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**ISAPP**

*Valencia, 2008*

## **Neutrino Oscillation Phenomenology - I**

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CERN

# *Outline*

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## Lecture 1:

- Neutrino oscillations  
oscillations in vacuum and matter
- Present neutrino oscillation experiments  
solar, atmospheric, reactor, accelerator

## Lecture 2:

- Global three flavour analysis  
discussion of three flavour effects  
summary of present status and open questions
- the LSND puzzle and recent MiniBooNE results

# *The Standard Model*

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Flavours:	1	2	3	
Quarks:	$u$ $d$	$c$ $s$	$t$ $b$	
Leptons:		$\nu_e$ $e$	$\nu_\mu$ $\mu$	$\nu_\tau$ $\tau$

The Fermions in the Standard Model come in three generations (“Flavours”)

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The Fermions in the Standard Model come in three generations (“Flavours”)

Neutrinos are the “partners” of the charged leptons  
(precisely: form a doublet under the SU(2) gauge symmetry)

# *Flavour neutrinos*

---

A neutrino of flavour  $\alpha$  is **defined** by the charged current interaction with the corresponding charged lepton:

$$\mathcal{L}_{\text{CC}} = -\frac{g}{\sqrt{2}} W^\rho \sum_{\alpha=e,\mu,\tau} \bar{\nu}_{\alpha L} \gamma_\rho \ell_{\alpha L} + \text{h.c.}$$

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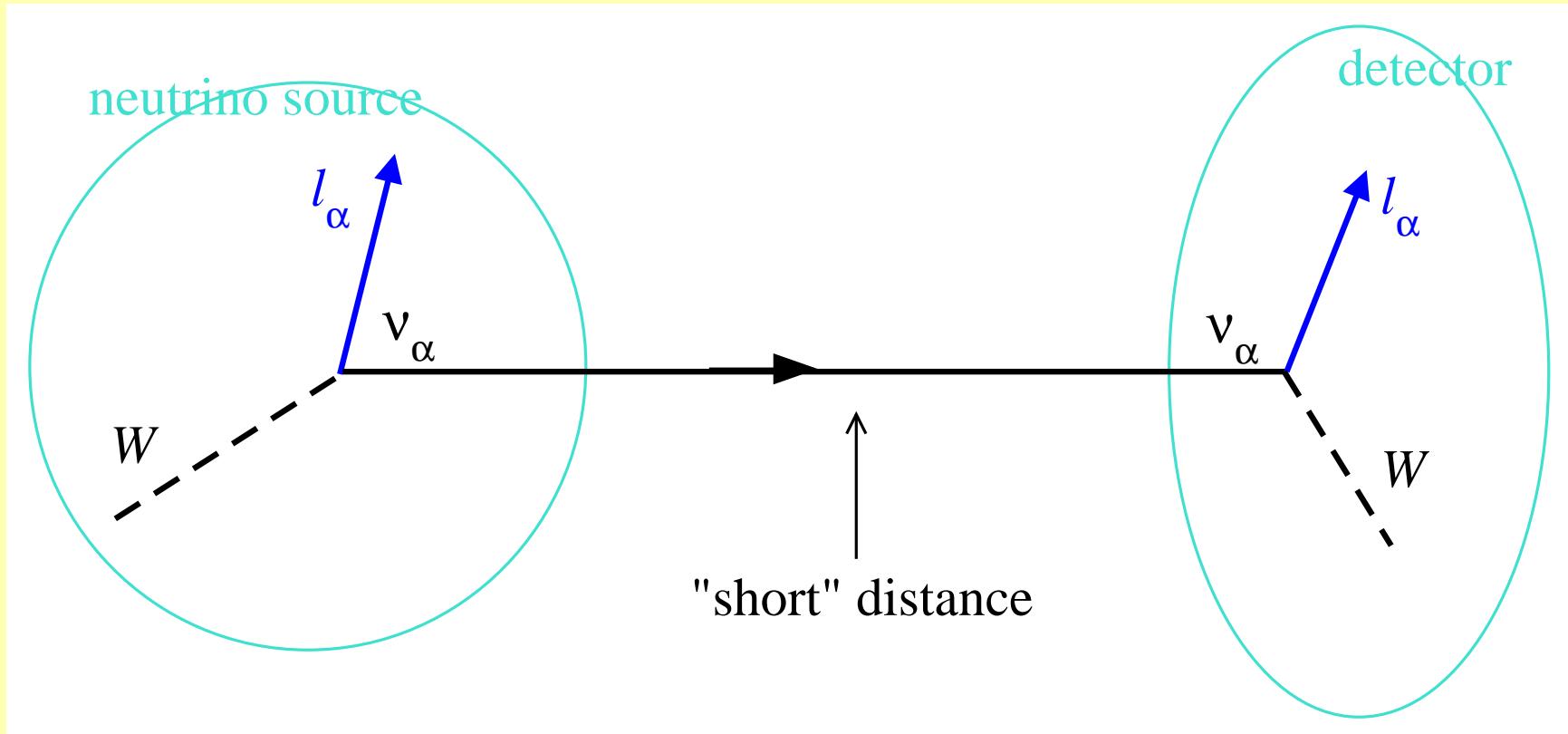
for example

$$\pi^+ \rightarrow \mu^+ \nu_\mu$$

the muon neutrino  $\nu_\mu$  comes together with the charged muon  $\mu^+$

# Flavour neutrinos

---



# *Let's give mass to the neutrinos*

---

Majorana mass term:

$$\mathcal{L}_M = -\frac{1}{2} \sum_{\alpha, \beta = e, \mu, \tau} \nu_{\alpha L}^T C^{-1} \mathcal{M}_{\alpha \beta} \nu_{\beta L} + \text{h.c.}$$

$\mathcal{M}$ : symmetric mass matrix

In the basis where the CC interaction is diagonal the mass matrix is in general not a diagonal matrix

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In the basis where the CC interaction is diagonal the mass matrix is in general not a diagonal matrix

any complex symmetric matrix  $\mathcal{M}$  can be diagonalised by a unitary matrix as

$$U_\nu^T \mathcal{M} U_\nu = m, \quad m : \text{diagonal}, \quad m_i \geq 0$$

# Lepton mixing

---

$$\mathcal{L}_{\text{CC}} = -\frac{g}{\sqrt{2}} W^\rho \sum_{\alpha=e,\mu,\tau} \sum_{i=1}^3 \bar{\nu}_{iL} U_{\alpha i}^* \gamma_\rho \ell_{\alpha L} + \text{h.c.}$$

$$\mathcal{L}_{\text{M}} = -\frac{1}{2} \sum_{i=1}^3 \nu_{iL}^T C^{-1} \nu_{iL} m_i^\nu - \sum_{\alpha=e,\mu,\tau} \bar{\ell}_{\alpha R} \ell_{\alpha L} m_\alpha^\ell + \text{h.c.}$$

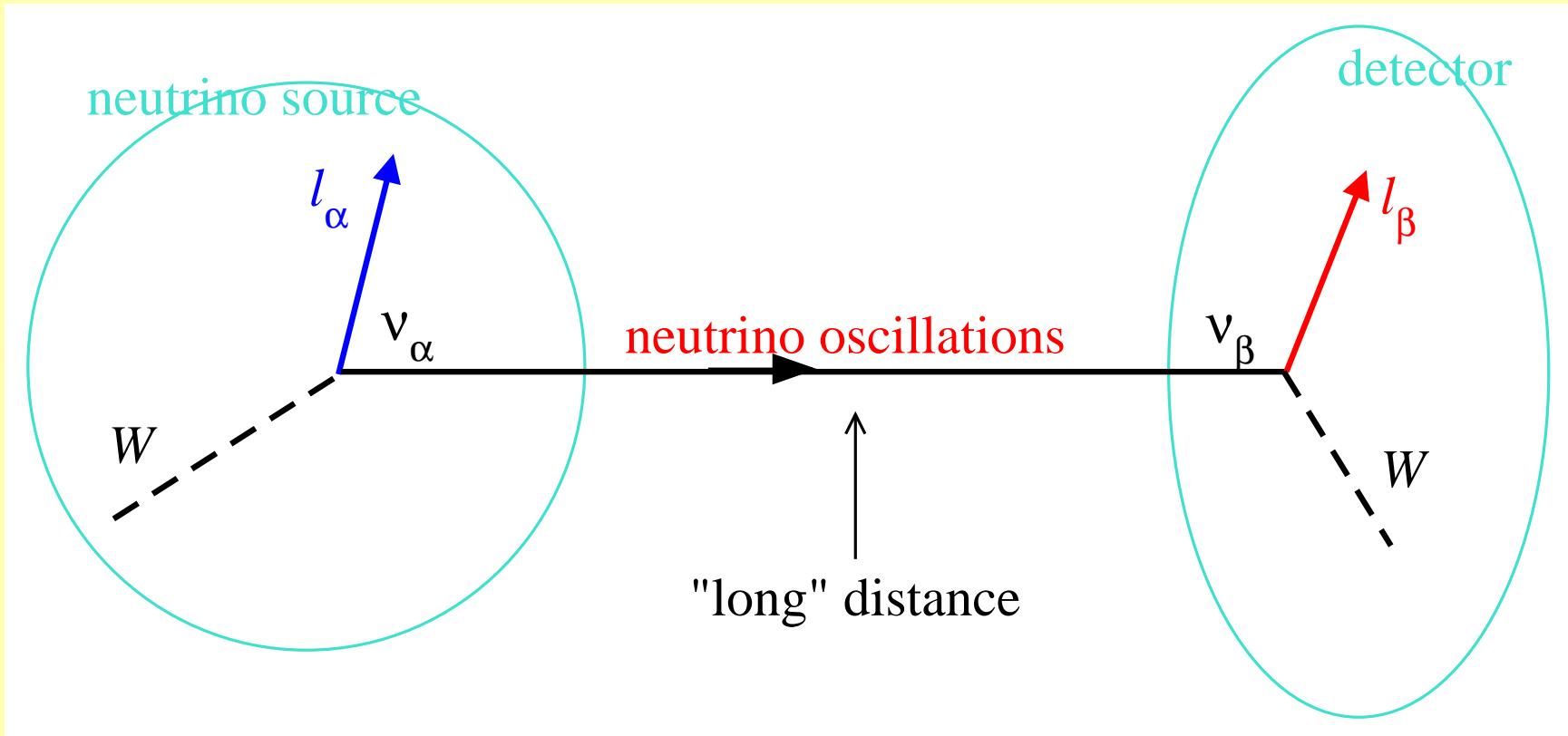
The unitary lepton mixing matrix:

$$(U_{\alpha i}) \equiv U_{\text{PMNS}} = V^{\text{Dirac}} D^{\text{Maj}}$$

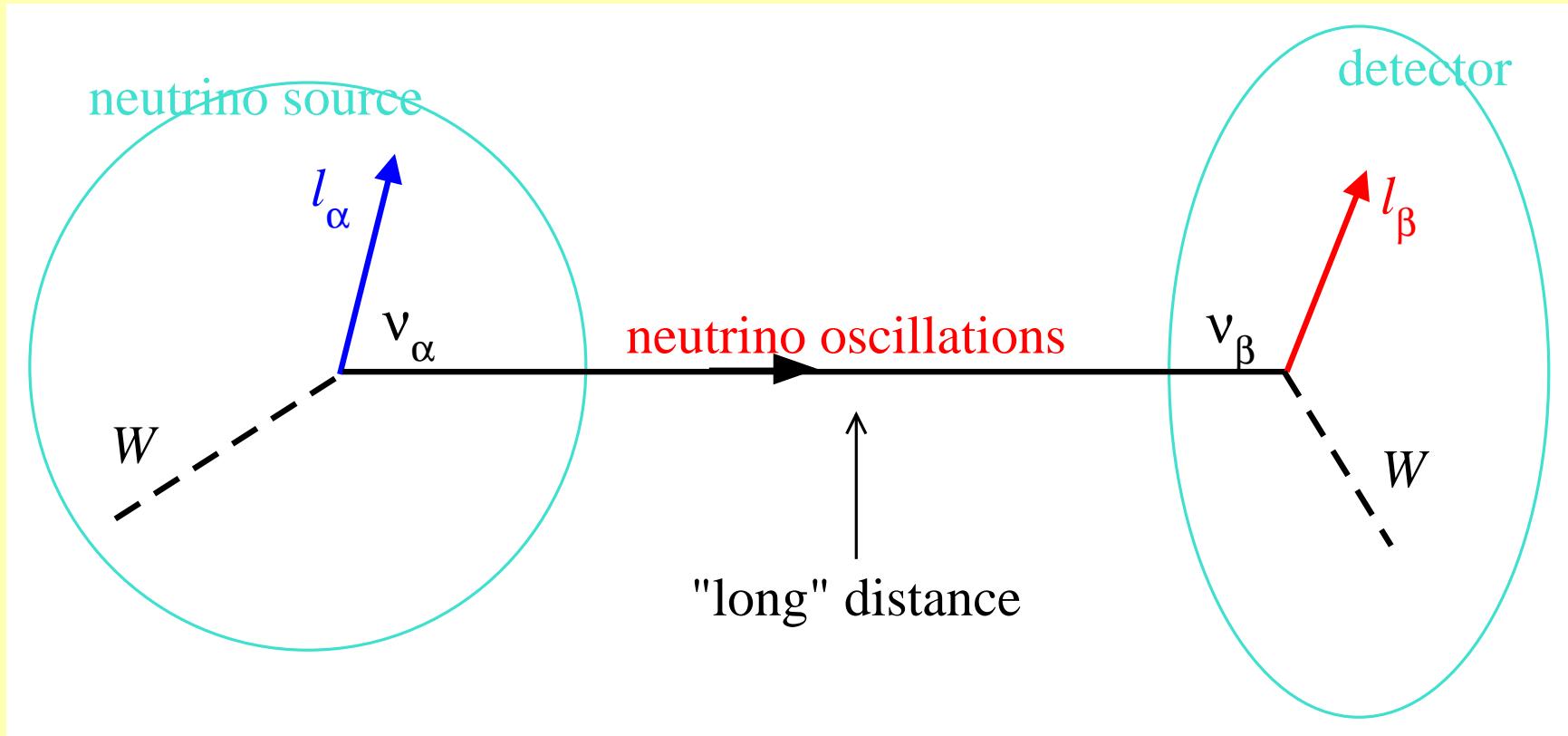
$$D^{\text{Maj}} = \text{diag}(e^{i\alpha_i/2})$$

# *Neutrino oscillations*

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# Neutrino oscillations



$$|\nu_\alpha\rangle = U_{\alpha i}^* |\nu_i\rangle$$

$$e^{-i(Et - p_i x)}$$

$$|\nu_\beta\rangle = U_{\beta i}^* |\nu_i\rangle$$

propagating states are states with definite mass

# *Neutrino oscillations (in vacuum)*

---

oscillation amplitude:

$$\begin{aligned}\mathcal{A}_{\nu_\alpha \rightarrow \nu_\beta} &= \langle \nu_\beta | \text{propagation} | \nu_\alpha \rangle \\ &= \sum_{i,j} \langle \nu_j | U_{\beta j} e^{-i(Et - p_i x)} U_{\alpha i}^* | \nu_i \rangle\end{aligned}$$

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$$Et - p_i x = Et - \sqrt{E^2 - m_i^2} x \approx Et - Ex + \frac{m_i^2 x}{2E}$$

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oscillation probability:

$$P_{\nu_\alpha \rightarrow \nu_\beta} = |\mathcal{A}_{\nu_\alpha \rightarrow \nu_\beta}|^2$$

# *The oscillation probability in vacuum*

---

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L) = \sum_{jk} U_{\alpha j} U_{\beta j}^* U_{\alpha k}^* U_{\beta k} \exp \left[ -i \frac{\Delta m_{kj}^2 L}{2 E_\nu} \right]$$

$\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$ : oscillations are sensitive only to mass-squared differences (not to absolute mass!)

