Theoretical Models of neutrino parameters.

G.G.Ross, Paris, September 2008

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Number of light neutrinos	3 ?
Masses + Mixing Angles	6 (15)
CP violating phases	3 (6)

Theoretical Models of neutrino parameters.

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- $q \leftrightarrow l$ — See-saw with sequential domination
- Symmetry — GUT; Family — Abelian, Non-Abelian, Discrete, String?
- Tri-bi-maximal mixing — Non-Abelian discrete symmetry

Theoretical Models of neutrino parameters.

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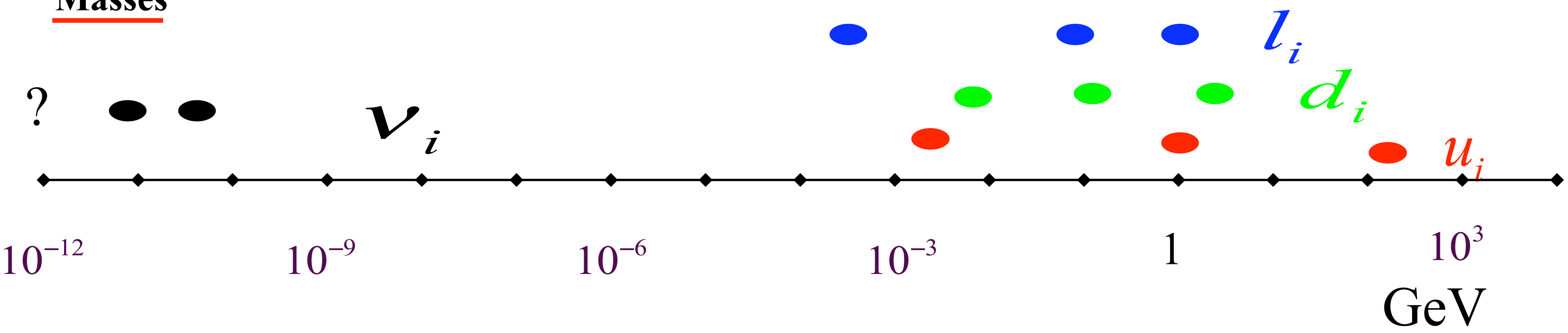
- Tri-bi-maximal mixing — Non-Abelian discrete symmetry

$\Rightarrow$ parameter constraints

DATA :

Masses



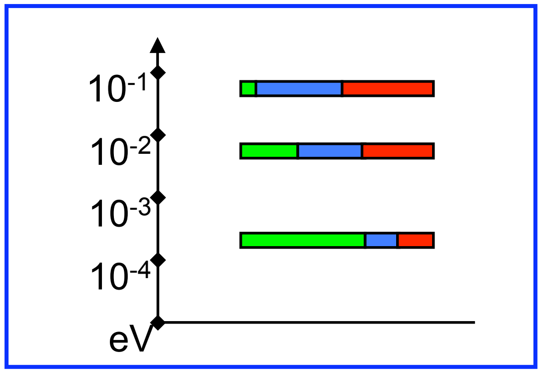
Mixing

Quarks

Leptons

$$V_{CKM} \approx \begin{pmatrix} 1 & 0.218-0.224 & 0.002-0.005 \\ 0.218-0.224 & 1 & 0.032-0.048 \\ 0.004-0.015 & 0.03-0.048 & 1 \end{pmatrix}$$

$$V_{MNS} = \begin{pmatrix} 0.79-0.88 & 0.48-0.61 & <0.2 \\ 0.27-0.49 & 0.45-0.71 & 0.52-0.82 \\ 0.28-0.5 & 0.51-0.65 & 0.57-0.81 \end{pmatrix}$$



$$\approx \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & \sim 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Bi-Tri Maximal Mixing …
Discrete
Non Abelian
Structure?

DATA :

$$L_{Yukawa} = Y^u_{ij}\ Q^i u^{c,j} H + Y^d_{ij}\ Q^i d^{c,j} \overline{H}$$

$$M^u_{ij} = Y^u_{ij} < H^0 > \qquad M^d_{ij} = Y^d_{ij} < \overline{H}^0 >$$

$$M^u = V_L^\dagger \underline{M^u_{Diag}} V_R$$

$$M^d = U_L^\dagger \underline{M^d_{Diag}} U_R$$

$$\underline{V_{CKM}} = V_L^\dagger U_L$$

The data for quarks is *consistent* with a very symmetric structure :

$$\boxed{\frac{M^{d,u}}{m^{b,t}} \simeq \begin{pmatrix} <\varepsilon^4 & \varepsilon^3 & -\varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & -\varepsilon^2 \\ -\varepsilon^3 & -\varepsilon^2 & 1 \end{pmatrix} \qquad \begin{matrix} \varepsilon^d = 0.15 \\ \\ \varepsilon^u = 0.05 \end{matrix}}$$

$q \leftrightarrow l$ symmetry?

