Decay Channel Optimization using MARS

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From work in collaboration with
C. Johnstone and **MARS** (i.e., Nikolia Mokhov)

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Pion/Muon Production Overview:

- Choosing a target...

  ...Mercury jet (high density, but difficult to work with)

  ...Graphite rod (lower density, but easy to work with)

  ...other possibilities?

- The pion decay channel...

  ...Dominant decay mode ($\sim$ 100%):

  $$\pi^+ \to \mu^+ \, \nu_\mu$$
  $$\pi^- \to \mu^- \, \bar{\nu}_\mu$$

  ...Lifetime: $\tau = 26$ ns (or $c\tau = 7.8$ m)

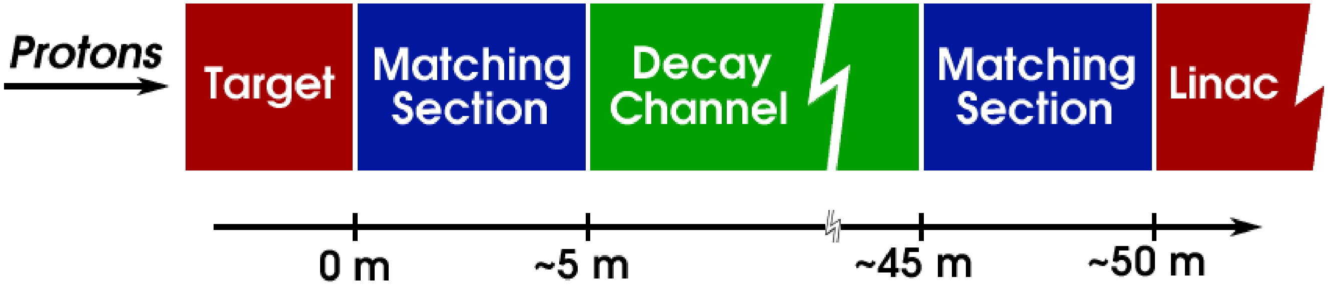
  ...After a 40 m decay channel:
  $\sim$ 99% of pions with $p = 150$ MeV/c decay
  $\sim$ 95% of pions with $p = 250$ MeV/c decay

...Use solenoids to contain the beam

- Matching sections...

  ...To match from target region into decay channel, use adiabatic magnetic matching sections

  ...Use something else to match into the buncher/linac (see C. Johnstone)



The Target Region:

- Using Feasibility Study I target design...

  ...Graphite rod (80 cm long, 1.5 cm diamater)
  at a 50 mrad angle w.r.t. central axis
  (parallel to incident proton beam)

  ...1 MW incident proton beam power with 16 GeV
  protons in pulses at 15 Hz

  ...Allows for $\sim 5 \times 10^{12}$ muons per pulse
  if ~ 0.2 muons produced per proton on target

  ...Currently, leaving target design *unchanged*

- “Large” capture solenoid...

  ...Chosen to capture pions of given p_T,

  $$(p_T)_{MAX} = e\,B_0\left(\frac{R_0}{2}\right)$$

  where B_0 is the strength of the capture solenoid, and R_0 is the radius of the beampipe (one-half the full aperture) at the target ($z = 0$ cm)

  ...Aperture limited to 60 cm at the end of the decay channel (beginning of buncher/linac)

  ...Field strength in buncher set at $B = 1.25$ T

  ...Hence, magnetic flux fixed at the end of the decay channel:

  $$\Phi \approx B\,(\pi R^2) = 0.353 \text{ Wb}$$

- The capture solenoid...

