagram, as shown in Fig. 3.

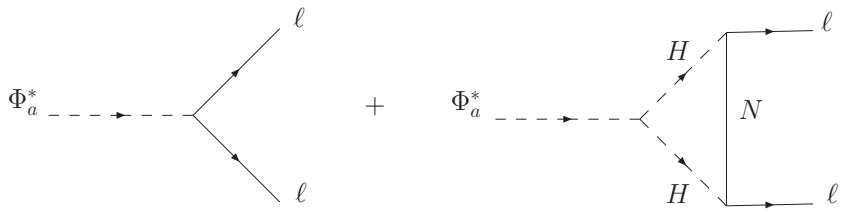

FIG. 3: Tree-level and one-loop diagrams contributing to the CP asymmetry in the scalar triplet decay.

Assuming $M_{\Phi_a} \ll M_N$, the resulting asymmetry is approximately given by [28]

$$\varepsilon \simeq \frac{1}{8\pi^2}\,\frac{M_{\Phi_a}^3}{v^4}\,\frac{\mathrm{Im}\,[\mathrm{Tr}\,M_\nu^{II} M_\nu^{I\dagger}]}{\Gamma_\Phi}\,. \tag{20}$$

Using the relation

$$16\pi v^2 \Gamma_\Phi \sqrt{\mathcal{B}_\ell\,\mathcal{B}_H} = M_{\Phi_a}^2 \sqrt{\mathrm{Tr}\,M_\nu^{II\dagger} M_\nu^{II}}\,, \tag{21}$$

and recalling that the effective seesaw neutrino mass matrix is $\mathcal{M}_\nu = M_\nu^I + M_\nu^{II}$, Eq. (20) can be recast in the form [29]

$$\varepsilon \simeq \frac{M_{\Phi_a}\sqrt{\mathcal{B}_\ell\,\mathcal{B}_H}}{4\pi v^2}\,\frac{\mathrm{Im}\,[\mathrm{Tr}\,M_\nu^{II}\mathcal{M}_\nu^\dagger]}{\sqrt{\mathrm{Tr}\,M_\nu^{II\dagger} M_\nu^{II}}}\,. \tag{22}$$

In order to discuss the bounds implied by leptogenesis, it is convenient to write the CP asymmetry ε as the product

$$\varepsilon = \varepsilon_{\max}\sin\delta_L\,, \tag{23}$$

where $\varepsilon_{\max}$ is the maximal CP asymmetry and δ_L is an effective leptogenesis phase. Using Eq. (22), it is then straightforward to show that the following upper bound holds[3]

$$\varepsilon_{\max} = \frac{M_{\Phi_a}}{4\pi v^2}\,\left(\mathcal{B}_\ell\,\mathcal{B}_H \sum m_{\nu_i}^2\right)^{1/2}\,, \tag{24}$$

where m_{ν_i} are the light neutrino masses. Clearly, the absolute maximum of the above expression is attained when $\mathcal{B}_\ell = \mathcal{B}_H = 1/2$. This situation, however, does not necessarily corresponds to

[3] This value is typically much smaller than the maximal value allowed by unitarity, $|\varepsilon| < 2\min(\mathcal{B}_\ell, \mathcal{B}_H)$.

a maximal baryon asymmetry, since the efficiency of leptogenesis, dictated by the solution of the relevant Boltzmann equations, is not necessarily maximal in such a case. In fact, a numerical study of these equations shows that the efficiency is minimal for $\mathcal{B}_\ell = \mathcal{B}_H = 1/2$ and maximal when either $\mathcal{B}_\ell \ll \mathcal{B}_H$ or $\mathcal{B}_\ell \gg \mathcal{B}_H$ [29].