Charged leptons are consistent with a similar form

$$\frac{M^{d,l,u}}{m^{b,\tau,t}} \simeq \begin{pmatrix} <\varepsilon^4 & \varepsilon^3 & -\varepsilon^3 \\ \varepsilon^3 & a\varepsilon^2 & -a\varepsilon^2 \\ -\varepsilon^3 & -a\varepsilon^2 & 1 \end{pmatrix}$$

$$\varepsilon^{\mathrm{d}} = 0.15, \quad \mathrm{a}^{\mathrm{d}} = 1$$
$$\varepsilon^{\mathrm{l}} = 0.15, \quad \mathrm{a}^{\mathrm{l}} = -3$$
$$\varepsilon^{\mathrm{u}} = 0.05, \quad \mathrm{a}^{\mathrm{u}} = 1$$

Symmetry 1.

GUT relations

$e.g.\quad SU(4)\subset SO(10)$

$$\psi_\alpha = \begin{pmatrix} d \\ d \\ d \\ l \end{pmatrix}$$

Symmetry

GUT relations

$$e.g.\quad SU(4)\subset SO(10)$$

$$\psi_\alpha = \begin{pmatrix} d \\ d \\ d \\ l \end{pmatrix}$$

$$Det(M^l) = Det(M^d)|_{M_X}$$

$$\overline{\psi}^\alpha \psi_\alpha$$

$$\frac{m_b}{m_\tau}(M_X) = 1$$

$$\frac{M^{d,l}}{m_3} = \begin{pmatrix} <\varepsilon^4 & \varepsilon^3 & -\varepsilon^3 \\ \varepsilon^3 & a\varepsilon^2 & -a\varepsilon^2 \\ -\varepsilon^3 & -a\varepsilon^2 & 1 \end{pmatrix}$$

$$\varepsilon^d = 0.15,\quad a^d = 1$$
$$\varepsilon^l = 0.15,\quad a^l = -3$$

Symmetry 1.

GUT relations

$e.g.\quad SU(4) \subset SO(10)$

$$\psi_\alpha = \begin{pmatrix} d \\ d \\ d \\ l \end{pmatrix}$$

$$\langle \Sigma_{45} \rangle = \bar{\psi}^\alpha \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix} \psi_\alpha$$

$$\frac{m_s}{m_\mu}(M_X) = \frac{1}{3}$$

Georgi Jarlskog

$$\frac{M^{d,l}}{m_3} = \begin{pmatrix} <\varepsilon^4 & \varepsilon^3 & -\varepsilon^3 \\ \varepsilon^3 & a\varepsilon^2 & -a\varepsilon^2 \\ -\varepsilon^3 & -a\varepsilon^2 & 1 \end{pmatrix}$$

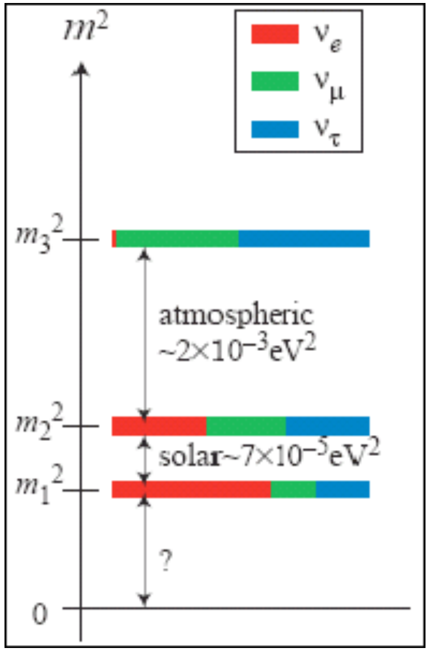
$\varepsilon^d = 0.15, \quad a^d = 1$

$\varepsilon^l = 0.15, \quad a^l = -3$

Neutrinos ???