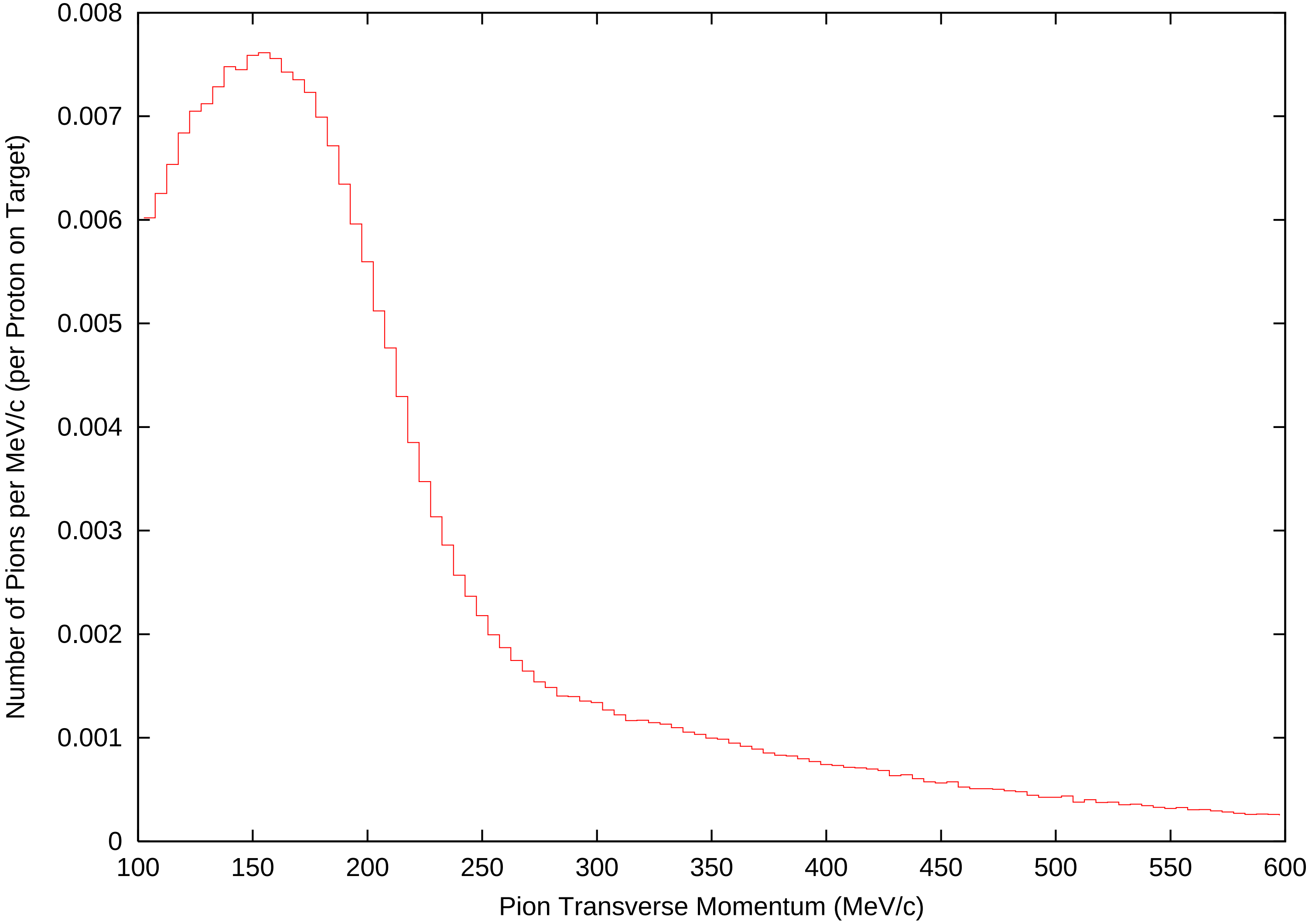
...By matching the sections adiabatically, magnetic flux is conserved throughout the decay channel:

$$\Phi_0 \approx B_0\,(\pi R_0^2) = \Phi$$

...Hence, increasing the captured pion $(p_T)_{MAX}$ means decreasing the initial aperture:

$$(p_T)_{MAX} = \frac{e\,\Phi}{2\pi\,R_0}$$

...How high should $(p_T)_{MAX}$ be?



...How about $\sim$ 225 MeV/c?

- The capture solenoid (*continued*)...