Assuming no pre-existing asymmetry, the total baryon asymmetry, obtained after partial lepton-to-baryon conversion through the sphalerons, is given by

$$\eta_B \simeq -3 \times 10^{-2}\, \varepsilon\, \kappa_f \,, \tag{25}$$

where κ_f is the efficiency factor, normalized in such a way that κ_f approaches one in the limit of thermal initial Φ_a abundance and no washout. The expression (25), when combined with Eq. (24) leads to a lower bound on the mass M_{Φ_a}. Indeed, using the observed value $\eta_B^{\rm min} = 5.8 \times 10^{10}$ (cf. Eq. (12)), one finds

$$M_{\Phi_a} \gtrsim \frac{1.5 \times 10^8 \text{ GeV}}{\kappa_f \sqrt{\mathcal{B}_\ell\, \mathcal{B}_H}}\, \frac{0.05 \text{ eV}}{\sqrt{\sum m_{\nu_i}^2}} \,. \tag{26}$$

It is possible to obtain a simple estimate of the efficiency of leptogenesis by comparing the total triplet decay rate with the expansion rate of the universe. We define, as usual, the decay parameter

$$K_\Phi \equiv \frac{\Gamma_\Phi}{H(T = M_{\Phi_a})} \,, \quad H(T) = 1.66\, g_*^{1/2} \frac{T^2}{M_P} \,, \tag{27}$$

where $H(T)$ is the Hubble rate, $M_P \simeq 1.2 \times 10^{19}$ GeV is the Planck mass and g_* is the effective number of relativistic degrees of freedom ($g_* = 106.75$ in the SM). Using Eq. (21), we can rewrite K_Φ in the form

$$K_\Phi = \frac{23}{\sqrt{\mathcal{B}_\ell\, \mathcal{B}_H}}\, \frac{\sqrt{\sum m_{\nu_i}^2}}{0.05 \text{ eV}} \,. \tag{28}$$

For not very large values of $K_\Phi \gtrsim 1$, an order-of-magnitude estimate of the efficiency factor is $\kappa_f \sim 1/K_\Phi$ and Eqs. (26) and (27) imply

$$M_{\Phi_a} \gtrsim \frac{3.4 \times 10^9 \text{ GeV}}{\mathcal{B}_\ell\, \mathcal{B}_H} \,, \tag{29}$$

which for $\mathcal{B}_\ell = \mathcal{B}_H = 1/2$ yields $M_{\Phi_a} \gtrsim 1.4 \times 10^{10}$ GeV. Clearly, a more precise analysis requires the solution of the full set of Boltzmann equations [29]. Nevertheless, for $\kappa_f \simeq 10^{-2} - 10^{-3}$ and $\mathcal{B}_\ell \sim \mathcal{B}_H$, Eq. (26) implies

$$M_{\Phi_a} \gtrsim 3 \times 10^{10-11} \text{ GeV} \left(\frac{0.05 \text{ eV}}{\sqrt{\sum m_{\nu_i}^2}} \right) \,. \tag{30}$$

For hierarchical light neutrinos, one has $\sqrt{\sum m_{\nu_i}^2} \simeq m_{atm}$ and the above bound leads to $M_{\Phi_a} \gtrsim 3 \times 10^{10-11}$ GeV. On the other hand, if neutrinos are quasi-degenerate in mass, then the WMAP constraint $\sum m_{\nu_i} < 0.69$ eV leads to the less restrictive lower limit $M_{\Phi_a} \gtrsim 4 \times 10^{9-10}$ GeV. We also note that this bound could be further reduced by about a factor of two, if the dark energy component of the universe is not in the form of a cosmological constant. In the latter case, assuming a dark energy equation of state $p = -w\rho$ with $w < -1$, the present cosmological bound on neutrino masses relaxes to $\sum m_{\nu_i} < 1.48$ eV [30].

Combining the above leptogenesis bounds with the ones shown in Fig. 2, we conclude that the natural implementation of the type-II seesaw mechanism and successful leptogenesis exclude the region of the parameter space where the mass of the triplet M_{Φ_a} is below $10^9 - 10^{10}$ GeV. This in turn implies that the leptoquark Φ_b must be light enough ($M_{\Phi_b} \lesssim 10^{6-7}$ GeV) to satisfy the cosmological constraints, thus opening the possibility to test our minimal non-supersymmetric $SU(5)$ GUT scenario at the next generation of collider experiments through the production of light leptoquarks, and particularly, at LHC.