$$L_{eff}^{\nu} = m_3\, \bar{\phi}_{23}^{i} \nu_i \bar{\phi}_{23}^{j} \nu_j + m_2\, \bar{\phi}_{123}^{i} \nu_i \bar{\phi}_{123}^{j} \nu_j$$

$$<\bar{\phi}_{23}>^i = (0,1,-1), \quad <\bar{\phi}_{123}>^i = (1,1,1)$$



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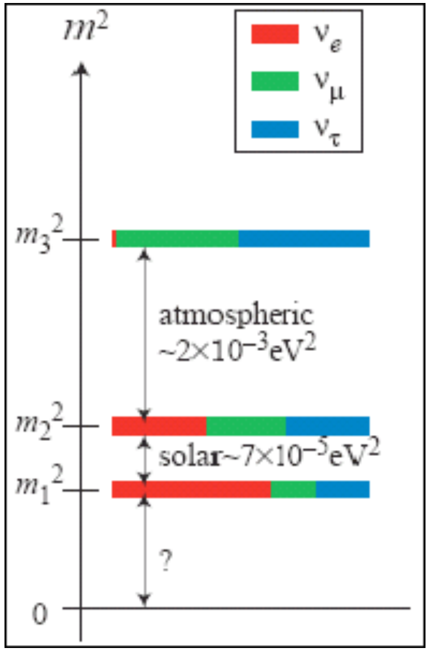

Can one have a unified description of quark, charged lepton and neutrinos?

$$c.f.\ \ L^{q,l}_{Dirac} = m_3\, \bar{\phi}^{\,i}_3 \psi_i \bar{\phi}^{\,j}_3 \psi^c_j + \ldots \qquad <\bar{\phi}_3>^i = (0,0,1) \quad ???$$

Neutrinos ???

$$L_{eff}^{\nu} = m_3\,\overline{\phi}^i_{23}\nu_i\overline{\phi}^j_{23}\nu_j + m_2\,\overline{\phi}^i_{123}\nu_i\overline{\phi}^j_{123}\nu_j$$

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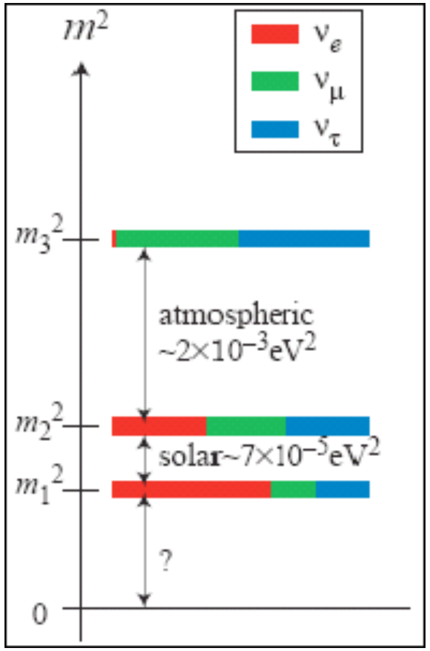


See-Saw

Quarks, charged leptons, neutrinos can have similar Dirac mass :

$$L_{Dirac}^{q,l,\nu} = \alpha\,\psi_i\overline{\phi}^i_3\psi^c_j\overline{\phi}^j_3 + \beta\left(\psi_i\overline{\phi}^i_{123}\psi^c{}_j\overline{\phi}^j_{23} + \psi_i\overline{\phi}^i_{23}\psi^c{}_j\overline{\phi}^j_{123}\right) + \gamma\,\psi_i\overline{\phi}^i_{123}\psi^c{}_j\overline{\phi}^j_{23}\Sigma_{45} \qquad \alpha > \beta$$

$$\frac{M^{Dirac}}{m_3} = \begin{pmatrix} <\varepsilon^4 & \varepsilon^3+\varepsilon^4 & -\varepsilon^3+\varepsilon^4 \\ \varepsilon^3+\varepsilon^4 & a\varepsilon^2+\varepsilon^3 & -a\varepsilon^2+\varepsilon^3 \\ -\varepsilon^3+\varepsilon^4 & -a\varepsilon^2+\varepsilon^3 & 1 \end{pmatrix}$$

$\varepsilon^d = 0.15,\ a^d = 1$

$\varepsilon^l = 0.15,\ a^e = -3$

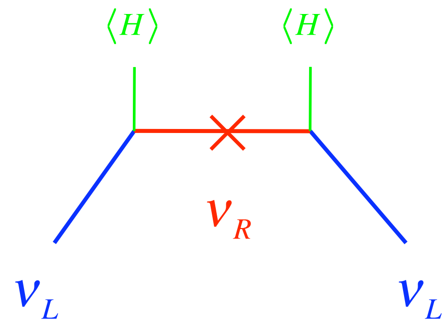
$\varepsilon^u = 0.05,\ a^u = 1$

$\varepsilon^\nu = 0.05,\ a^\nu = 0$

- "See-saw" with sequential domination $\quad M_\nu = M_D^\nu M_M^{-1} M_D^{\nu T}$