...With the choice $(p_T)_{MAX} = 225$ MeV/c, we find

$$R_0 = \frac{e\,\Phi}{2\pi\,(p_T)_{MAX}} = 7.5\ \text{cm}$$

$$B_0 = \frac{\Phi}{\pi\,R_0^2} = 20\ \text{T}$$

...Strong! But is it too strong? Let's hope not...

...If too strong, then only option is magnetic horns?

Matching Sections:

- How to match solenoids into solenoids...

...Use solenoids!

...Simultaneously change field strength and aperture with z, keeping the total flux through the beampipe fixed:

$$B_0 \, R_0^2 = B(z) \, R(z)^2$$

...Change must be “slow” in order to be adiabatic:

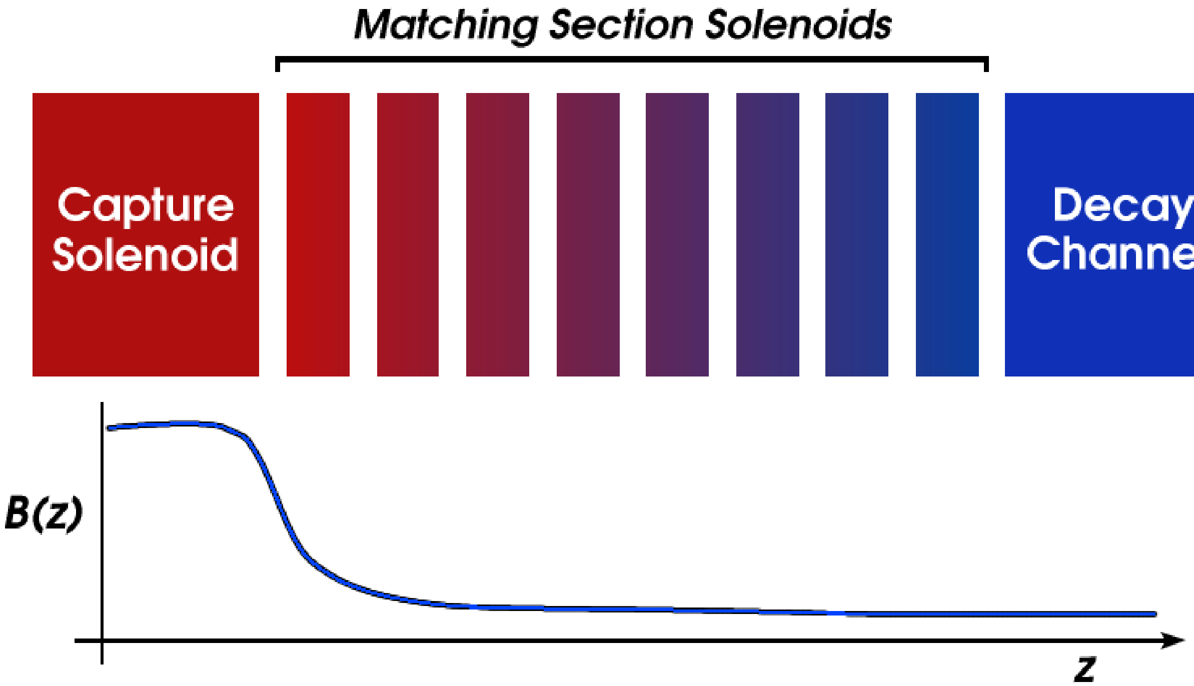
$$a \ll R_B, \; B \left(\frac{\partial B}{\partial z} \right)^{-1}$$

where a is the particle’s Larmor radius of orbit, and R_B is the radius of curvature of the field lines

...Design half-aperture, $R(z)$, with these constraints and fit $B(z)$ accordingly (using short solenoids) such that:

$$B(z) = B_0 \left(\frac{R_0}{R(z)}\right)^2$$

is the field strength *on axis*



- Adiabatic matching sections...