VI. SUMMARY

We have investigated in detail the constraints coming from unification of gauge interactions in the minimal extension of the Georgi-Glashow model, where the Higgs sector is composed by $\mathbf{5}_H$, $\mathbf{15}_H$ and $\mathbf{24}_H$. We have shown that the scalar leptoquark Φ_b has to be light in order to achieve unification in agreement with all experimental constraints. Using the constraints coming from triplet seesaw leptogenesis, the upper bound on the leptoquark mass is $M_{\Phi_b} \lesssim 10^{6-7}$ GeV. Therefore there is a hope that our scenario could be tested at the next generation of collider experiments through the production of these light leptoquarks.

We have also predicted an upper bound on the total proton decay lifetime which is $\tau_p \leq 1.4 \times 10^{36}$ years. Since at the next generation of proton decay experiments the bounds are expected to be improved by a few orders of magnitude, this minimal non-supersymmetric $SU(5)$ model will be certainly tested or ruled out.

The upper bound on the proton decay lifetime and the exciting possibility to verify the model at future collider experiments make our GUT scenario an appealing candidate for the testability of the idea of grand unification.

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APPENDIX A: TWO-LOOP GAUGE COUPLING RUNNING

The relevant two-loop equations for the running of the gauge couplings take the form

$$\mu\frac{\mathrm{d}\,\alpha_i(\mu)}{\mathrm{d}\,\mu}=\frac{b_i}{2\pi}\,\alpha_i^2(\mu)+\frac{1}{8\pi^2}\sum_{j=1}^{3}b_{ij}\,\alpha_i^2(\mu)\,\alpha_j(\mu)+\frac{1}{32\pi^3}\,\alpha_i^2(\mu)\sum_{l=U,D,E}\mathrm{Tr}\left[C_{il}Y_l^\dagger Y_l\right]. \qquad \text{(A1)}$$

The general formula for b_i and b_{ij} coefficients is given in [31]. Besides the well-known SM coefficients we have:

$$b_i^{\Sigma_8(\Sigma_3)}=\begin{pmatrix}0\\ 0\,(\frac{1}{3})\\ \frac{1}{2}\,(0)\end{pmatrix},\qquad b_i^{\Phi_b}=\begin{pmatrix}\frac{1}{30}\\ \frac{1}{2}\\ \frac{1}{3}\end{pmatrix},\qquad b_i^{\Phi_a}=\begin{pmatrix}\frac{3}{5}\\ \frac{2}{3}\\ 0\end{pmatrix},$$

$$b_{ij}^{\Sigma_8(\Sigma_3)}=\begin{pmatrix}0 & 0 & 0\\ 0 & 0\,(\frac{28}{3}) & 0\\ 0 & 0 & 21\,(0)\end{pmatrix},\quad b_{ij}^{\Phi_b}=\begin{pmatrix}\frac{1}{150} & \frac{3}{10} & \frac{8}{15}\\ \frac{1}{10} & \frac{13}{2} & 8\\ \frac{1}{15} & 3 & \frac{22}{3}\end{pmatrix},\quad b_{ij}^{\Phi_a}=\begin{pmatrix}\frac{108}{25} & \frac{72}{5} & 0\\ \frac{24}{5} & \frac{56}{3} & 0\\ 0 & 0 & 0\end{pmatrix},$$

which we incorporate at the appropriate scales. The C_{il} coefficients are [32]:

$$C_{il}=\begin{pmatrix}\frac{17}{10} & \frac{1}{2} & \frac{3}{2}\\ \frac{3}{2} & \frac{3}{2} & \frac{1}{2}\\ 2 & 2 & 0\end{pmatrix}.$$

To insure the proper inclusion of boundary conditions [33] at M_{GUT} we set $\alpha_i^{-1}\big|_{GUT}=\alpha_{GUT}^{-1}-\lambda_i/(12\pi)$, where $\{\lambda_1,\lambda_2,\lambda_3\}=\{5,3,2\}$. The one-loop equations for the Yukawa couplings can be found, for example, in Ref. [32].

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