Minkowski
Gell-Mann,
Ramond,
Slansky;
Yanagida,
King

$$M_M \approx \begin{pmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{pmatrix} \qquad M_1 < M_2 << M_3$$



$$L^\nu_{Dirac} = \alpha\, \psi_i \bar{\phi}_3^i \psi^c_j \bar{\phi}_3^j + \beta\left(\psi_i \bar{\phi}^i_{123} \psi^c{}_j \bar{\phi}^j_{23} + \psi_i \bar{\phi}^i_{23} \psi^c{}_j \bar{\phi}^j_{123} \right) \quad \alpha > \beta$$

$$\boxed{L^\nu_{eff} = \frac{\beta^2}{M_1} \psi_i \phi^i_{123} \psi_j \phi^j_{123} + \frac{\beta^2}{M_2} \psi_i \phi^i_{23} \psi_j \phi^j_{23} + \frac{(\alpha+\beta)^2}{M_3} \psi_i \phi^i_3 \psi_j \phi^j_3}$$

- "See-saw" with sequential domination $M_\nu = M_D^\nu M_M^{-1} M_D^{\nu T}$

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small

$$L^\nu_{Dirac} = \alpha\, \psi_i \bar{\phi}^i_3 \psi^c_j \bar{\phi}^j_3 + \beta \left(\psi_i \bar{\phi}^i_{123} \psi^c{}_j \bar{\phi}^j_{23} + \psi_i \bar{\phi}^i_{23} \psi^c{}_j \bar{\phi}^j_{123} \right) \quad \alpha > \beta$$

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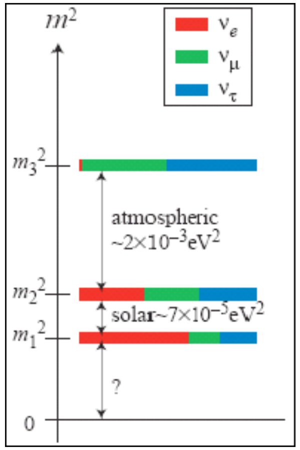
Symmetry 2.

Family symmetry



Non-Abelian family symmetry

Promote ϕ_i to fields transforming under $SU(3)_{family}$

Vacuum alignment $\quad \phi_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \phi_{23} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \phi_{123} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$\Rightarrow$ Discrete non Abelian symmetry

List of models with discrete flavour symmetry

(incomplete, by symmetry)

S_3: Pakvasa et al.(1978), Derman(1979), Ma(2000), Kubo et al.(2003), Chen et al.(2004), Grimus et al.(2005), Dermisek et al (2005), Mohapatra et al.(2006), Morisi(2006), Caravaglios et al.(2006), Haba et al(2006),…

S_4: Pakvasa et al.(1979), Derman(1979), Lee et al.(1994), Mohapatra et al.(2004),Ma(2006), Hagedorn et al.(2006), Caravaglios et al.(2006), Lampe(2007), Sawanaka(2007), …

A_4: Wyler(1979), Ma et al.(2001), Babu et al.(2003), Altarelli et al.(2005-8), He et al.(2006), Bazzocchi, Morisi, et al.(2007/8), King et al.(2007),…

D_4: Seidl(2003), Grimus et al.(2003/4), Kobayashi et al.(2005), …

D_5: Ma(2004), Hagedorn et al.(2006), …

D_n: Chen et al.(2005), Kajiyama et al.(2006), Frampton et al.(1995/6,2000), Frigerio et al (2005), Babu et al.(2005), Kubo(2005),...

T': Frampton et al.(1994,2007), Aranda et al.(1999,2000),Feruglio et al.(2007), Chen et al.(2007), …

Δ_n: Kaplan et al.(1994), Schmaltz(1994), Chou et al.(1997), de Madeiros Varzielas et al(2006/7).

T_7: Luhn et al.(2007).