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to observe oscillations one needs

- non-trivial mixing  $U_{\alpha i}$
- non-zero mass-squared differences  $\Delta m_{kj}^2$
- a suitable value for  $L/E_\nu$

# *The oscillation phase*

---

$$\phi = \frac{\Delta m^2 L}{4E_\nu} = 1.27 \frac{\Delta m^2 [\text{eV}^2] L [\text{km}]}{E_\nu [\text{GeV}]}$$

# The oscillation phase

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$$\phi = \frac{\Delta m^2 L}{4E_\nu} = 1.27 \frac{\Delta m^2 [\text{eV}^2] L [\text{km}]}{E_\nu [\text{GeV}]}$$

- “short” distance:  $\phi \ll 1$ : no oscillations can develop and  $P_{\nu_\alpha \rightarrow \nu_\beta} = \delta_{\alpha\beta}$  because of  $\sum_j U_{\alpha j} U_{\beta j}^* = \delta_{\alpha\beta}$ .
- “long” distance:  $\phi \gtrsim \pi/2$ : oscillations are observable
- “very long” distance:  $\phi \gg 2\pi$ : oscillations are averaged out (indep. of  $L$  and  $E_\nu$ ):

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \sum_j |U_{\alpha j}|^2 |U_{\beta j}|^2$$

# *2-neutrino oscillations*

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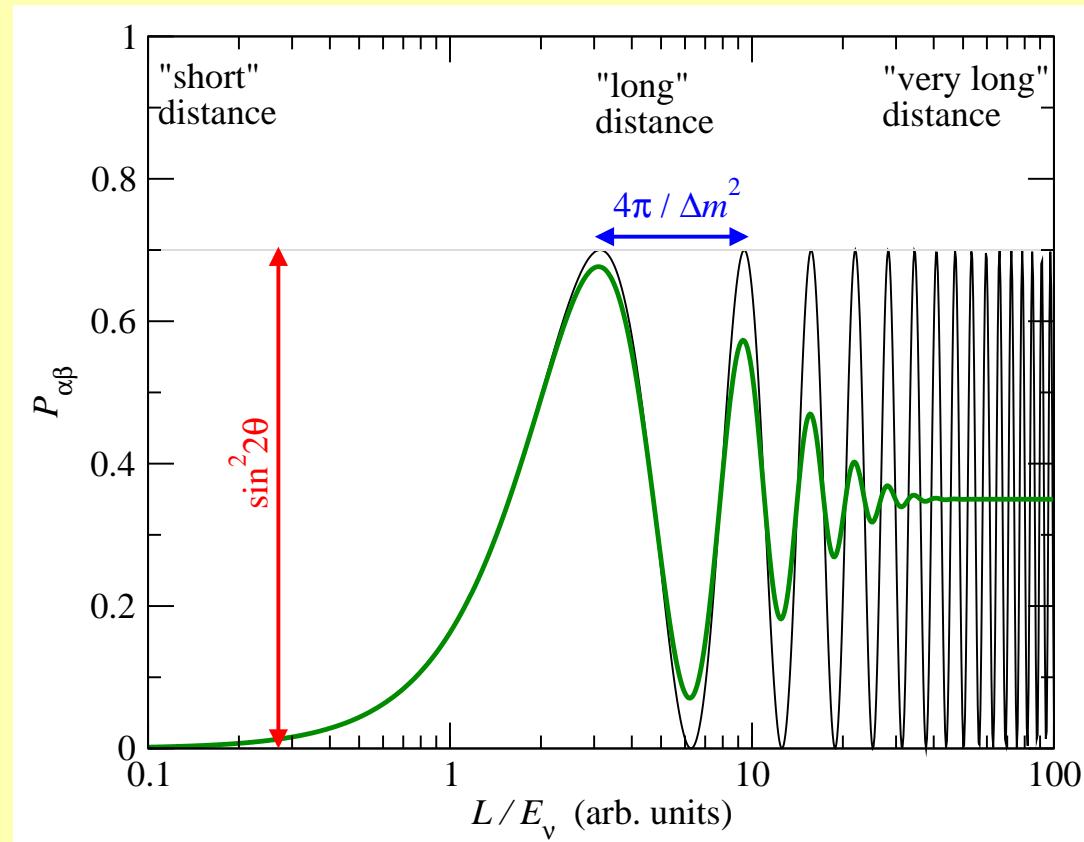
Two-flavour limit:

$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \quad P = \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E_\nu}$$

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# *Appearance vs. disappearance*

---

- appearance experiments:

$$P_{\nu_\alpha \rightarrow \nu_\beta} , \quad \alpha \neq \beta$$

“appearance” of a neutrino of a new flavour  $\beta \neq \alpha$   
in a beam of  $\nu_\alpha$

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- disappearance experiments:

$$P_{\nu_\alpha \rightarrow \nu_\alpha}$$

measurement of the “survival” probability of a neutrino of given flavour

# *General properties of vacuum oscillations*

---

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \sum_{jk} U_{\alpha j} U_{\beta j}^* U_{\alpha k}^* U_{\beta k} \exp \left[ -i \frac{\Delta m_{kj}^2 L}{2 E_\nu} \right]$$

- Unitarity:  $\sum_\beta P_{\nu_\alpha \rightarrow \nu_\beta} = 1$

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- Phases in  $U$  induce CP violation:  $P_{\nu_\alpha \rightarrow \nu_\beta} \neq P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta}$
- there is no CP violation in disappearance experiments:

$$P_{\nu_\alpha \rightarrow \nu_\alpha} = P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\alpha} = \sum_{k,j} |U_{\alpha k}|^2 |U_{\alpha j}|^2 e^{-i \Delta m_{kj}^2 L / 2E}$$

(but  $P_{\alpha\alpha}$  may still depend on  $\cos \delta, \sin^2 \delta, \dots$ )

# *Eff. Schrödinger equation*

---

The evolution of the flavour state can be described by an effective Schrödinger equation:

$$i \frac{d}{dt} \begin{pmatrix} a_e \\ a_\mu \\ a_\tau \end{pmatrix} = H_{\text{vac}} \begin{pmatrix} a_e \\ a_\mu \\ a_\tau \end{pmatrix}$$

where

$$H_{\text{vac}}^\nu = U \mathbf{diag} \left( 0, \frac{\Delta m_{21}^2}{2E_\nu}, \frac{\Delta m_{31}^2}{2E_\nu} \right) U^\dagger$$

$$H_{\text{vac}}^{\bar{\nu}} = U^* \mathbf{diag} \left( 0, \frac{\Delta m_{21}^2}{2E_\nu}, \frac{\Delta m_{31}^2}{2E_\nu} \right) U^T$$

---

# **Neutrino oscillations in matter**

# *The matter effect*

---

When neutrinos pass through matter the interactions with the particles in the background induce an effective potential for the neutrinos

The coherent forward scattering amplitude leads to an index of refraction for neutrinos

L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); *ibid.* D **20**, 2634 (1979)

# *Effective Hamiltonian in matter*

---

$$H_{\text{mat}}^\nu = U \mathbf{\text{diag}} \left( 0, \frac{\Delta m_{21}^2}{2E_\nu}, \frac{\Delta m_{31}^2}{2E_\nu} \right) U^\dagger + \mathbf{\text{diag}}(\sqrt{2}G_F N_e, 0, 0)$$
$$H_{\text{mat}}^{\bar{\nu}} = \underbrace{U^* \mathbf{\text{diag}} \left( 0, \frac{\Delta m_{21}^2}{2E_\nu}, \frac{\Delta m_{31}^2}{2E_\nu} \right) U^T}_{H_{\text{vac}}} - \underbrace{\mathbf{\text{diag}}(\sqrt{2}G_F N_e, 0, 0)}_{V_{\text{mat}}}$$

$N_e(x)$ : electron density along the neutrino path

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$N_e(x)$ : electron density along the neutrino path

for non-constant matter the Hamiltonian depends on time:

$$i \frac{d}{dt} a = H_{\text{mat}}(t) a$$

# *Effective Hamiltonian in matter*

---

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$N_e(x)$ : electron density along the neutrino path

Remember:  $U = V^{\text{Dirac}} D^{\text{Maj}}$

⇒ Majorana phases do not show up in oscillations

# *Effective matter potential - 1*

---

Effective 4-point interaction Hamiltonian in the SM

$$H_{\text{int}}^{\nu_\alpha} = \frac{G_F}{\sqrt{2}} \bar{\nu}_\alpha \gamma_\mu (1 - \gamma_5) \nu_\alpha \underbrace{\sum_f \bar{f} \gamma^\mu (g_V^{\alpha,f} - g_A^{\alpha,f} \gamma_5) f}_{J_{\text{mat}}^\mu}$$

# *Effective matter potential - 1*

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ordinary matter:  $e^-$ ,  $p$ ,  $n$

non-relativistic, unpolarised, neutral

$$\langle \bar{f} \gamma^\mu f \rangle = \frac{1}{2} N_f \delta_{\mu 0} , \quad \langle \bar{f} \gamma_5 \gamma^\mu f \rangle = 0 , \quad N_e = N_p$$

# *Effective matter potential - 2*

---

$$\begin{aligned} J_{\text{mat}}^{\mu} &= \frac{1}{2} \delta_{\mu 0} \sum_{f=e,p,n} N_f g_V^{\alpha,f} \\ &= \frac{1}{2} \delta_{\mu 0} [N_e (g_V^{\alpha,e} + g_V^{\alpha,p}) + N_n g_V^{\alpha,n}] \end{aligned}$$

# Effective matter potential - 2

---

$$\begin{aligned}
 J_{\text{mat}}^\mu &= \frac{1}{2} \delta_{\mu 0} \sum_{f=e,p,n} N_f g_V^{\alpha,f} \\
 &= \frac{1}{2} \delta_{\mu 0} [N_e (g_V^{\alpha,e} + g_V^{\alpha,p}) + N_n g_V^{\alpha,n}]
 \end{aligned}$$

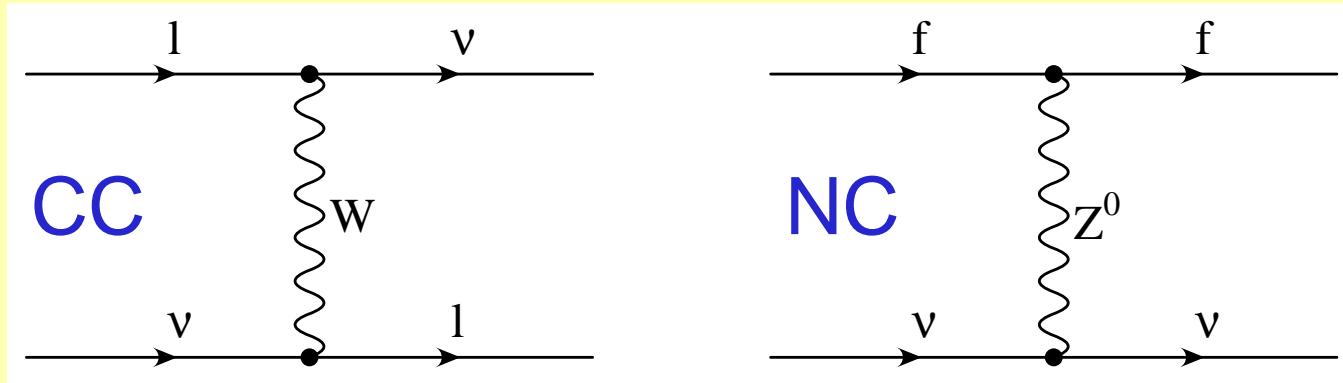
$g_V$	$e^-$	$p$	$n$
$\nu_e$	$2 \sin^2 \Theta_W + \frac{1}{2}$	$-2 \sin^2 \Theta_W + \frac{1}{2}$	$-\frac{1}{2}$
$\nu_{\mu,\tau}$	$2 \sin^2 \Theta_W - \frac{1}{2}$	$-2 \sin^2 \Theta_W + \frac{1}{2}$	$-\frac{1}{2}$

$$\Rightarrow V_{\text{mat}} \propto \left( N_e - \frac{1}{2} N_n, -\frac{1}{2} N_n, -\frac{1}{2} N_n \right)$$

# *Effective matter potential - 3*

---

$$V_{\text{mat}} = \sqrt{2}G_F \text{diag} (N_e - N_n/2, -N_n/2, -N_n/2)$$



- only  $\nu_e$  feel CC (there are no  $\mu, \tau$  in normal matter)
- NC is the same for all flavours  $\Rightarrow$  potential proportional to identity has no effect on the evolution
- NC has no effect for 3-flavour active neutrinos, but is important in the presence of sterile neutrinos

# *Neutrino oscillations in constant matter*

---

diagonalize the Hamiltonian in matter:

$$\begin{aligned} H_{\text{mat}}^{\nu} &= U \text{diag} \left( 0, \frac{\Delta m_{21}^2}{2E_{\nu}}, \frac{\Delta m_{31}^2}{2E_{\nu}} \right) U^{\dagger} + \text{diag}(\sqrt{2}G_F N_e, 0, 0) \\ &= U_m \text{diag}(\lambda_1, \lambda_2, \lambda_3) U_m^{\dagger} \end{aligned}$$

Same expression for oscillation probability, but  
replace “vacuum” parameters by “matter” parameters

# *2-neutrino oscillations in constant matter*

---

Two-flavour case:

$$P_{\text{mat}} = \sin^2 2\theta_{\text{mat}} \sin^2 \frac{\Delta m_{\text{mat}}^2 L}{4E}$$

with

$$\sin^2 2\theta_{\text{mat}} = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

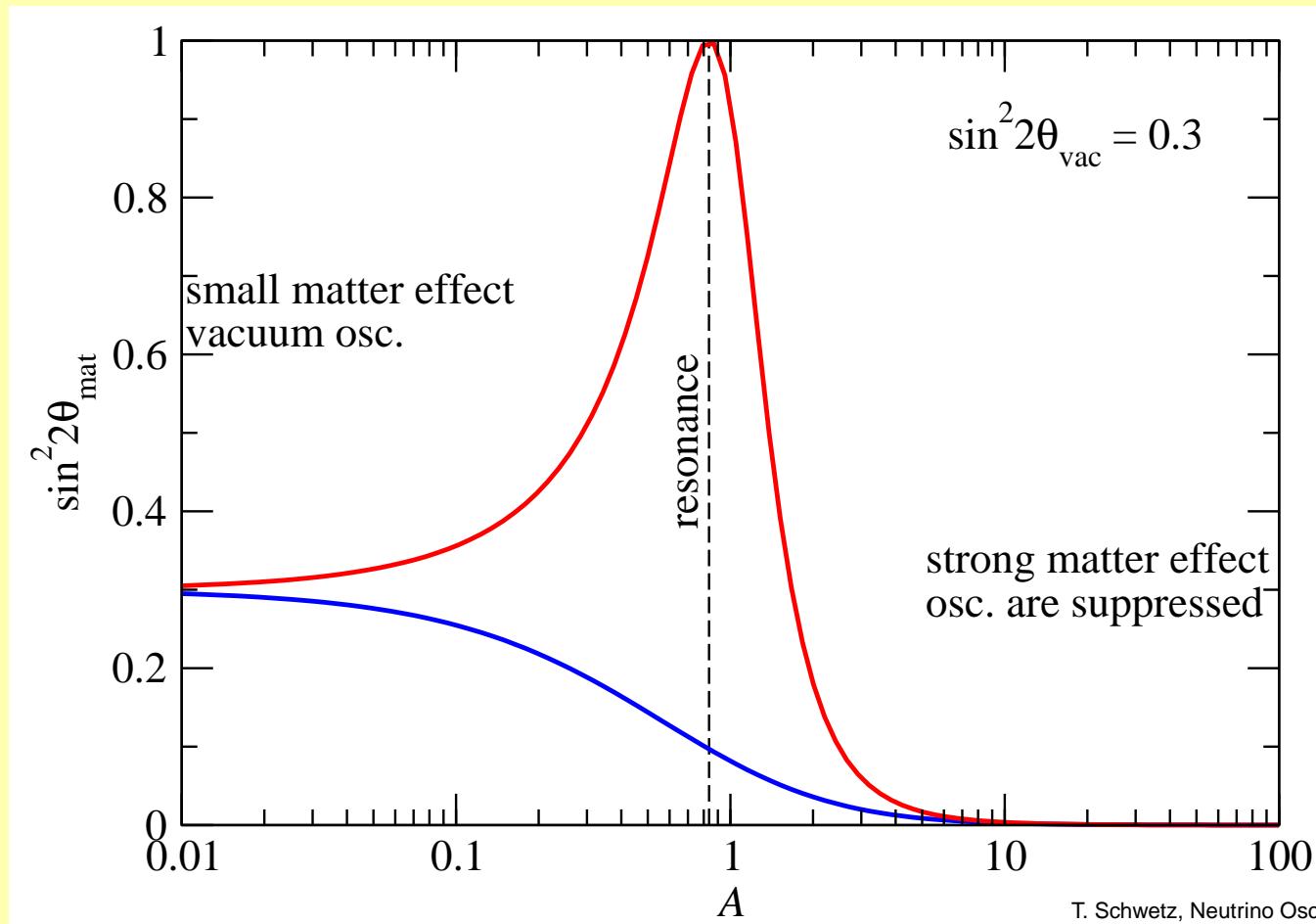
$$\Delta m_{\text{mat}}^2 = \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

$$A \equiv \frac{2EV}{\Delta m^2}$$

# 2-neutrino oscillations in constant matter

$$\sin^2 2\theta_{\text{mat}} = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2} \quad A \equiv \frac{2EV}{\Delta m^2}$$

resonance for  $\cos 2\theta = A$ : “MSW resonance”