...Should hold pions with $P_T < 225$ MeV/c

...Easily understood in terms of the adiabatic invariants:

$$Ba^2 \qquad \frac{p_T^2}{B}$$

(See *Jackson*, Sect. 12.6)

...As the particle moves along z, B decreases, p_T decreases and a increases

...Since kinetic energy is conserved, the longitudinal momentum p_z increases

BOTTOM LINE:

It decreases the divergence of the beam
at the cost of increased spot size!

- Designing adiabatic matching sections...

...Determine constraints on aperture of adiabatic region:

$$R(z_1) \equiv R_1 \qquad \left(\frac{\partial R}{\partial z}\right)|_{z_1} \equiv \lambda_1$$

$$R(z_2) \equiv R_2 \qquad \left(\frac{\partial R}{\partial z}\right)|_{z_2} \equiv \lambda_2$$

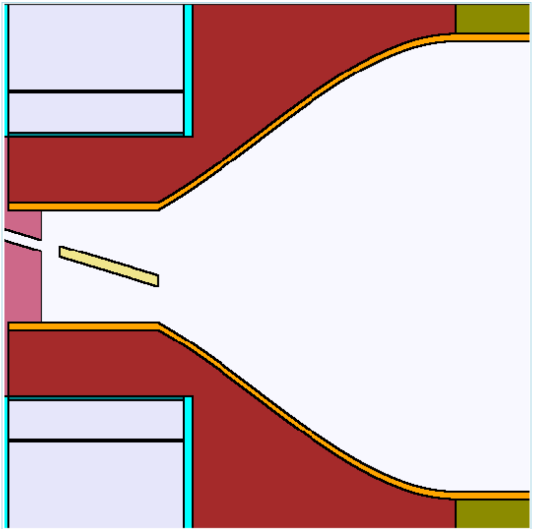
...Simple choice for $R(z)$:

$$R(z) \;\equiv\; \left(\alpha_0 + \alpha_1 z + \alpha_2 z^2 + \alpha_3 z^3\right)^{\frac{1}{k}}$$

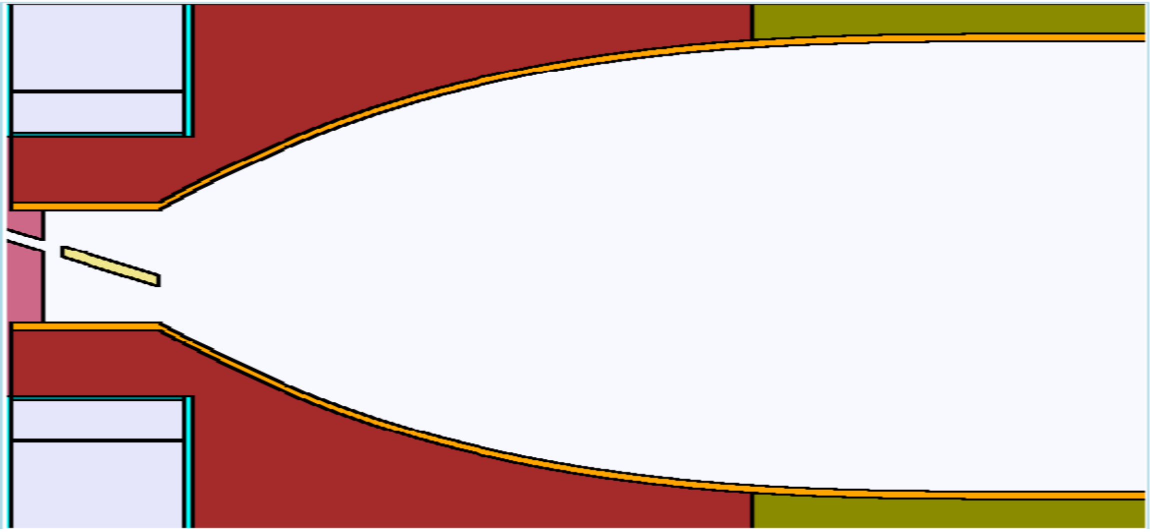
Solve for $\alpha_i(k)$'s

...Easy! But how do we choose k?
Minimize curvature & maximize length!
(i.e., choose $k \approx 1$)