Adapted from Lindner, Manchester

Symmetry 2.   Family symmetry

Non-Abelian family symmetry

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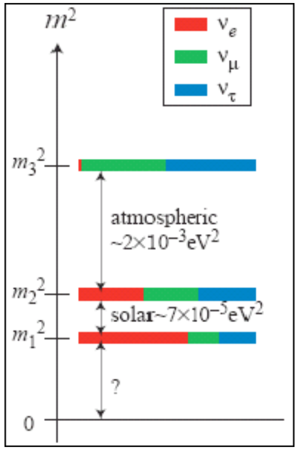

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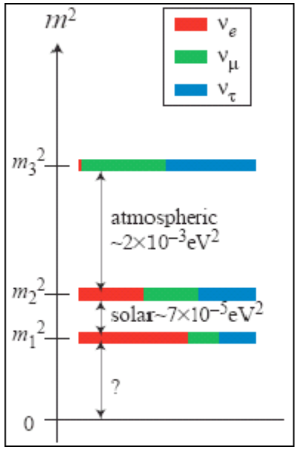
$\Rightarrow$ Discrete non Abelian symmetry e.g. $\Delta(27) \subset SU(3)_f$, ϕ_i triplets

ϕ_i	$Z_3\phi_i$	$Z_3'\phi_i$
ϕ_1	$\rightarrow \phi_2$	$\rightarrow \phi_1$
ϕ_2	$\rightarrow \phi_3$	$\rightarrow \alpha\phi_2$
ϕ_3	$\rightarrow \phi_1$	$\rightarrow \alpha^2\phi_3$

Symmetry 2.

Family symmetry



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ϕ_2	$\rightarrow \phi_3$	$\rightarrow \alpha\phi_2$
ϕ_3	$\rightarrow \phi_1$	$\rightarrow \alpha^2\phi_3$

$$\Delta(27) \rightarrow \begin{cases} Z_3, \quad <\phi> = (1,1,1) \quad \lambda > 0 \\ Z_3', \quad <\phi> = (0,0,1) \quad \lambda < 0 \end{cases}$$

$$V(\phi) = m^2\ \phi^\dagger\phi + \ldots + \lambda\ m^2 \phi^\dagger\phi_i\phi^\dagger\phi_i$$

A complete model

$$\Delta(27) \otimes SO(10) \otimes G \qquad (G = R \otimes U(1))$$

Varzielas, GGR

- $\psi_i^c, \psi_i \subset (16,3) \Rightarrow$ No mass while SU(3) unbroken

- Spontaneous symmetry breaking

c.f. Georgi-Jarskog

$$\overline{\phi}_3^{\,i}, \quad \overline{\phi}_{23}^{\,i}, \quad \overline{\phi}_{123}^{\,i}, \quad H_{45}$$

$$(1,\overline{3}) \qquad (1,\overline{3}) \qquad (1,\overline{3}) \qquad (45,1)$$

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}\varepsilon M \qquad \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\varepsilon^2 M \qquad M$$

- $$P_Y = \frac{1}{M^2}\overline{\phi}_3^{\,i}\psi_i\overline{\phi}_3^{\,j}\psi_j^c H + \frac{1}{M^3}\overline{\phi}_{23}^{\,i}\psi_i\overline{\phi}_{23}^{\,j}\psi_j^c H H_{45} + \frac{1}{M^2}\overline{\phi}_{23}^{\,i}\psi_i\overline{\phi}_{123}^{\,j}\psi_j^c H + \frac{1}{M^2}\overline{\phi}_{123}^{\,i}\psi_i\overline{\phi}_{23}^{\,j}\psi_j^c H$$