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# **Evidences for neutrino oscillations**

# *Neutrino oscillation experiments*

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natural neutrino sources:

- solar neutrinos
  - Homestake, SAGE+GNO, Super-K, SNO, Borexino
- atmospheric neutrinos
  - Super-Kamiokande

artificial neutrino sources:

- reactor neutrinos
  - Chooz (1 km), KamLAND (180 km)
- long-baseline accelerator experiments
  - K2K (250 km), MINOS (735 km)

# 3-flavour oscillation parameters

---

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\Delta m_{31}^2$	$\Delta m_{21}^2$
atmospheric+LBL	Chooz
solar+KamLAND	

# *3-flavour oscillation parameters*

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atmospheric+LBL      
 Chooz      
 solar+KamLAND

3-flavour effects are suppressed because

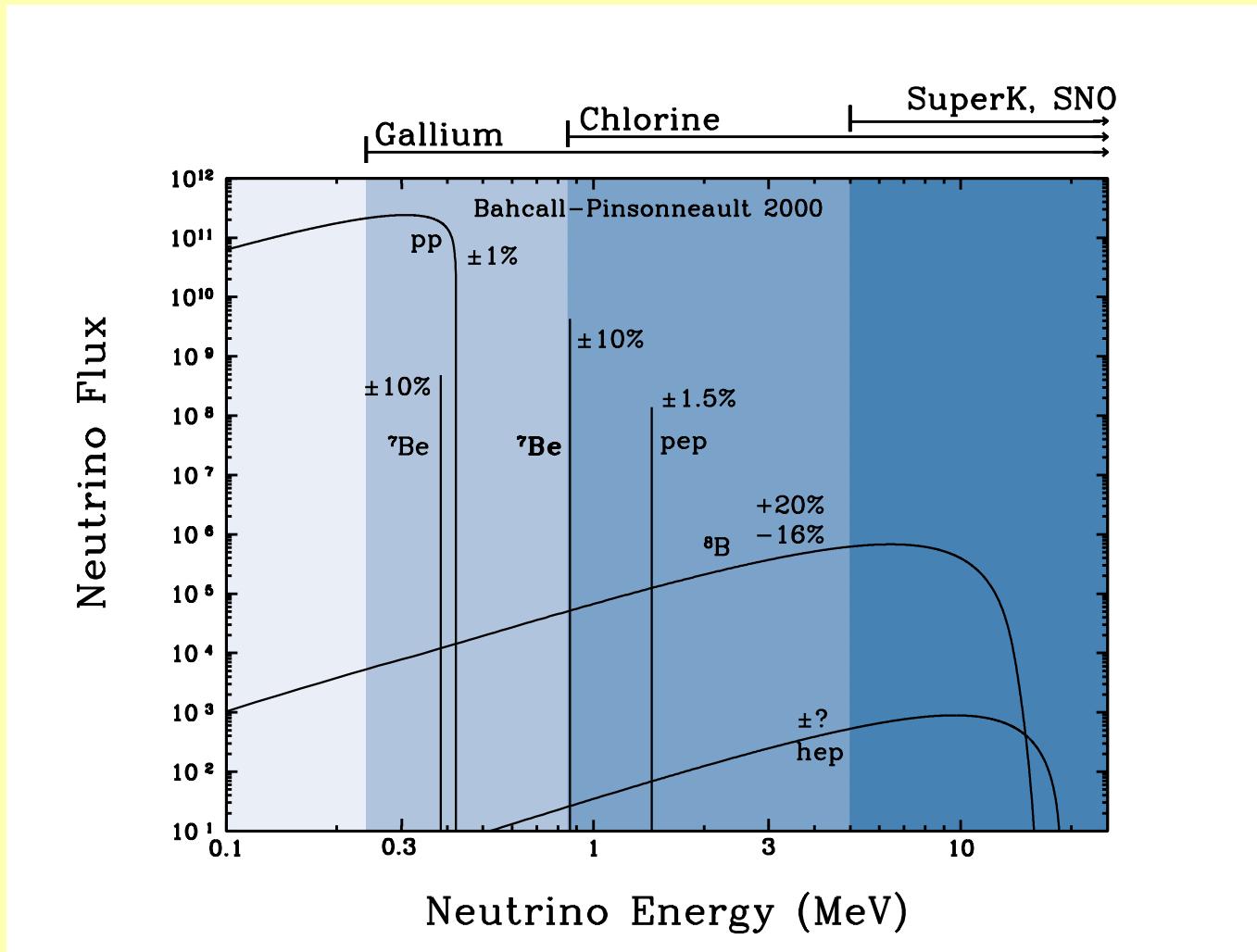
$$\theta_{13} \ll 1 \text{ und } \Delta m^2_{21} \ll \Delta m^2_{31}$$

⇒ dominant oscillations are well described by effective two-flavour oscillations

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## **Solar neutrinos and the parameters $\Delta m_{21}^2$ , $\theta_{12}$**

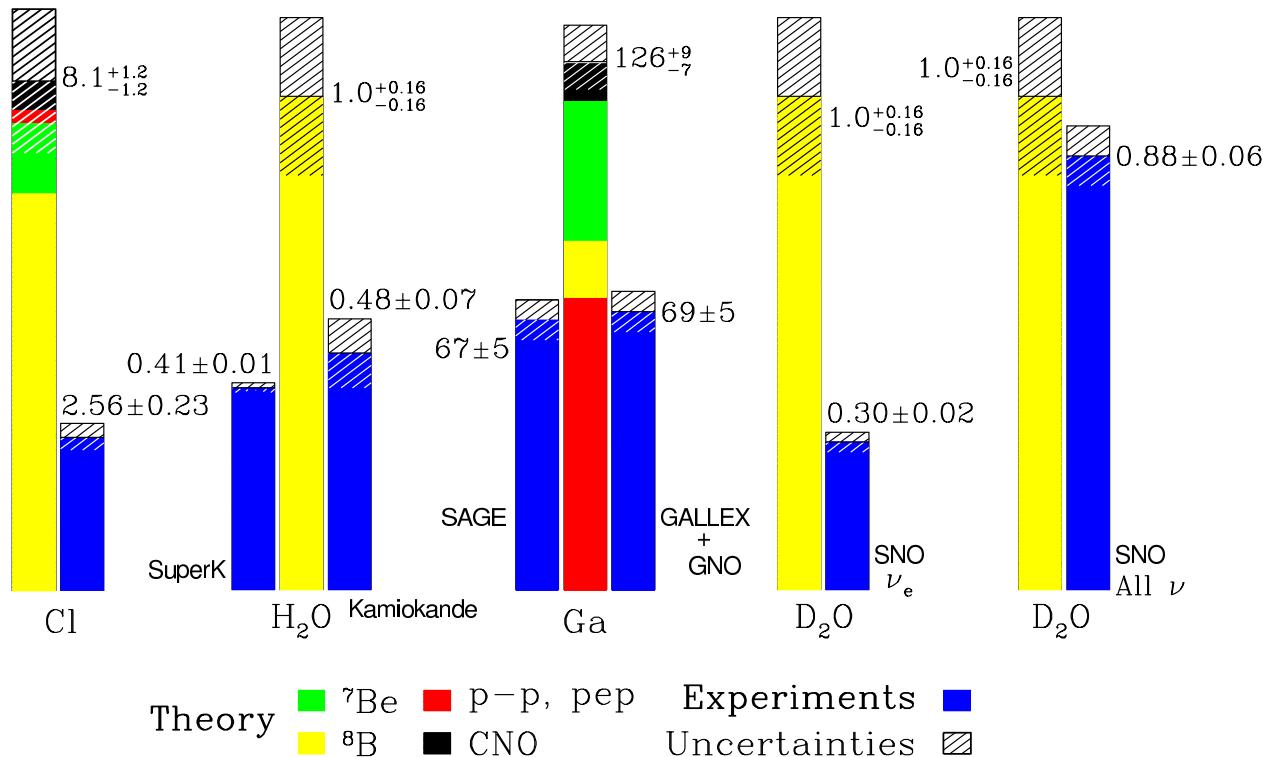
# *The solar neutrino flux*



# Solar neutrino experiments

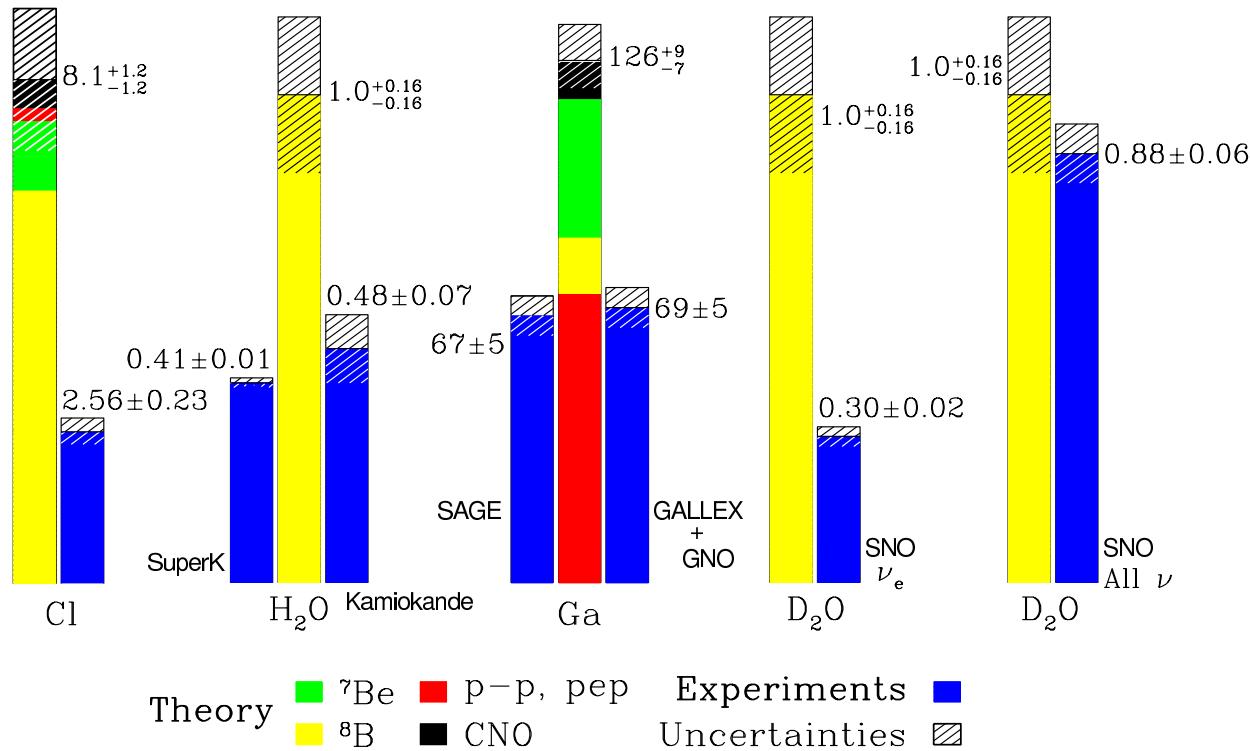
## summary of solar neutrino experiments

Total Rates: Standard Model vs. Experiment  
Bahcall–Serenelli 2005 [BS05(OP)]



# Solar neutrino experiments

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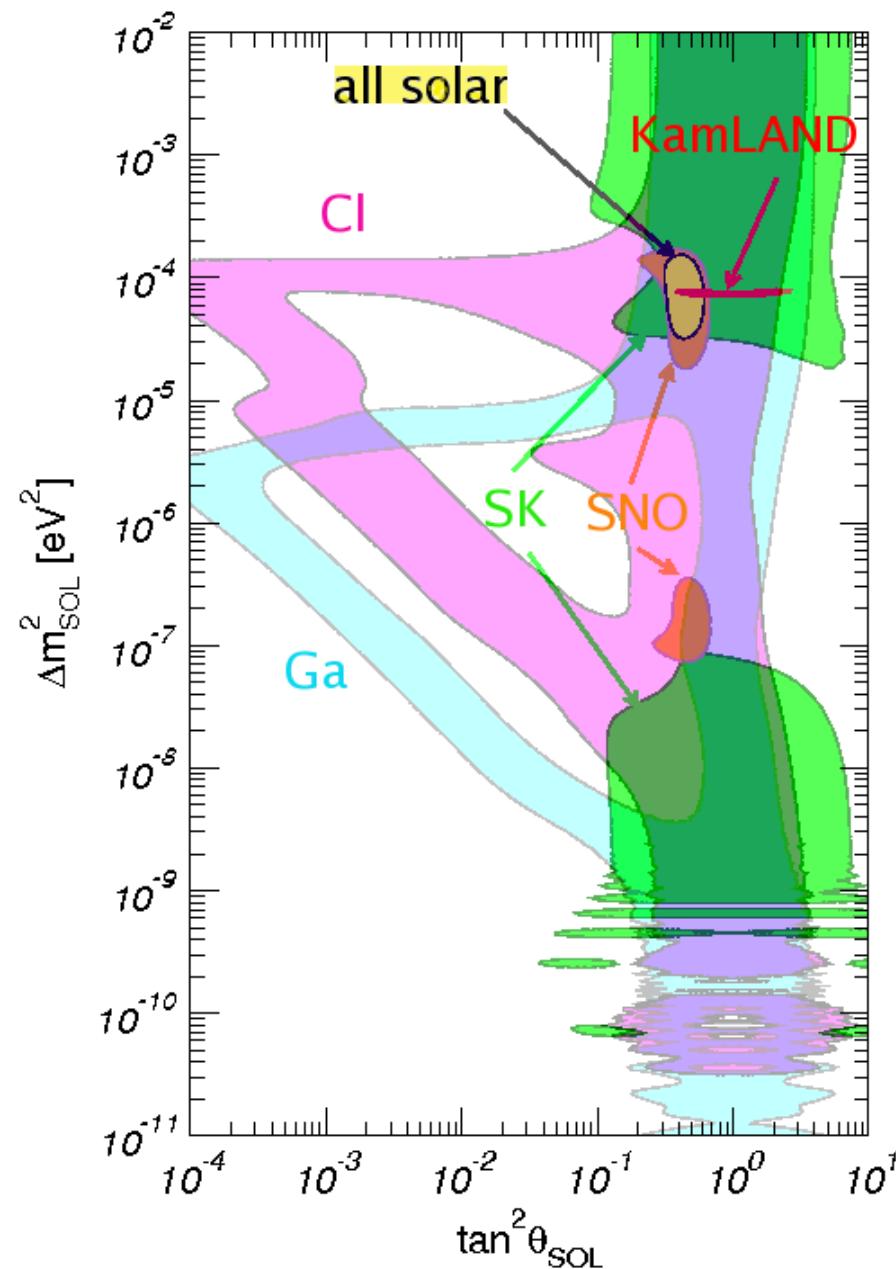