...Consider a “short” section (240 cm):

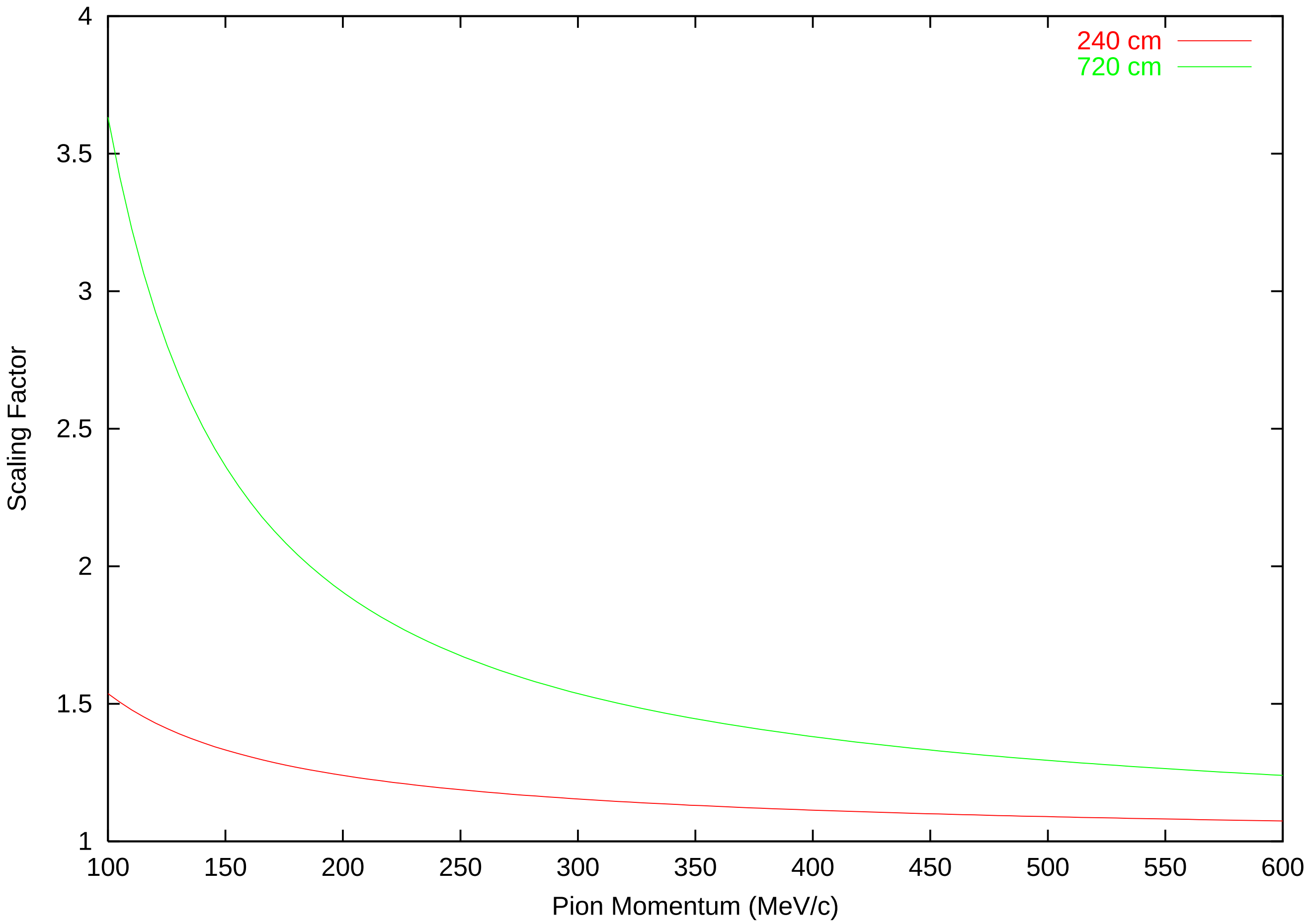


...And a “long” section (720 cm):

