

When $\delta_{CP} = 0^\circ$ and Matter Effects are neglected then,

$$P_{\nu_\mu \rightarrow \nu_e} \simeq 4c_{13}^2 s_{13}^2 s_{23}^2 \sin^2 \Delta_{31} \quad (1)$$

$$+ (8c_{12}s_{12}c_{13}^2 s_{13}s_{23}c_{23} - 8s_{12}^2 c_{13}^2 s_{13}^2 s_{23}^2) \cos \Delta_{23} \sin \Delta_{31} \sin \Delta_{21} \quad (2)$$

As $\Delta_{23} = \Delta_{21} + \Delta_{13}$ then

$$\cos \Delta_{23} = \cos \Delta_{21} \cos \Delta_{13} - \sin \Delta_{21} \sin \Delta_{13}$$

and in numerical application $\Delta_{21} = 1.27\delta m_{21}^2 L/E \approx O(10^{-2})$. So,

$$P_{\nu_\mu \rightarrow \nu_e} \simeq 4c_{13}^2 s_{13}^2 s_{23}^2 \sin^2 \Delta_{31} \quad (3)$$

$$+ (8c_{12}s_{12}c_{13}^2 s_{13}s_{23}c_{23} - 8s_{12}^2 c_{13}^2 s_{13}^2 s_{23}^2) \Delta_{21} \cos \Delta_{13} \sin \Delta_{31} \quad (4)$$

If one uses $s_{12}^2 = 0.314$ and $s_{23}^2 = 0.44$ then one realizes that in the parenthesis the second term is of the order s_{13} compared to the first term, so it may be neglected hereafter as we will focus on $s_{13} < 10^{-2}$. Then, it yields

$$P_{\nu_\mu \rightarrow \nu_e} \simeq \alpha \cos \beta \sin^2 \Delta_{31} + \alpha \sin \beta \cos \Delta_{13} \sin \Delta_{31} \quad (5)$$

with

$$\alpha \cos \beta \equiv 4c_{13}^2 s_{13}^2 s_{23}^2 \quad (6)$$

$$\alpha \sin \beta \equiv 8c_{12}s_{12}c_{13}^2 s_{13}s_{23}c_{23} \quad (7)$$

It is remarkable that now the oscillation probability may be written as

$$P_{\nu_\mu \rightarrow \nu_e} \simeq \frac{\alpha}{2} [\cos \beta - \cos (2\Delta_{13} + \beta)] \quad (8)$$

The maximum of the probability is obtained at

$$\Delta_{31} = \frac{\pi}{2} - \frac{\beta}{2} \quad (9)$$

$$\text{with } \tan \beta = 2\Delta_{21} \frac{c_{12}s_{12}c_{23}}{s_{13}s_{23}} \quad (10)$$

The "usual" 2-families case is obtain with $\beta = 0$. In case of $E \sim 0.3$ GeV, $L \sim 130$ km and $\sin^2 2\theta_{13} = 10^{-3}$ then

$$(\delta m_{31}^2)_{max} = 2.9 \cdot 10^{-3} \text{ if } \beta = 0 \quad (11)$$

$$= 1.8 \cdot 10^{-3} \quad (12)$$