**SNO:**  $\nu_e + d \rightarrow p + p + e^-$        $\frac{\phi_{CC}}{\phi_{NC}} = 0.301 \pm 0.033$

$$\nu_x + d \rightarrow p + n + \nu_x$$

7 $\sigma$  evidence for a non-zero  $\nu_{\mu,\tau}$  flux from the sun

# 'Solar' parameters



# *Probing solar properties with neutrinos*

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Neutrinos as messengers from the center of the sun:

$$\text{boron-8 neutrino flux } \Phi \propto T^{20}$$

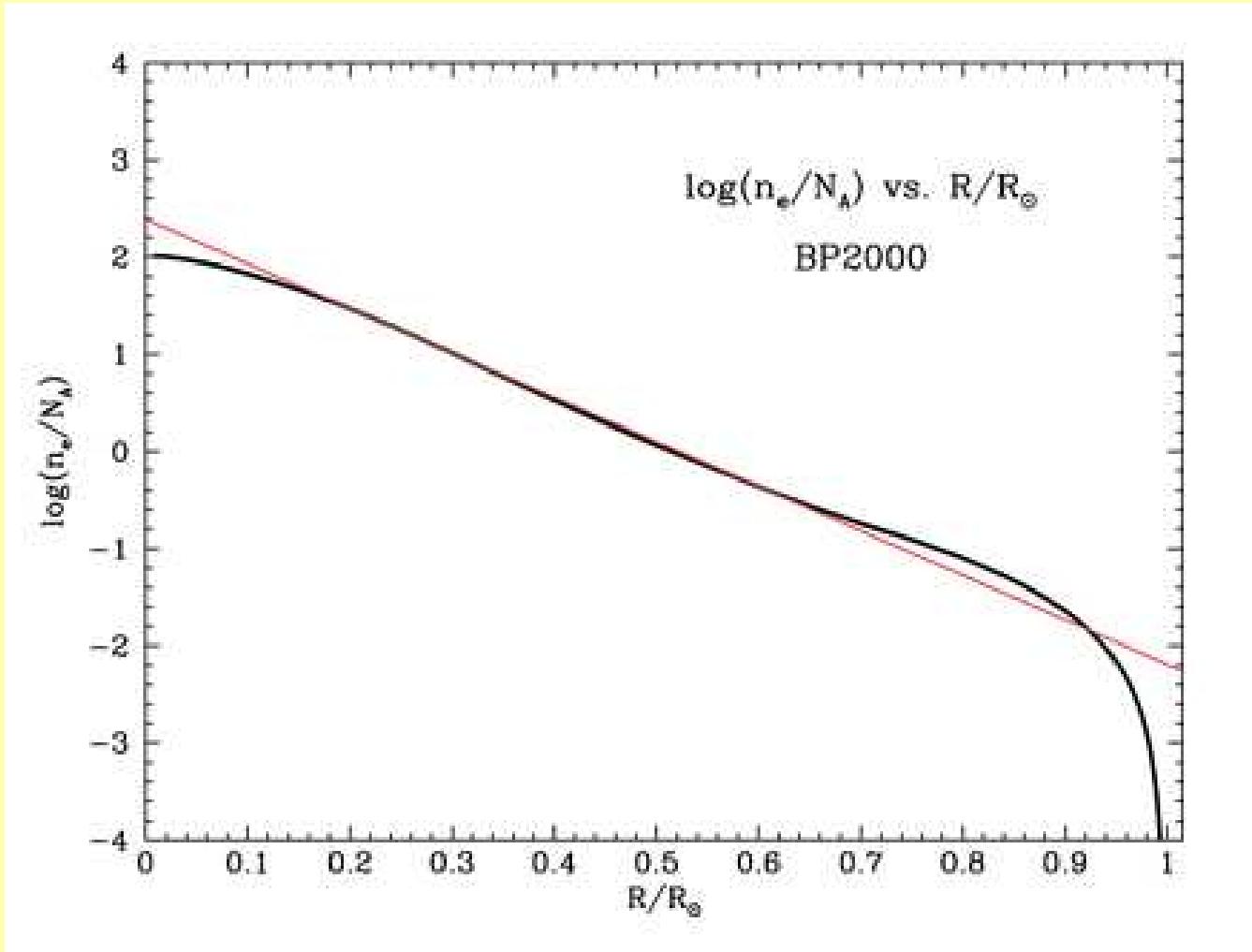
where  $T$  is the temperature at the center of the sun

the measurement of the solar neutrino flux allows a determination of  $T$  with an accuracy of 1%:

$$T = 15.7(1 \pm 0.01) \times 10^6 \text{ K}$$

# *The electron density in the sun*

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# *The LMA-MSW mechanism*

---

evolution is adiabatic if  $\left( \frac{1}{\theta_m} \frac{d\theta_m}{dx} \right)^{-1} \gg L_{\text{osc}}$

using  $\Delta m^2 = 8 \times 10^{-5}$  eV<sup>2</sup> the oscillation length is

$$L_{\text{osc}} = \frac{4\pi E}{\Delta m^2} \simeq 30 \text{ km} \left( \frac{E}{\text{MeV}} \right)$$

for large mixing angles ( $\sin^2 \theta_{12} \simeq 0.3$ ):

$$\left( \frac{1}{\theta_m} \frac{d\theta_m}{dx} \right)^{-1} \sim \left( \frac{1}{V} \frac{dV}{dx} \right)^{-1} \sim \text{size of sun} \gg 30 \text{ km}$$

⇒ **adiabatic evolution**

# *The LMA-MSW mechanism*

---

the electron neutrino is born at the center of the sun as

$$|\nu_e\rangle = \cos\theta_m |\nu_1\rangle + \sin\theta_m |\nu_2\rangle$$

then  $|\nu_1\rangle$  and  $|\nu_2\rangle$  evolve adiabatically to the Earth

# *The LMA-MSW mechanism*

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$$P_{ee} = P_{e1}^{\text{prod}} P_{1e}^{\text{det}} + P_{e2}^{\text{prod}} P_{2e}^{\text{det}}$$

$P_{e3}^{\text{prod}} \approx \sin^2 \theta_{13} \approx 0$ , interference term averages out

$$P_{e1}^{\text{prod}} = \cos^2 \theta_m, \quad P_{1e}^{\text{det}} = \cos^2 \theta$$

$$P_{e2}^{\text{prod}} = \sin^2 \theta_m, \quad P_{2e}^{\text{det}} = \sin^2 \theta$$

$$\Rightarrow \quad P_{ee} = \cos^2 \theta_m \cos^2 \theta + \sin^2 \theta_m \sin^2 \theta$$

# *The LMA-MSW mechanism*

---

in the center of the sun we have

$$A \equiv \frac{2EV}{\Delta m^2} \simeq 0.2 \left( \frac{E_\nu}{\text{MeV}} \right) \left( \frac{8 \times 10^{-5} \text{ eV}^2}{\Delta m^2} \right)$$

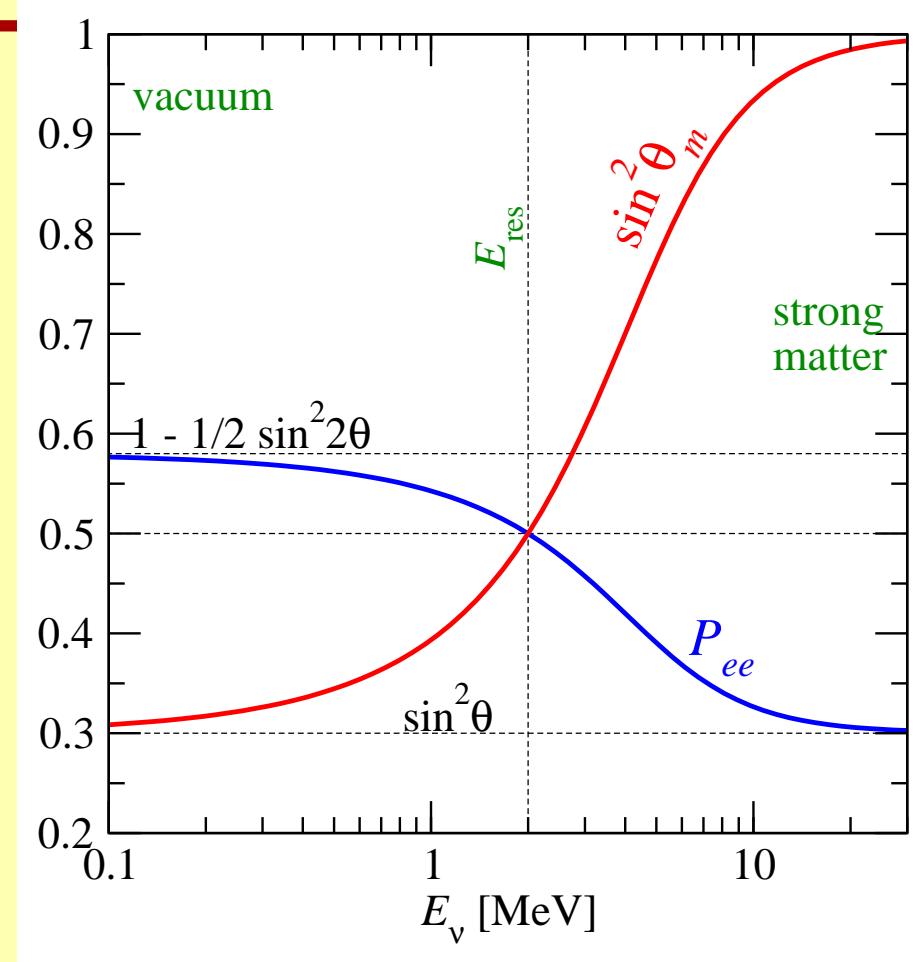
resonance occurs for

$$A = \cos 2\theta = 0.4$$

$$\Rightarrow E_{\text{res}} \simeq 2 \text{ MeV}$$

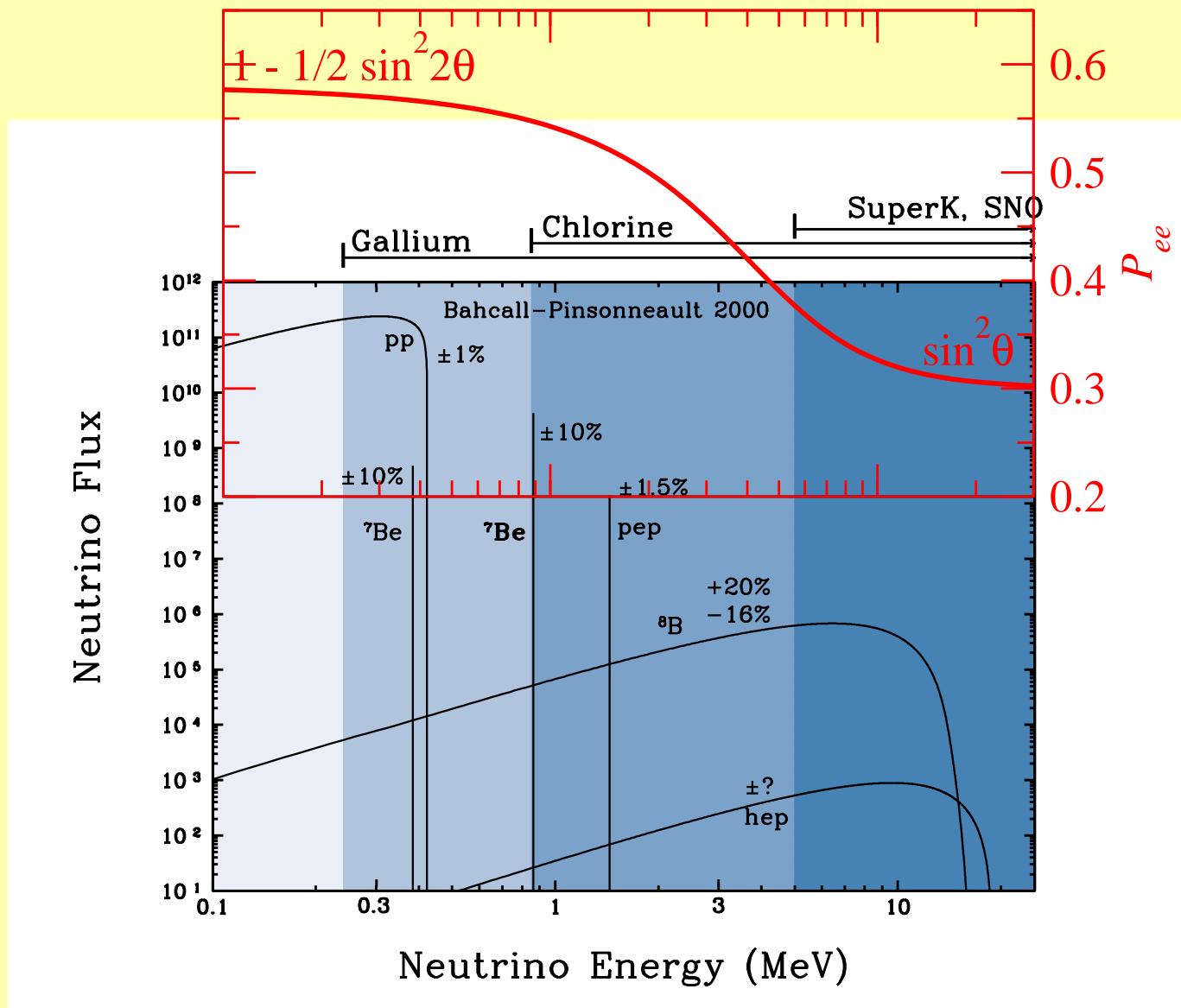
# The LMA-MSW mechanism

$$P_{ee} = \cos^2 \theta_m \cos^2 \theta + \sin^2 \theta_m \sin^2 \theta$$



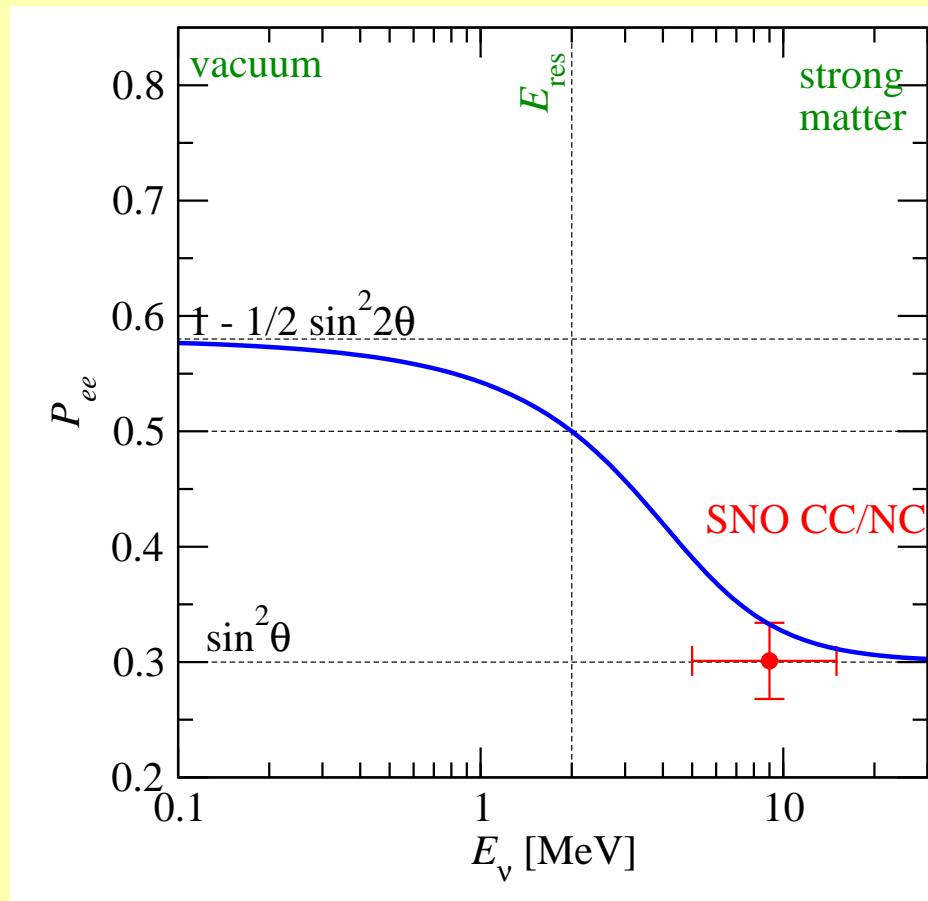
$$P_{ee} = \begin{cases} c^4 + s^4 = 1 - \frac{1}{2} \sin^2 2\theta & \text{vacuum} \quad (\theta_m = \theta) \\ \sin^2 \theta & \text{strong matter} \quad (\theta_m = \pi/2) \end{cases}$$

# The LMA-MSW mechanism



# *SNO evidence for the MSW effect*

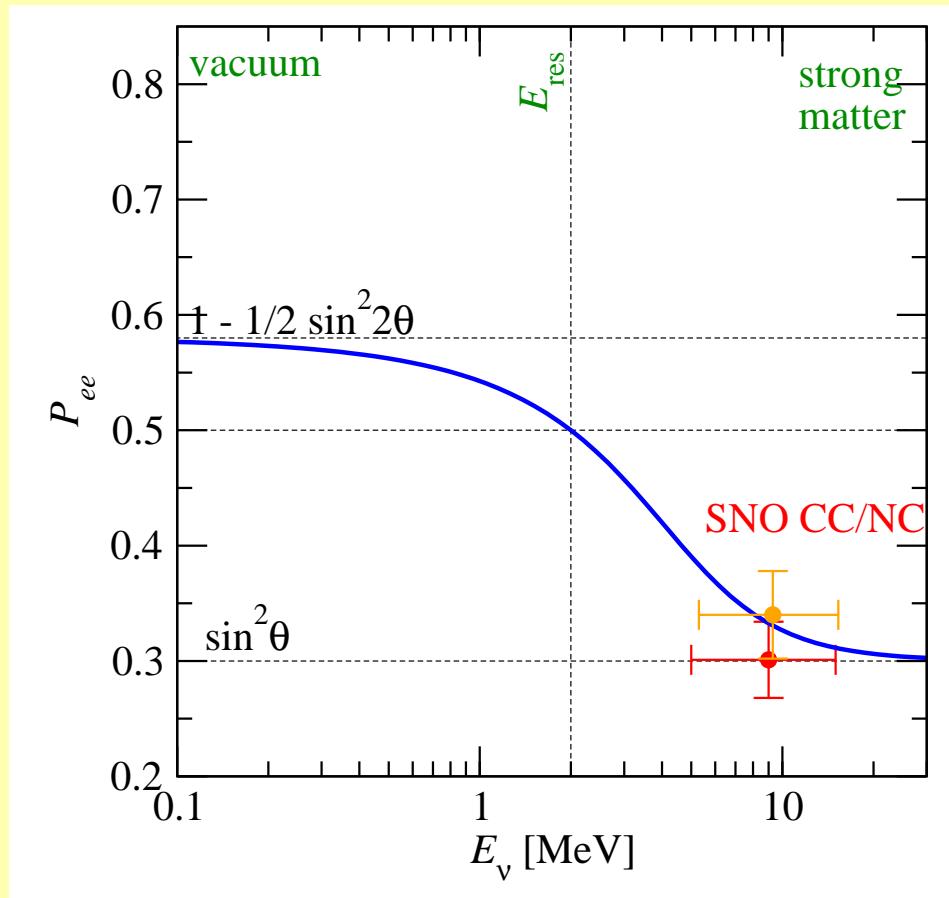
CC/NC measurement of SNO:



- constrains  $\sin^2 \theta_{12}$ :  $\phi_{CC}/\phi_{NC} \approx P_{ee}^{\text{SNO}} \approx \sin^2 \theta_{12}$
- $\phi_{CC}/\phi_{NC} < 1/2$ : evidence for matter eff. and MSW reson.