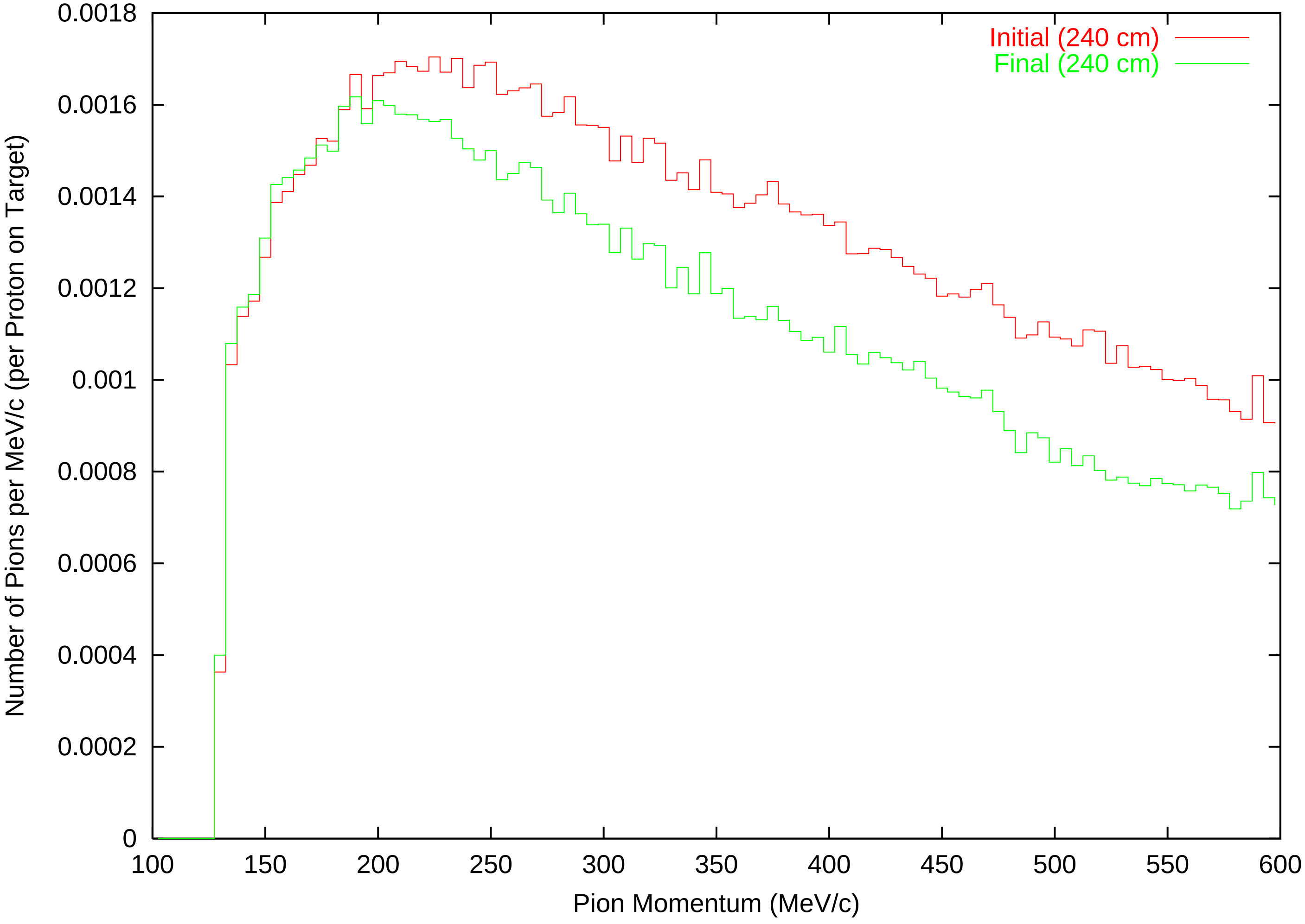


...Distributions at the end of the sections
include losses due to pion decay