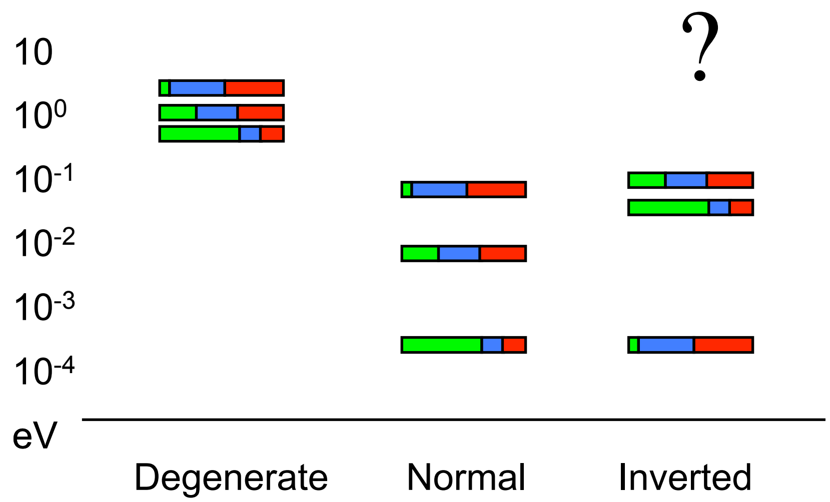
only terms allowed by G

Neutrino Parameters

Neutrino Parameters

- Tri-bi-maximal mixing

  Degenerate and normal spectra favoured



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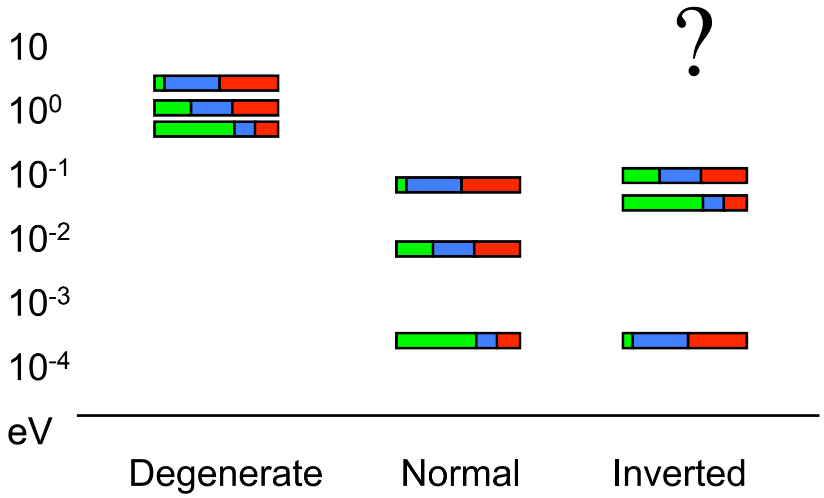

- Mixing angles

$$\sin^2\theta_{12} \approx \frac{1}{3} \pm 0.03$$

$$\sin^2\theta_{23} \approx \frac{1}{2} \pm 0.03$$

$$\sin\theta_{13} \approx \sqrt{\frac{m_e}{2m_\mu}} = 0.053 \pm 0.05 \quad (3 \pm 3^o)$$

From charged lepton mixing

King; GGR, Varzielas

Neutrino Parameters

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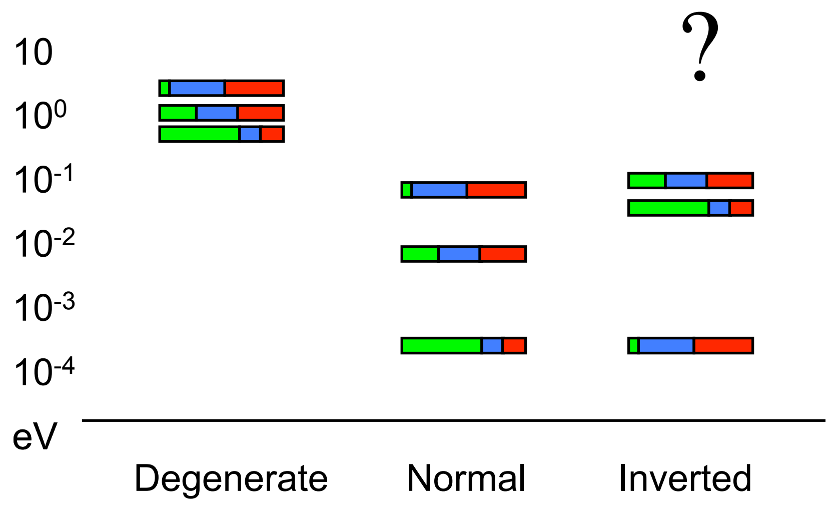

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King; GGR, Varzielas

$$\longrightarrow \quad \theta_{12} + \frac{1}{\sqrt{2}}\frac{\theta_C}{3}\cos(\delta - \pi) \simeq 35.26 \pm 2^o$$