# *new CC/CN from SNO NCD phase*

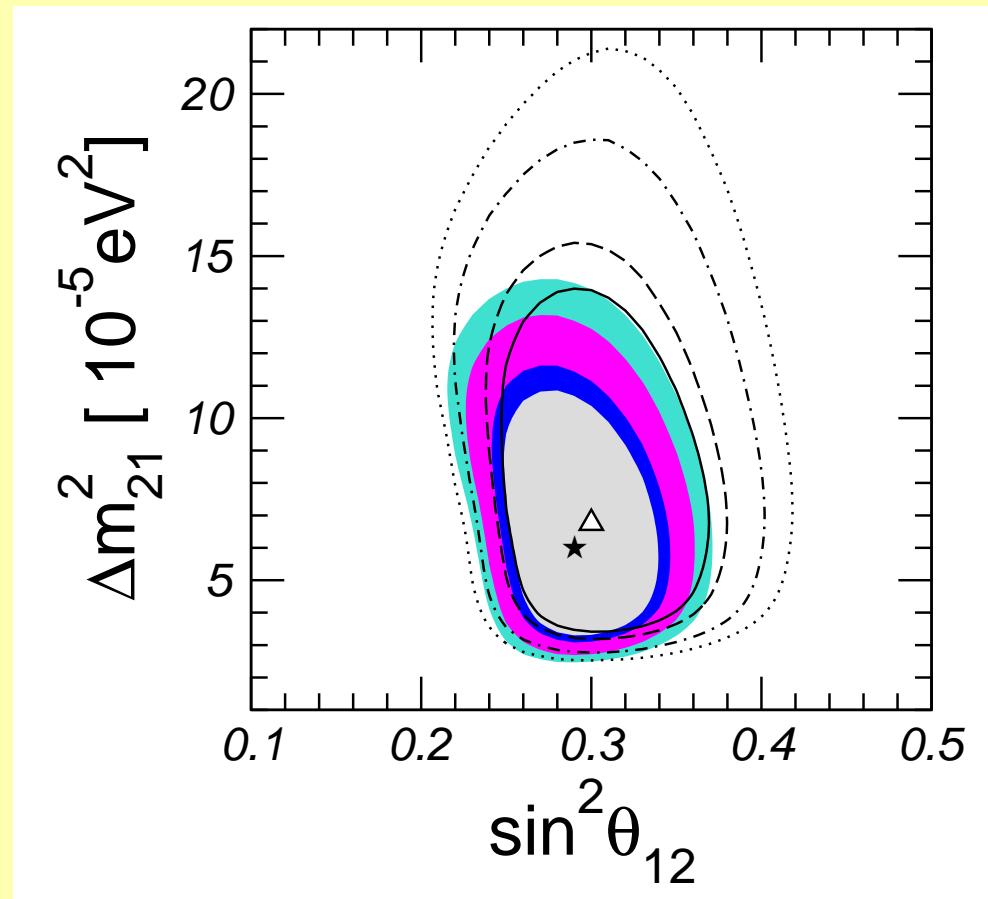
$$\frac{\phi_{CC}}{\phi_{NC}} = 0.340 \pm 0.038 \rightarrow 0.301 \pm 0.033$$



# *new CC/CN from SNO NCD phase*

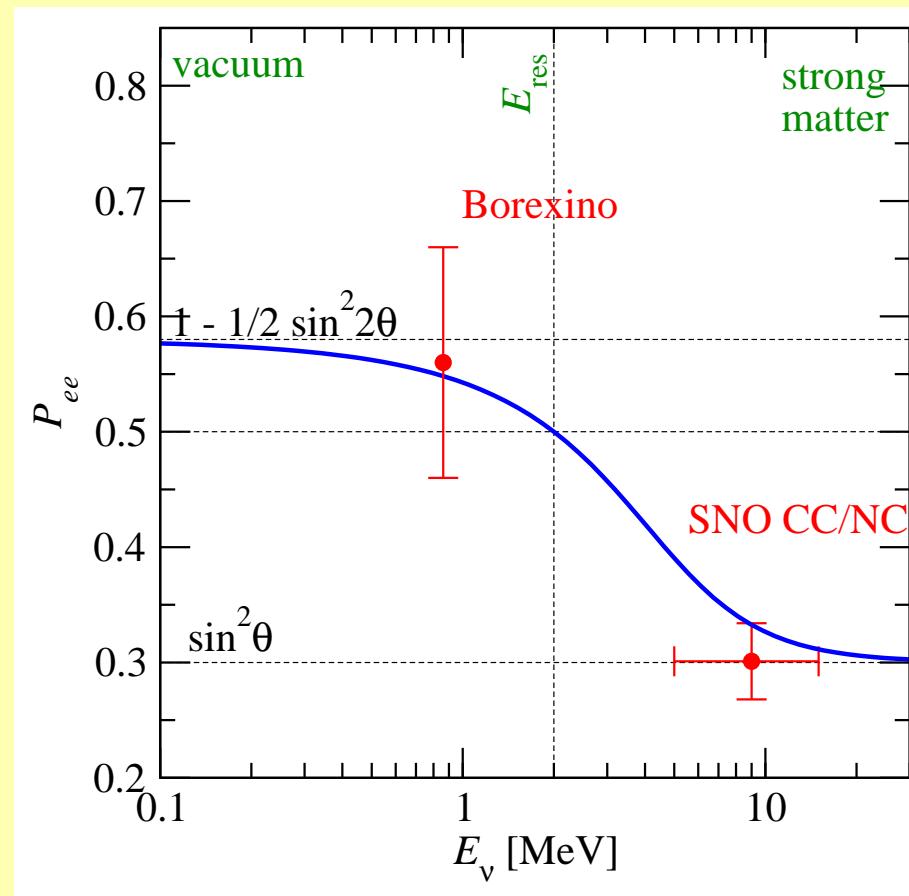
---

$$\frac{\phi_{CC}}{\phi_{NC}} = 0.340 \pm 0.038 \rightarrow 0.301 \pm 0.033$$



# *Testing the transition region*

BOREXINO: measurement of the Be7 neutrino line at 0.862 MeV  
by  $e\nu \rightarrow e\nu$  scattering ( $\Rightarrow$ )



---

# **Reactor neutrino experiments**

# *Reactor experiments*

---

**... have played always an important role in neutrino physics**

Starting from the discovery of the neutrino in the **Reines-Cowan experiment** C.L. Cowan et al., Science 124 (1956) 103

there have been many important experiments, e.g.:

**Gösgen** G. Zacek et al., Phys. Rev. D34 (1986) 2621

**Bugey** Y. Declais et al., Nucl. Phys. B434 (1995) 503

**CHOOZ** M. Apollonio et al., Phys. Lett. B466 (1999) 415

**Palo Verde** F. Boehm et al., Phys. Rev. D64 (2001) 112001

**KamLAND** Eguchi et al., Phys. Rev. Lett. 90 (2003) 021802

...

# *Reactor experiments*

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- Nuclear power reactors are an intense source of  $\bar{\nu}_e$
- Inverse  $\beta$ -decay offers a detection process with a clear experimental signature:

$$\bar{\nu}_e + p \rightarrow e^+ + n$$

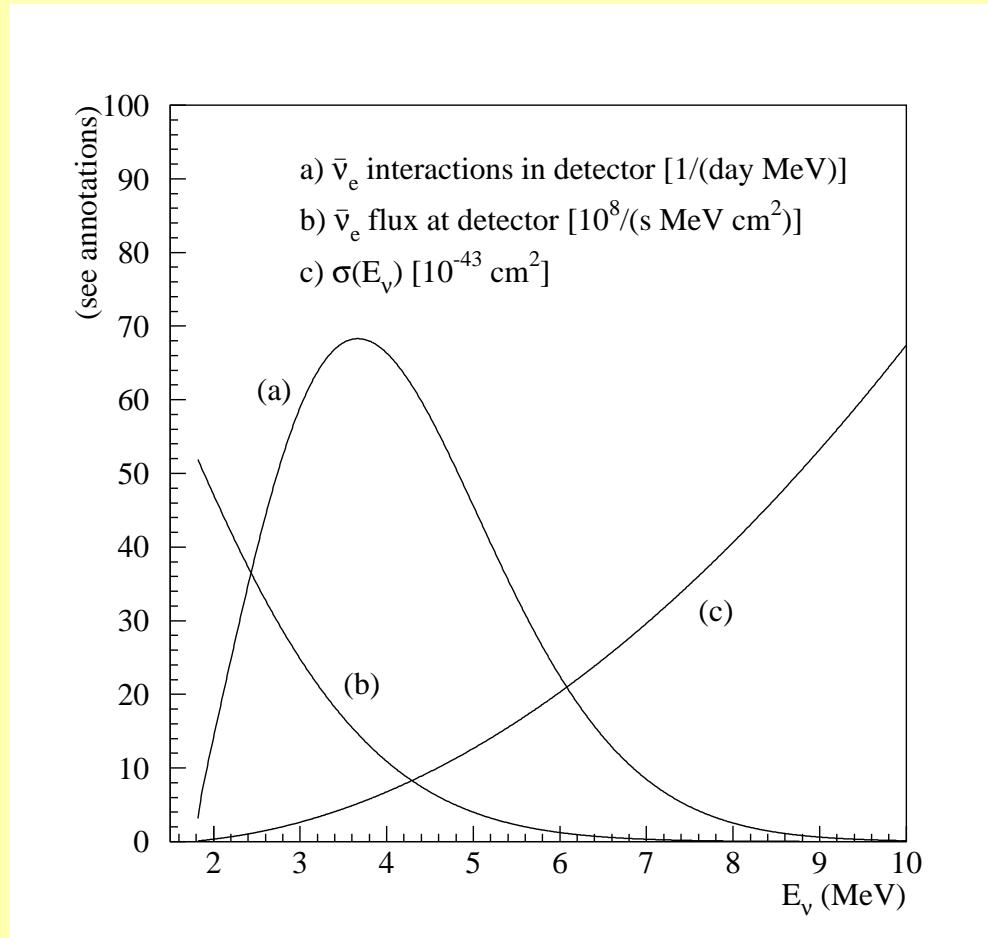
prompt positron + delayed neutron capture.

Energy threshold:  $E_\nu \geq m_e + m_n - m_p \approx 1.8 \text{ MeV}$

# *Reactor experiments*

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Bemporad, Gratta, Vogel, Rev.Mod.Phys.74(2002)297 [hep-ph/0107277]



$$\langle E_\nu \rangle \approx 3 - 4 \text{ MeV}$$

# *Reactor experiments*

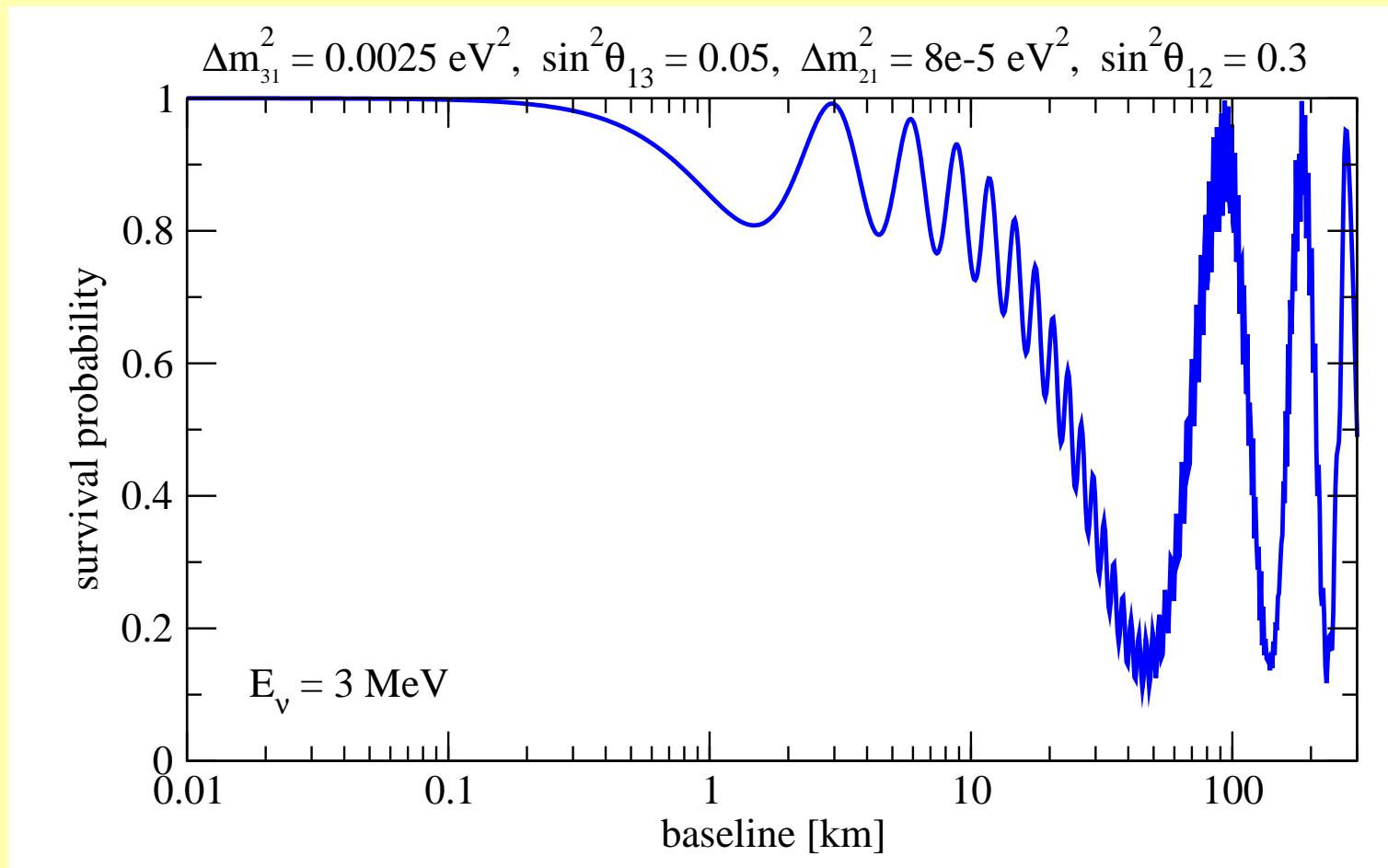
---

A reactor neutrino experiment can only be a  
 $\bar{\nu}_e$  disappearance experiment,

since for  $E_\nu \sim 4$  MeV neither  $\mu$  nor  $\tau$  can be produced  
in the detector