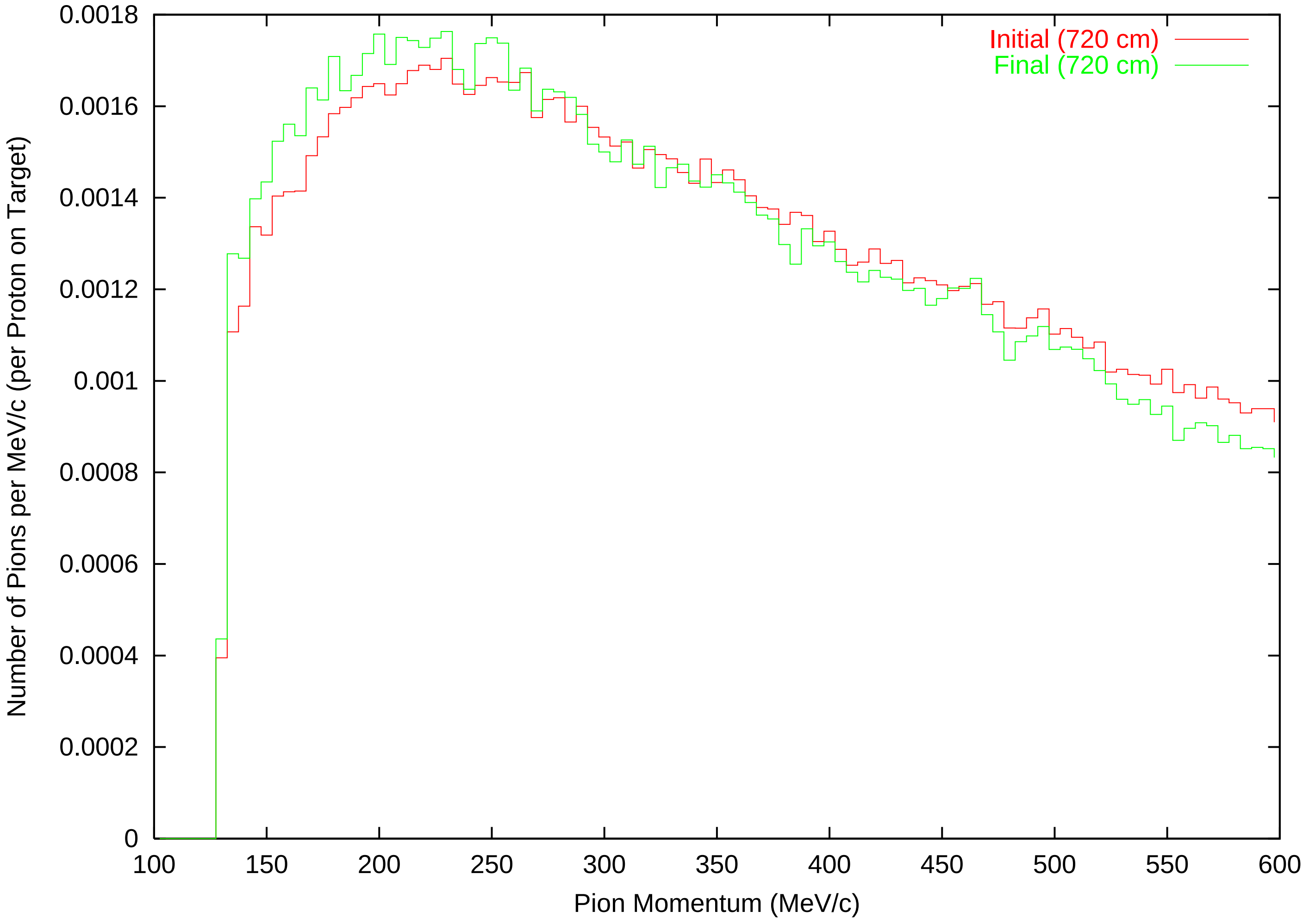
...Scale them according to the pion's momentum!