$3\cos(\delta - \pi)$

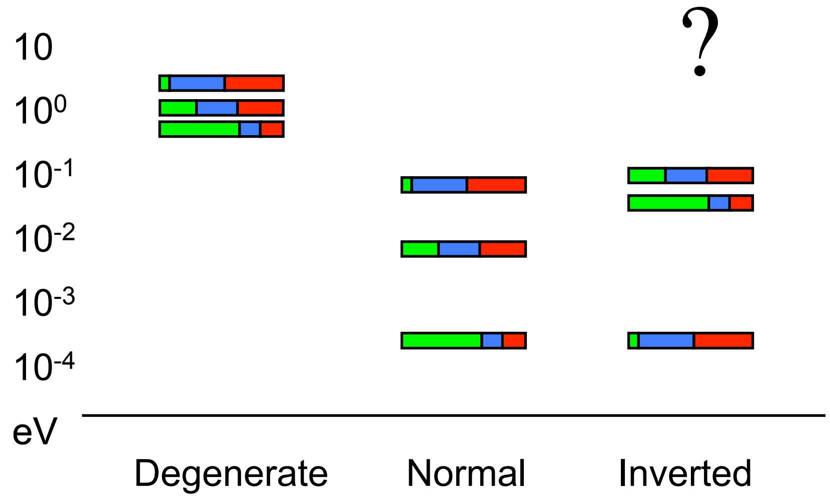
From charged lepton mixing

Antusch, King

Neutrino Parameters

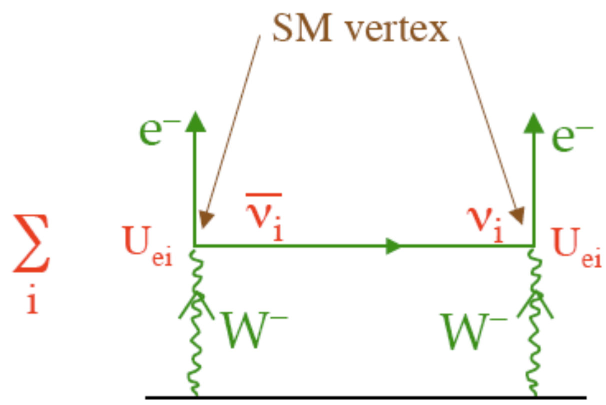
- Tri-bi-maximal mixing

  Degenerate and normal spectra possible



- Neutrinoless double β decay

  (3 light neutrinos)



Mass (ν_i)

$$\mathrm{Amp}[0\nu\beta\beta] \propto \left|\sum m_i U_{ei}^2\right| \equiv m_{\beta\beta}$$

Neutrino Parameters

- Tri-bi-maximal mixing

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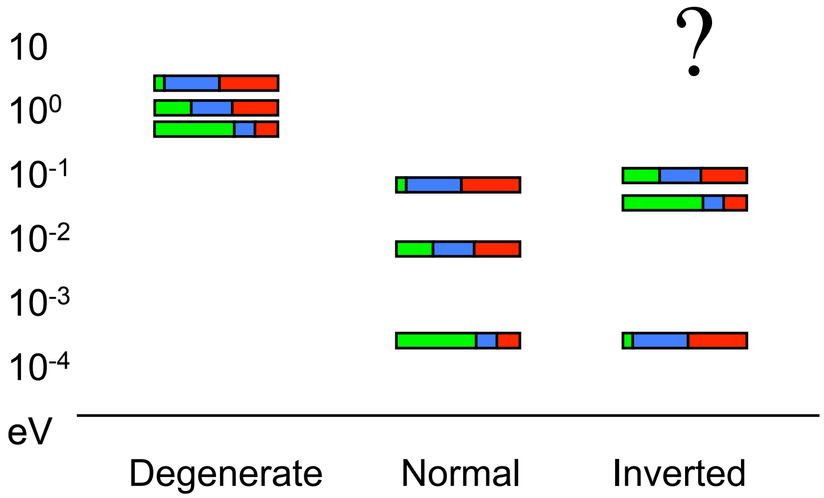