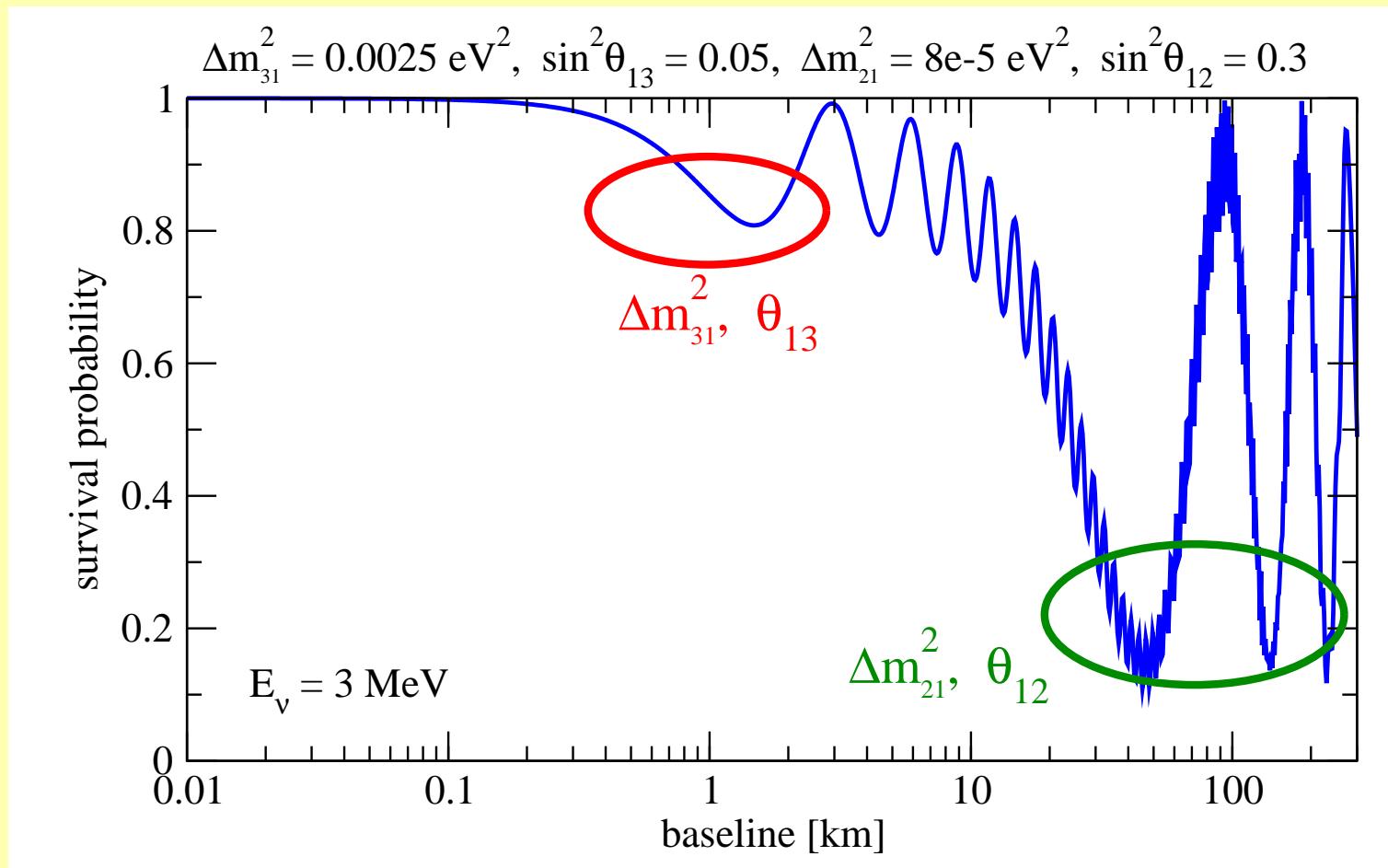
# *P<sub>ee</sub> at reactors*

The 3-flavour  $\bar{\nu}_e \rightarrow \bar{\nu}_e$  survival probability:



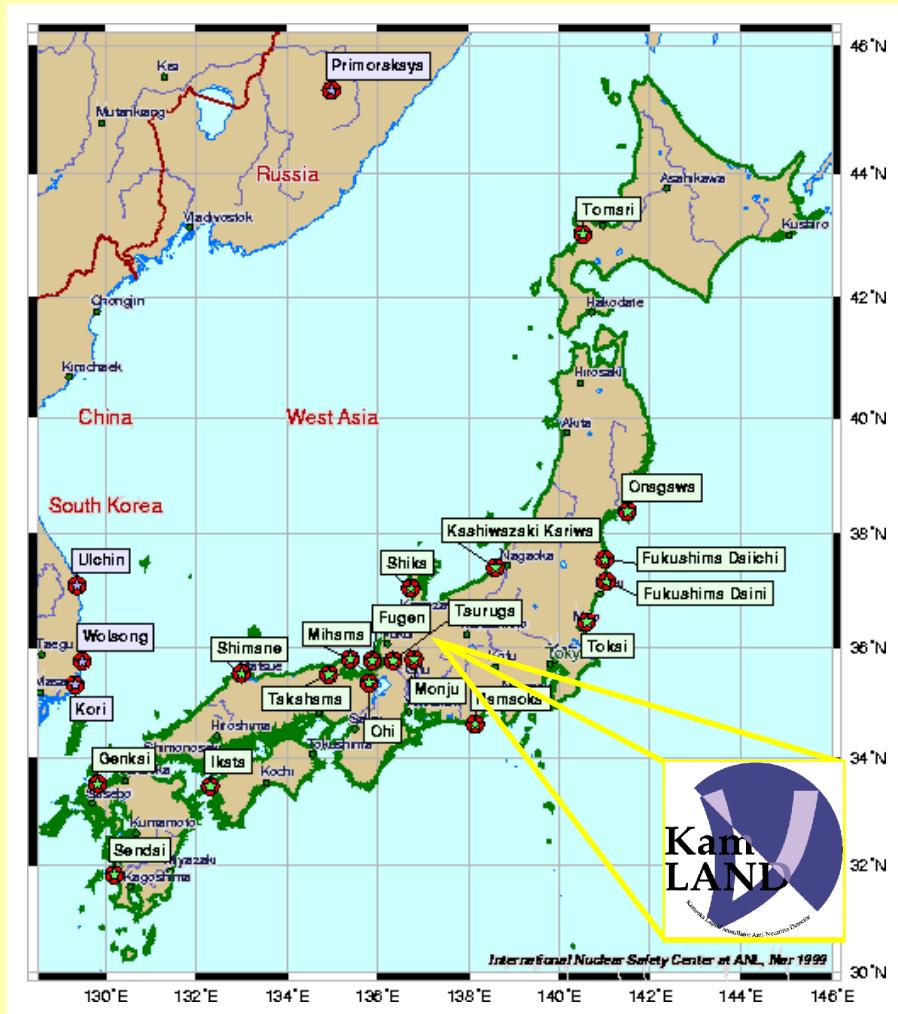
# $P_{ee}$ at reactors

The 3-flavour  $\bar{\nu}_e \rightarrow \bar{\nu}_e$  survival probability:



# *The KamLAND reactor neutrino experiment*

## Kamioka Liquid scintillator Anti-Neutrino Detector



detection of  $\bar{\nu}_e$  produced in surrounding nuclear power plants

70 GW of nuclear power (7% of world total) is generated at a distance  $175 \pm 30$  km from Kamioka

# *The KamLAND reactor neutrino experiment - 2*

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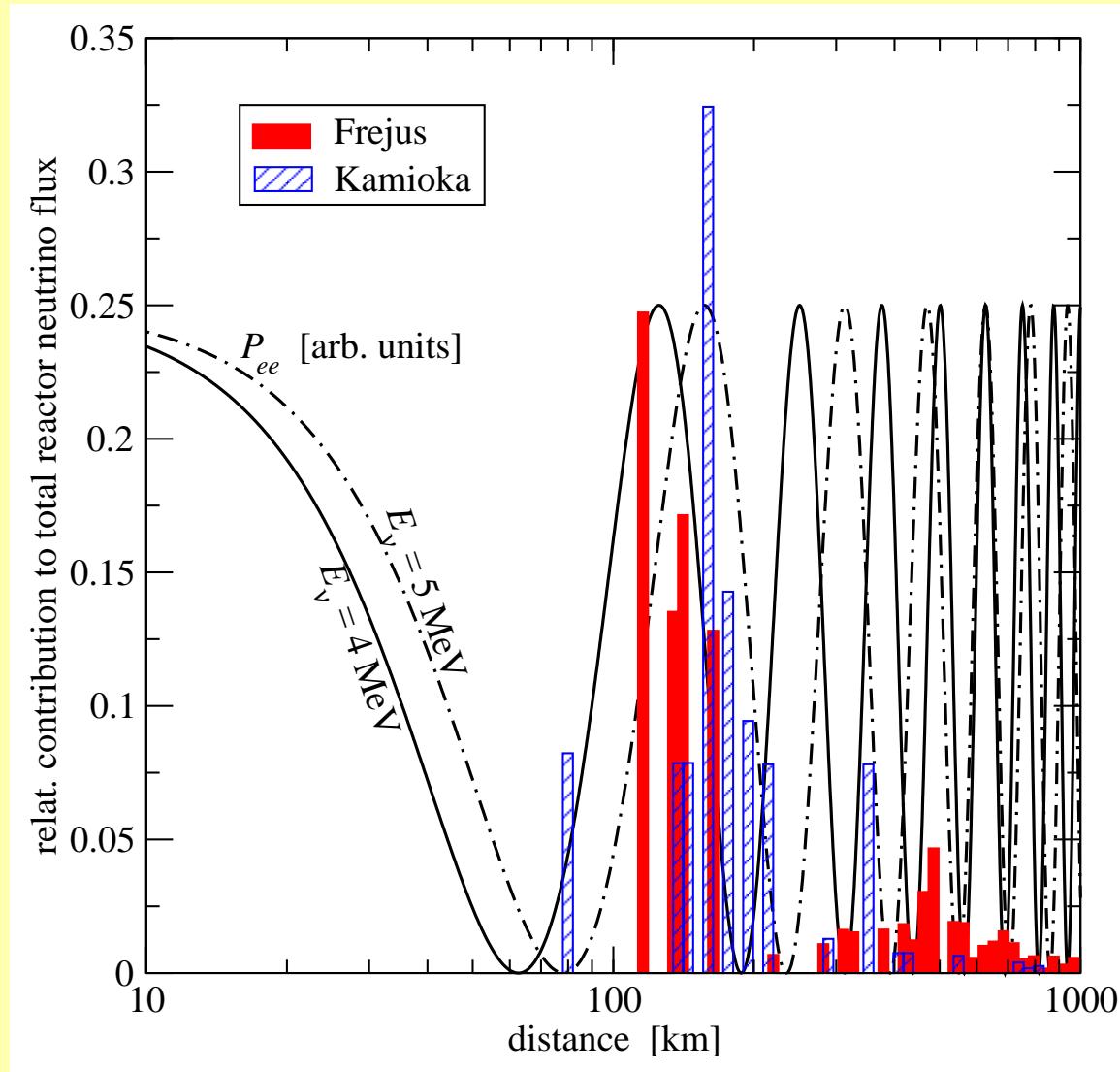
neutrino energy from nuclear reactors:  $E_\nu \simeq 4 \text{ MeV}$

$$\Rightarrow \frac{E_\nu}{L} \sim \frac{4 \text{ MeV}}{175 \text{ km}} \sim 2 \times 10^{-5} \text{ eV}^2$$

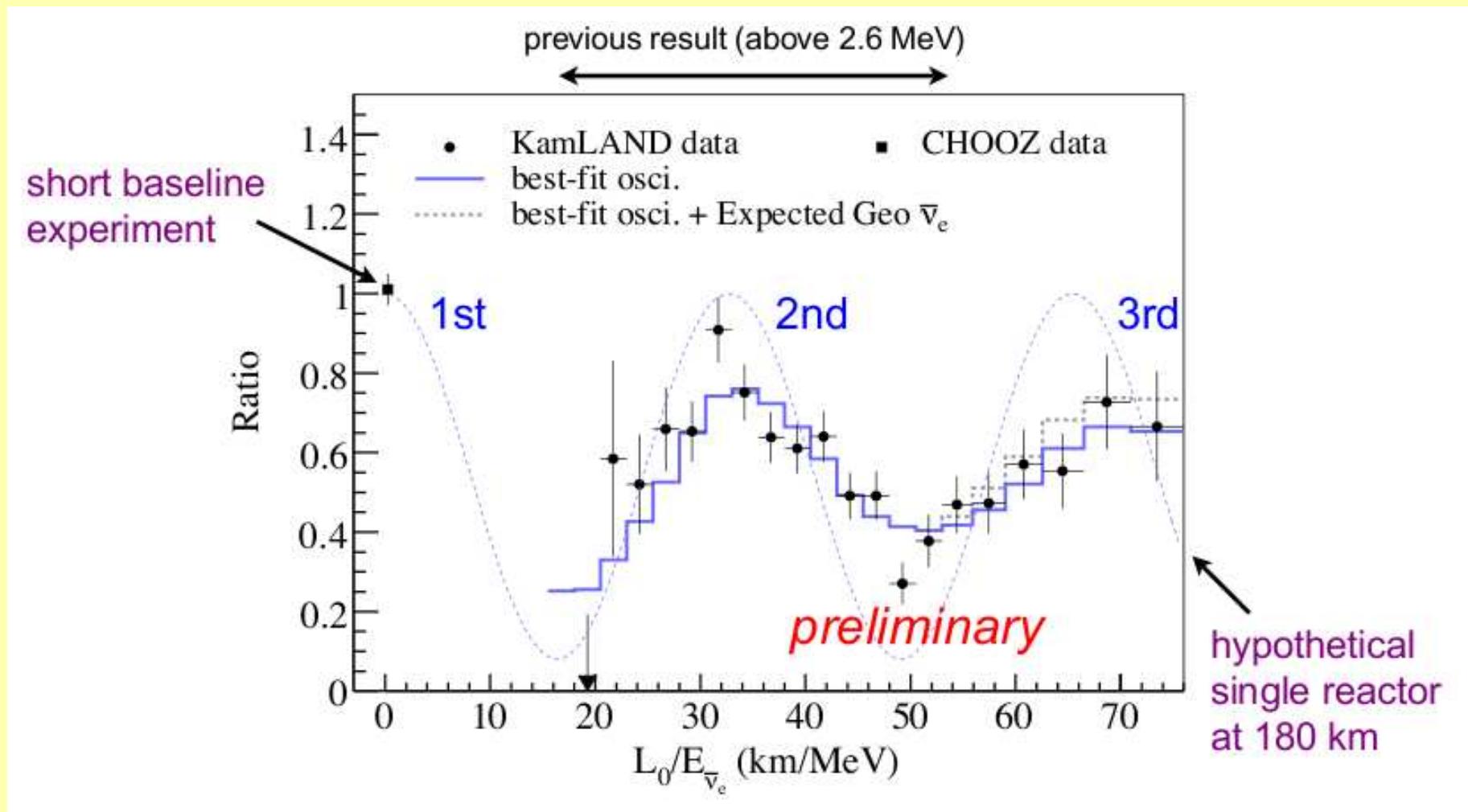
just the correct order of magnitude to test the LMA-MSW solution for the solar neutrino problem

# *Reactor distribution in KamLAND*

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# The KamLAND energy spectrum

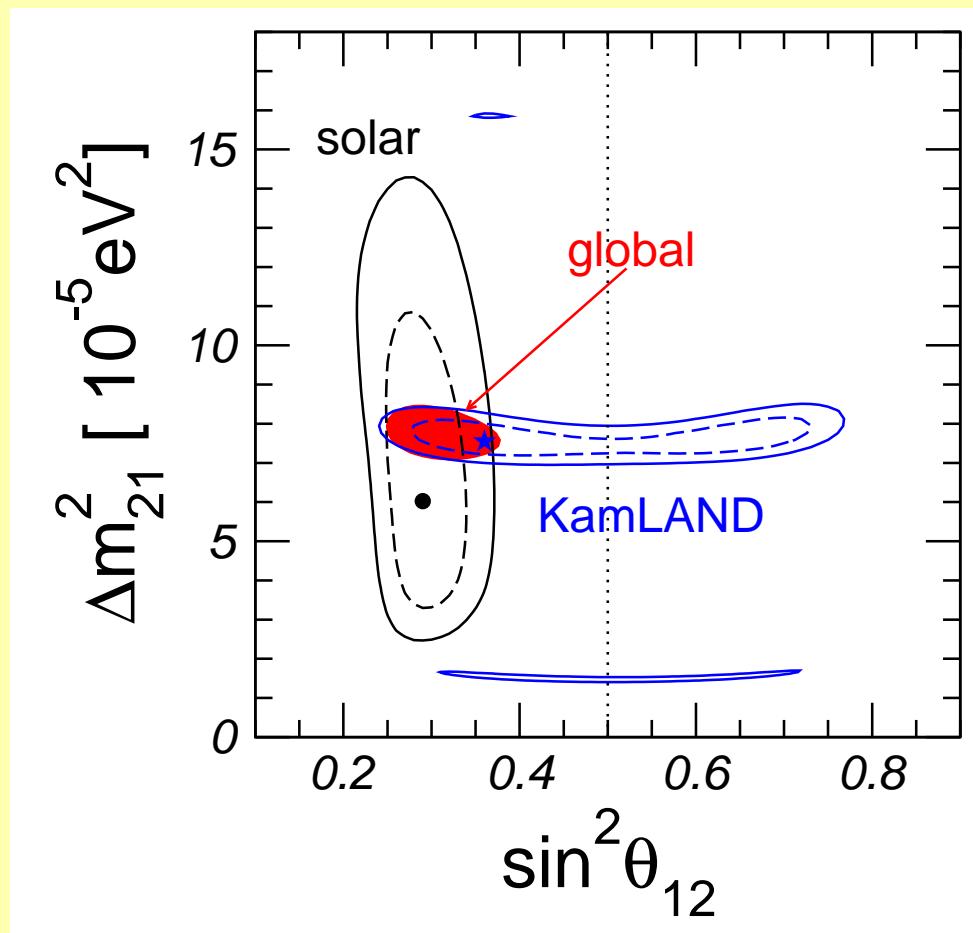


evidence for flux suppression and oscillations in  $1/E_\nu$



# KamLAND vs solar data

90% and 99.73% CL contours



$\Delta m_{21}^2$ :  
measured by  
KamLAND

$\sin^2 \theta_{12}$ :  
measured by SNO

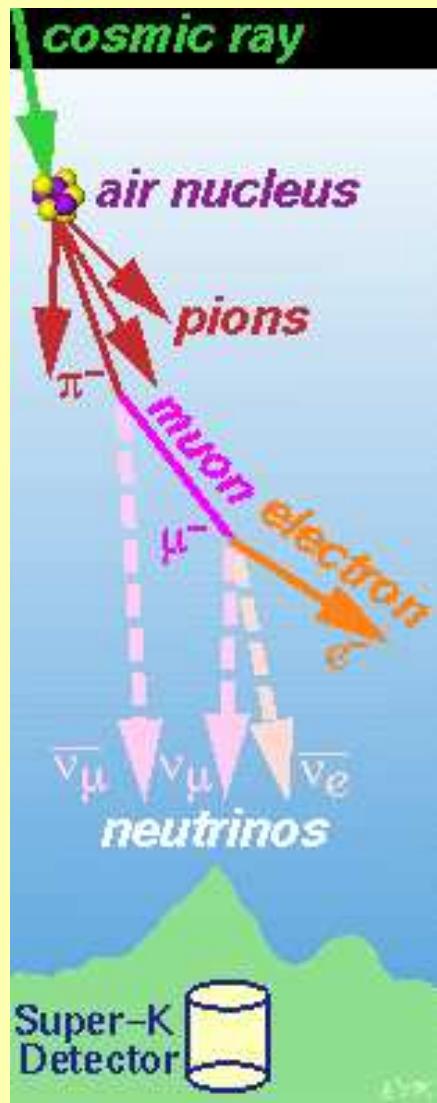
$$\Delta m_{21}^2 = 7.6 \pm 0.2 \times 10^{-5} \text{ eV}^2, \sin^2 \theta_{12} = 0.31^{+0.016}_{-0.023}$$

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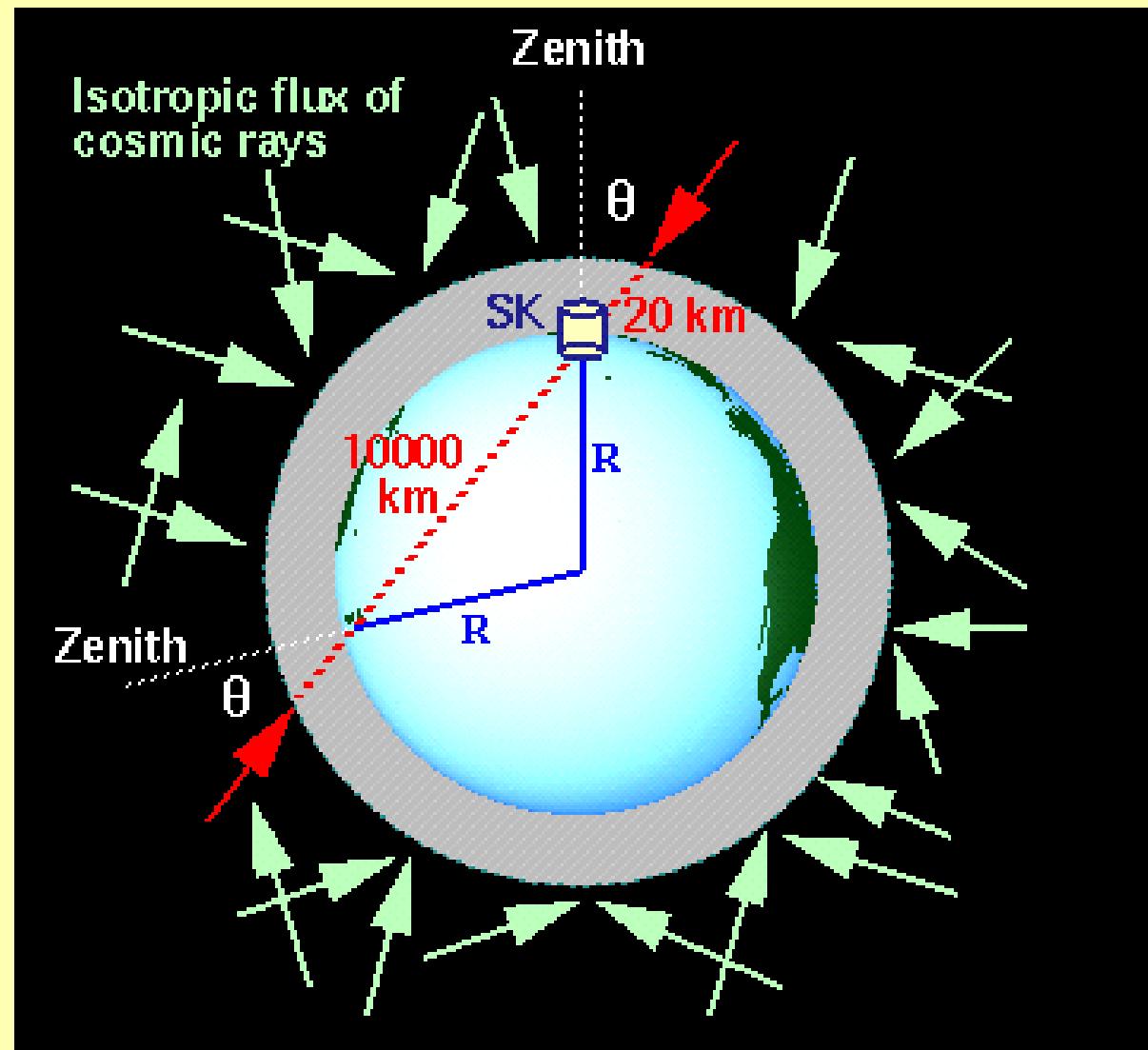
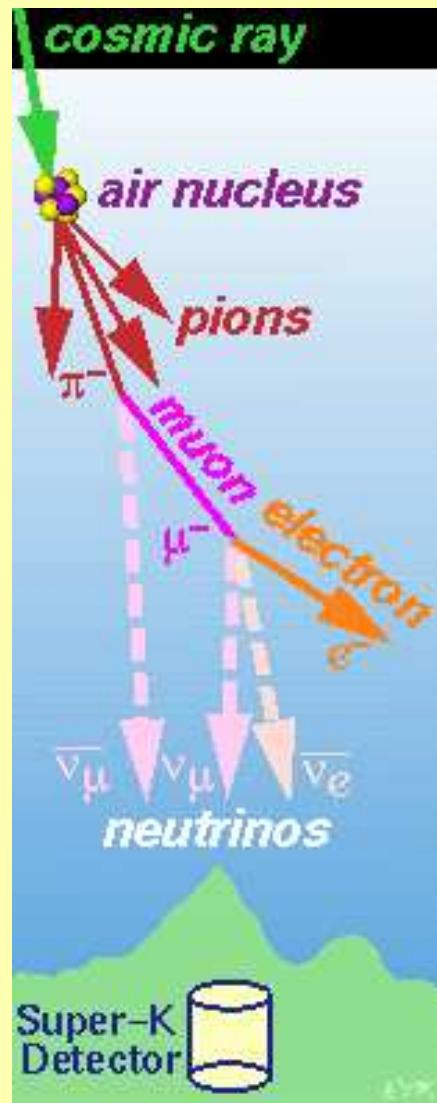
**The “atmospheric” parameters  $\Delta m_{31}^2$ ,  $\theta_{23}$**

# Atmospheric neutrinos

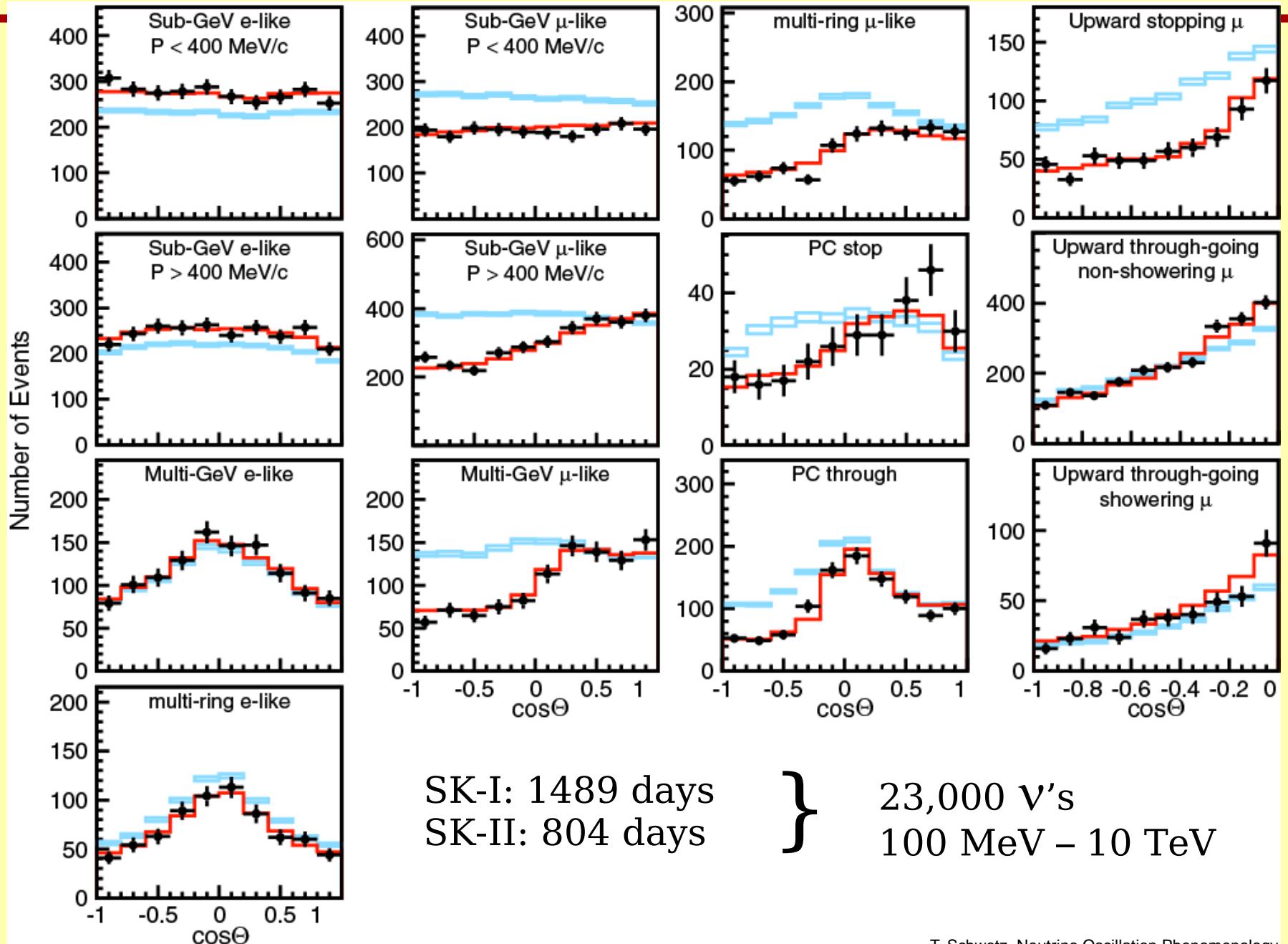
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# Atmospheric neutrinos



# Super-K atmospheric neutrino data



# Oscillations of atmospheric neutrinos

---

$$P_{ee}^{\text{atm}} \approx 1 - \mathcal{O}(\theta_{13}, \Delta m_{12}^2)$$

$$P_{\mu\mu}^{\text{atm}} \approx 1 - P_{\mu\tau}^{\text{atm}} \approx 1 - \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{31}^2 L}{4E_\nu}$$

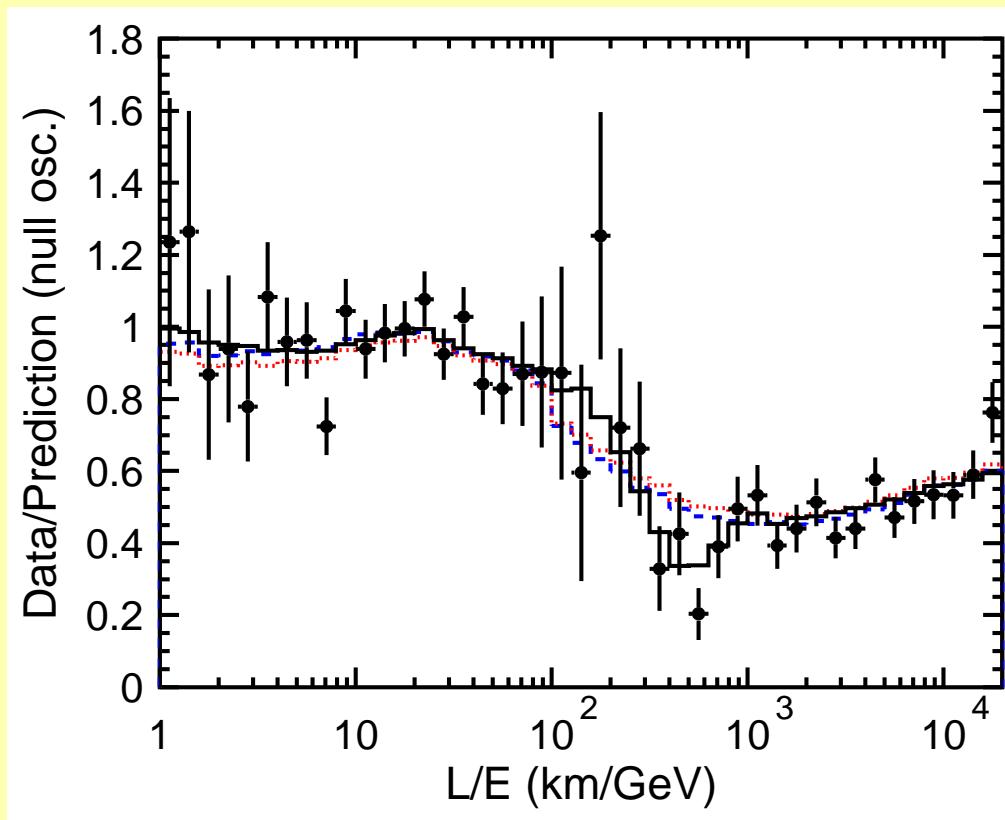
with

$$\sin^2 2\theta_{23} \approx 1 , \quad |\Delta m_{31}^2| \simeq 0.0024 \text{ eV}^2$$