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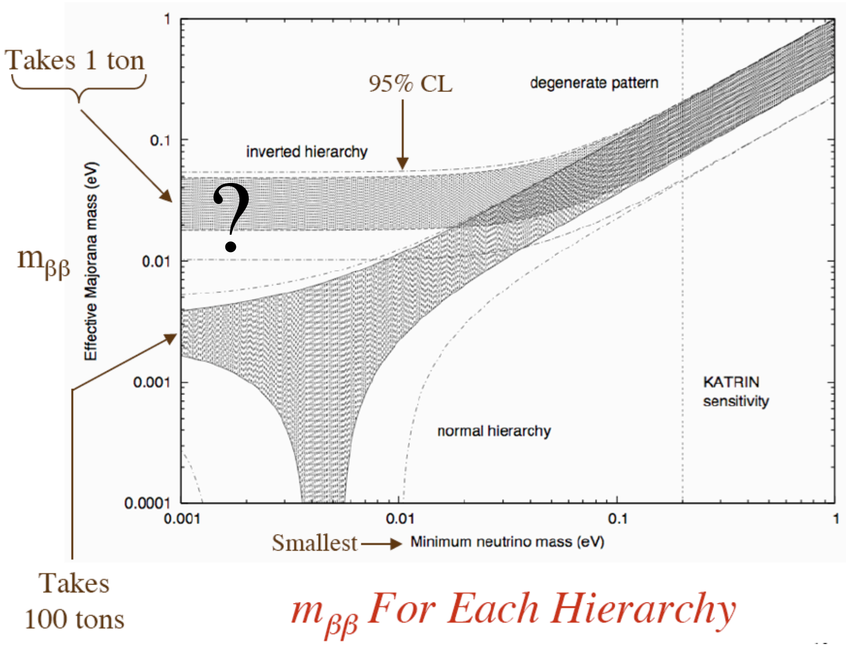
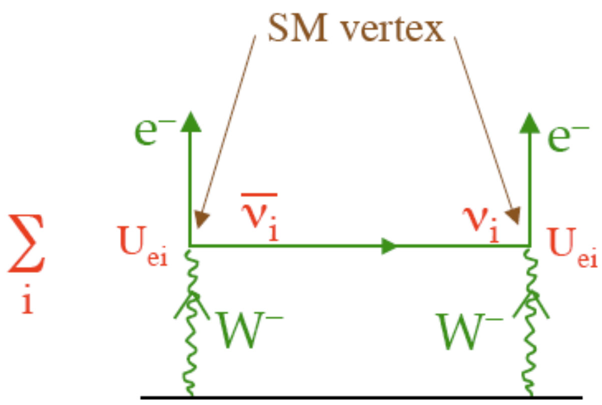


$m_{\beta\beta}$ For Each Hierarchy



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See saw parameters

See saw parameters

Masses + Mixing Angles	6	(15)
CP violating phases	3	(6)

$$\kappa = Y_\nu^T M^{-1} Y_\nu$$

Insufficient information

$$P = Y_\nu^\dagger Y_\nu$$

RGE for sleptons (SUSY)

In *principle* all parameters determined by neutrino structure and radiative corrections (assuming degenerate sleptons at GUT scale)

Davidson, Ibarra

See saw parameters

Masses + Mixing Angles 6 (15)
CP violating phases 3 (6)

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Davidson, Ibarra

Thermal Leptogenesis (Baryogenesis)

$$M_{\nu_R} > 10^9 \, GeV, \quad T_{\text{Reheat}} > 10^{10} \, GeV \quad \text{(hierarchical spectrum)}$$

$\beta\beta$ phase

$$\cancel{C}P = \cancel{C}P(\delta, \phi', \phi_{i=1,2,3})$$ see saw phases

ν factory phase

See saw parameters

Masses + Mixing Angles	6	(15)	(8)
CP violating phases	3	(6)	(3)

$$\kappa = Y_\nu^T M^{-1} Y_\nu$$

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Fukugita, Yanagida

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$\beta\beta$ phase

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see saw phases

ν factory phase

Sequential see saw and symmetry (texture zero) simplifies structure…

Sequential see saw and (1,1) texture zero

(1,1) texture zero

$$\frac{M^{Dirac}}{m_3} = \begin{pmatrix} <\varepsilon^4 & \varepsilon^3+\varepsilon^4 & -\varepsilon^3+\varepsilon^4 \\ \varepsilon^3+\varepsilon^4 & a\varepsilon^2+\varepsilon^3 & -a\varepsilon^2+\varepsilon^3 \\ -\varepsilon^3+\varepsilon^4 & -a\varepsilon^2+\varepsilon^3 & 1 \end{pmatrix}$$

Sequential see saw and (1,1) texture zero

Quark sector zero $\Rightarrow$ $\left|V_{us}\right| = \left|\sqrt{\frac{m_d}{m_s}} - e^{i\delta}\sqrt{\frac{m_u}{m_c}}\right|$

(1,1) texture zero

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Sequential see saw and (1,1) texture zero

Lepton sector zero $\Rightarrow \sin^2\theta_{23} \simeq \frac{1}{2}, \quad \sin\theta_{13} \simeq \sqrt{\frac{m_e}{2m_\mu}}$

(1,1) texture zero

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(1,1) texture zero

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$$\boxed{Y_B \propto \sin^2\theta_{13}\sin(2\delta-\phi)}$$

Abada et al

Summary

- Sequential see saw consistent with quark-lepton unification

Summary

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- Other model implications

  SUSY models: sparticles have related mass structure

  $(m^2\phi_i^\dagger\phi_i \ldots$ degeneracy split by small fermion mass related terms)

  $FCNC: \quad L_i, B_i$ *violation – close to present bounds*

  ($\mu \to e\gamma$, mercury EDM within a factor of 10 of present limits)