# Oscillatory signal in atmospheric neutrinos

Super-K Coll., Phys. Rev. Lett. 93 (2004) 101801

⇒

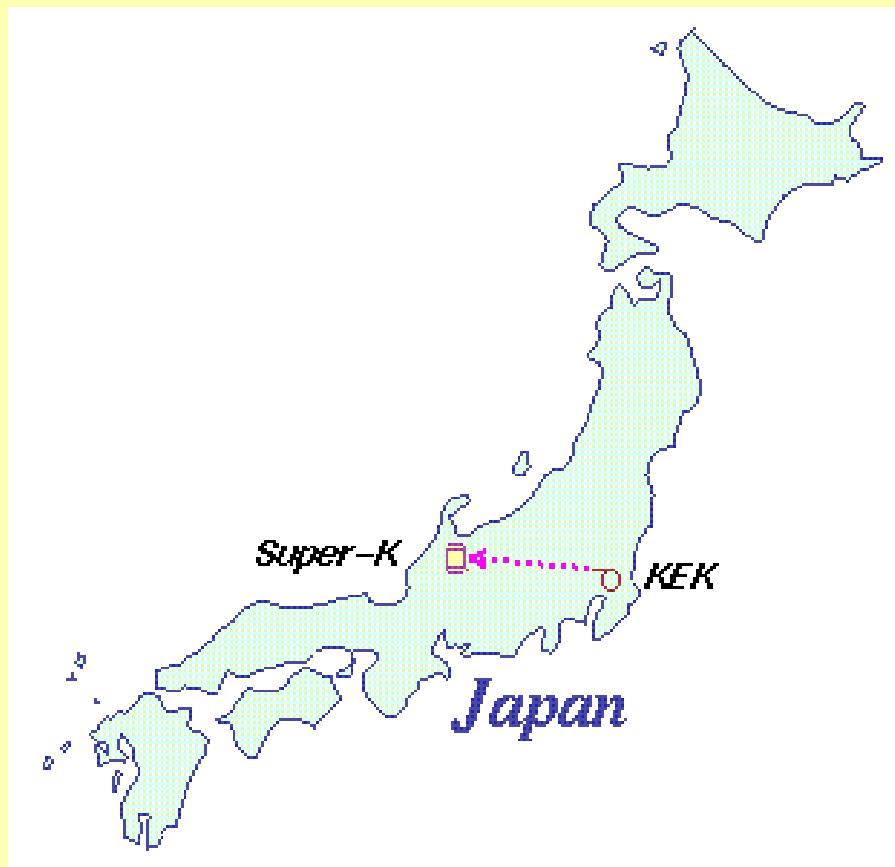


$$P_{2\nu} = 1 - \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2}{4} \frac{L}{E_\nu} \right)$$

# *Long-baseline experiments*

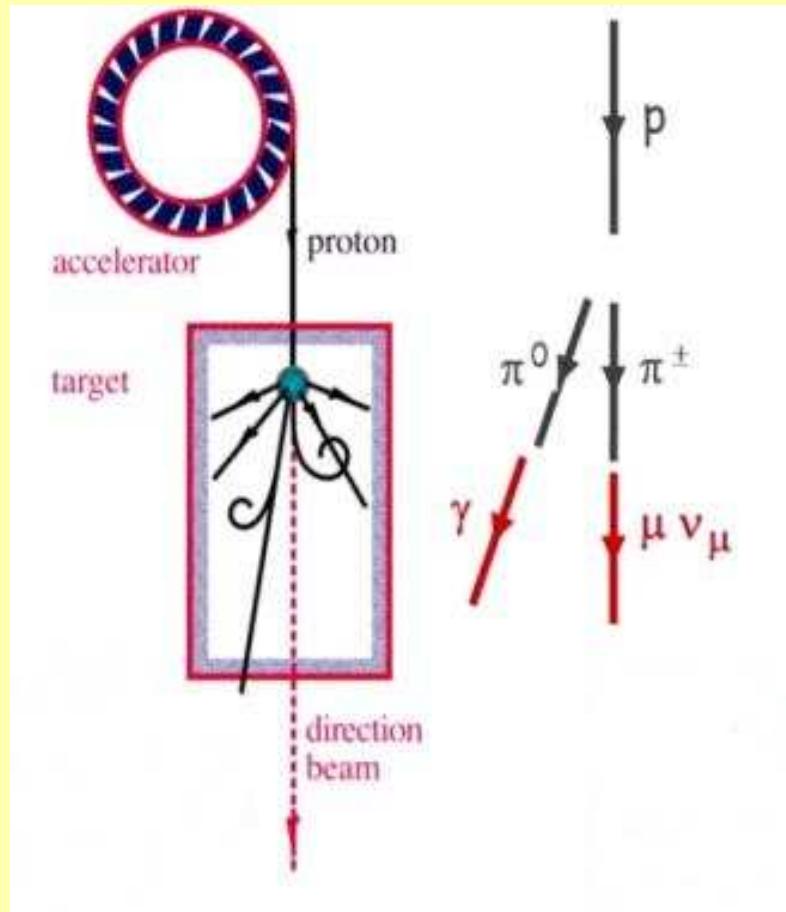
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first generation of LBL experiments  
( $\nu_\mu$ -disappearance)



# *The neutrino source*

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neutrino production via pion decay  $\Rightarrow \nu_\mu$  beam

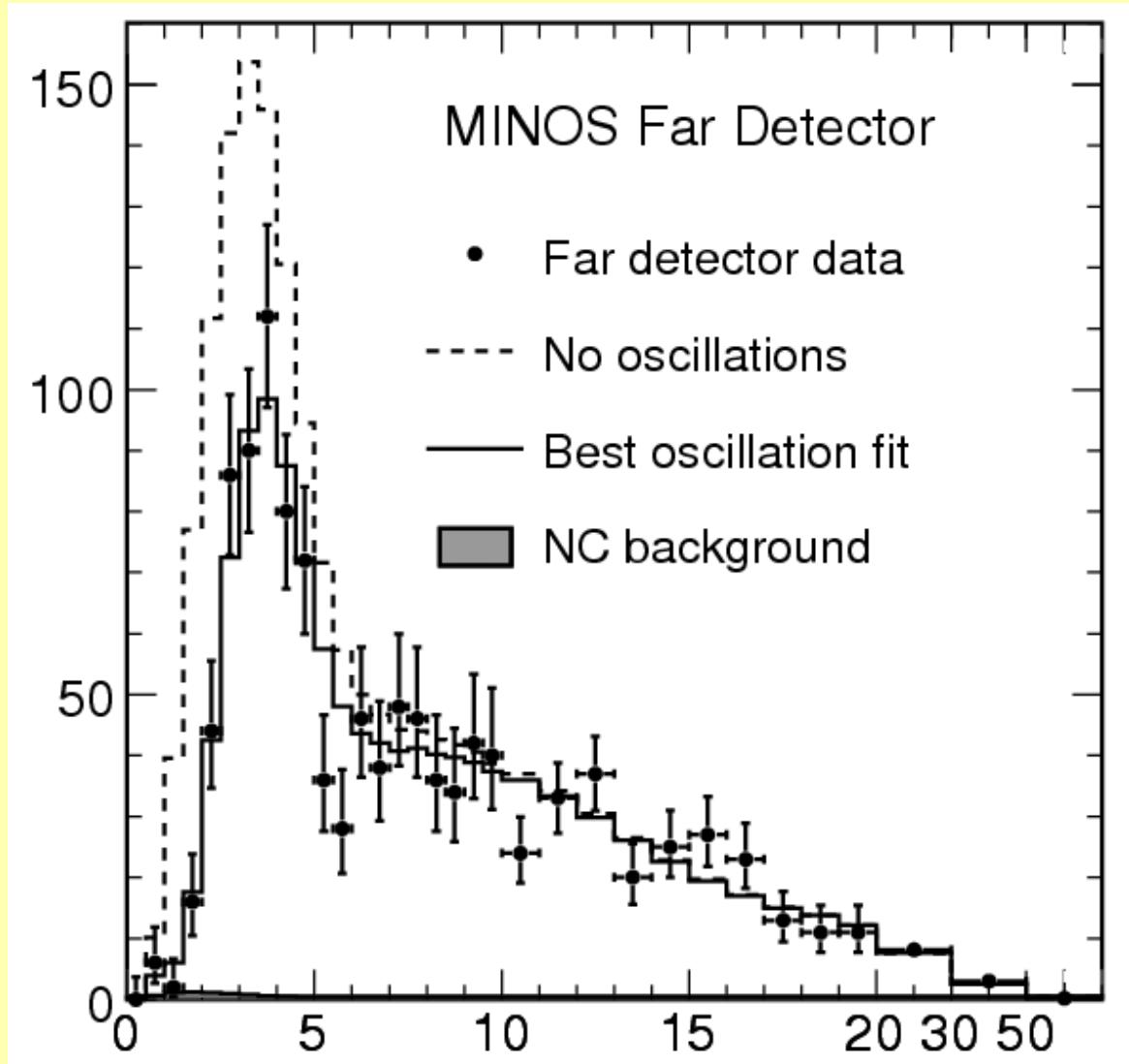
# *K2K vs MINOS*

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	K2K	MINOS
source	KEK	Fermilab
detector	Super-K	Soudan
baseline	250 km	735 km
neutrino energy	1.3 GeV	3 GeV
$E_\nu/L \text{ [eV}^2]$	$5.2 \times 10^{-3}$	$4.1 \times 10^{-3}$
channel	$\nu_\mu \rightarrow \nu_\mu$	$\nu_\mu \rightarrow \nu_\mu$
obs. events	112	848
expect. w/o osc.	$158.1^{+9.2}_{-8.6}$	$1065 \pm 60$

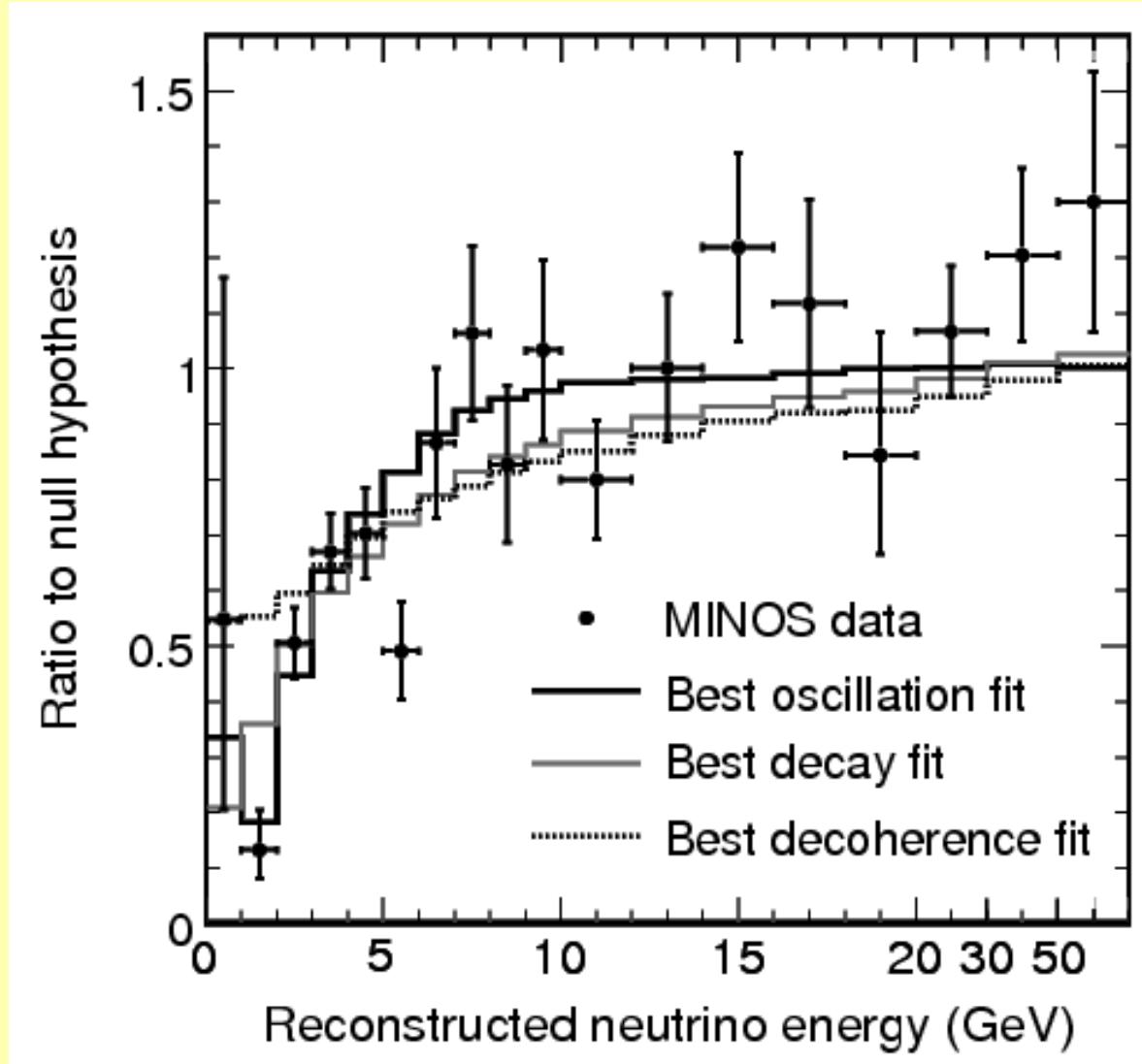
# *MINOS energy spectrum*

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# *MINOS survival probability*

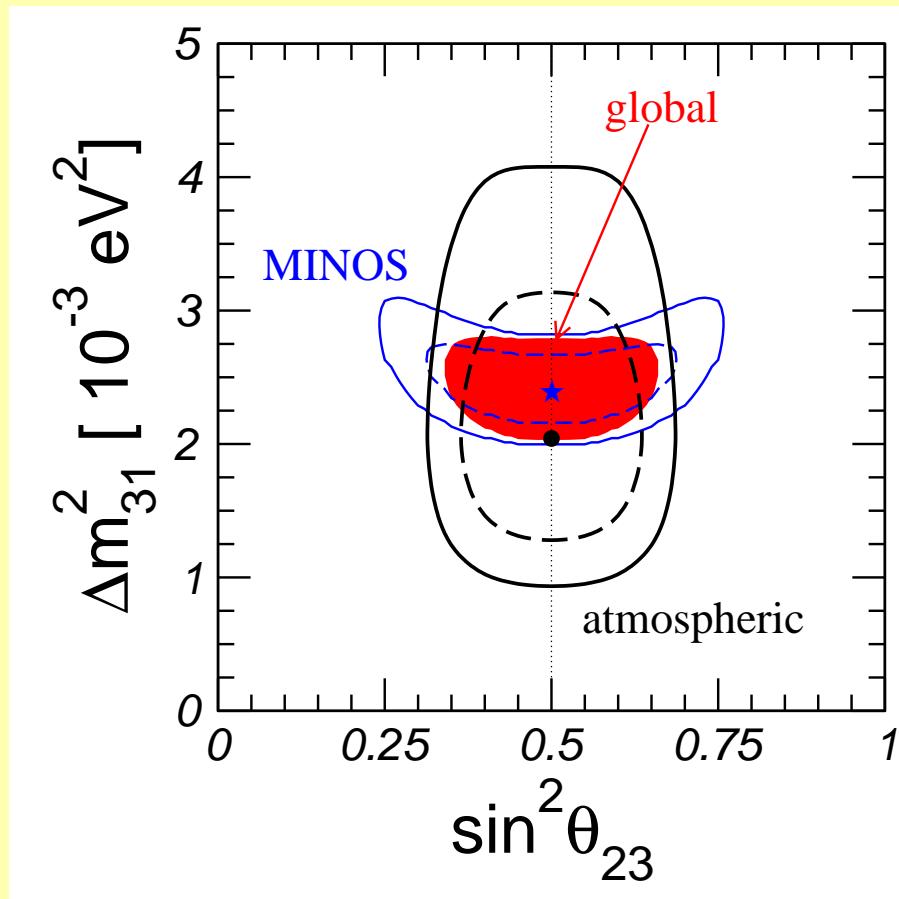
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# *Super-K + K2K + MINOS*

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90%, 99.73% CL regions



$$\Delta m_{31}^2 = 2.4 \pm 0.15 \times 10^{-3} \text{ eV}^2, \sin^2 \theta_{23} = 0.50 \pm 0.063$$