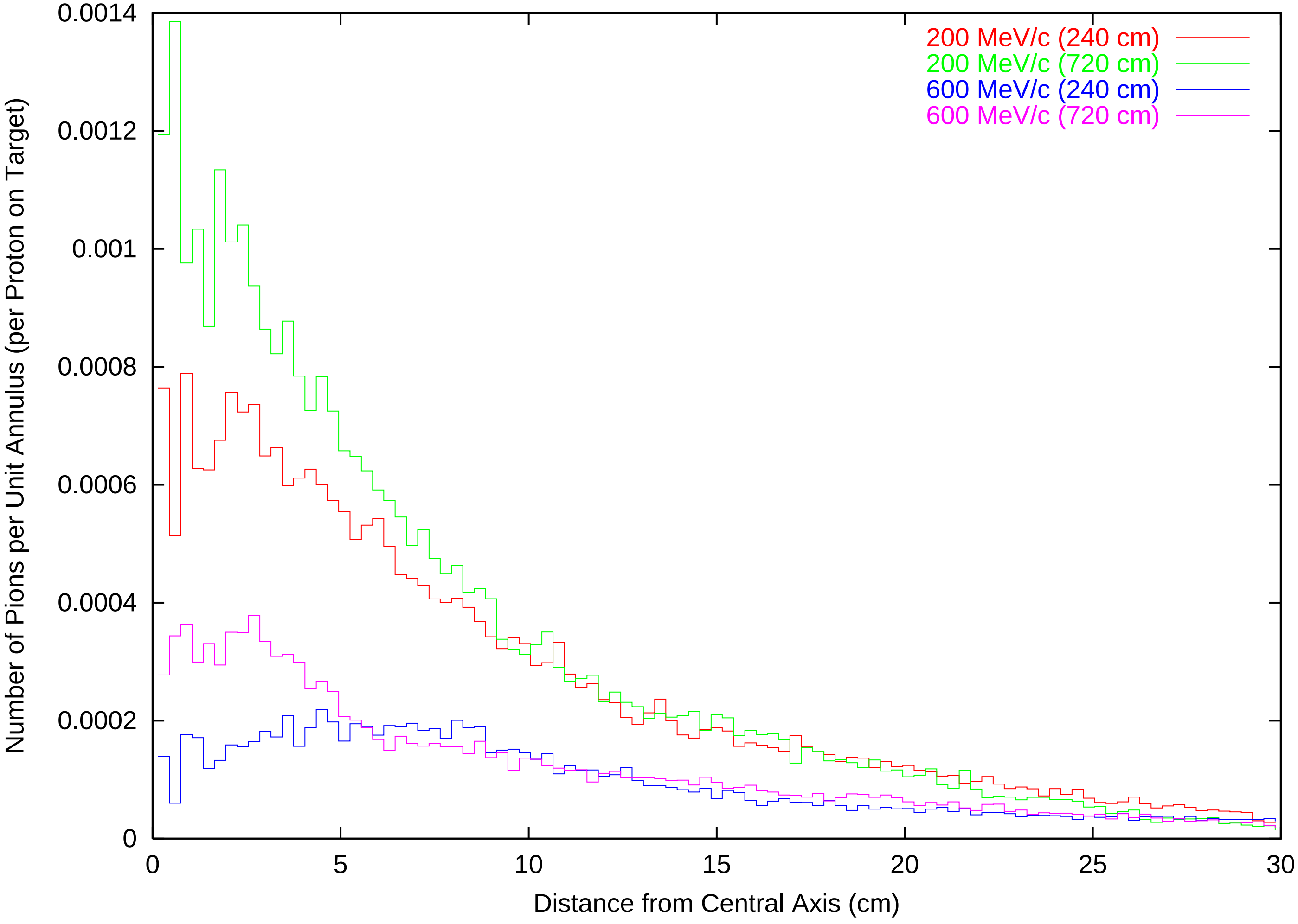
...Scaled momentum distributions in short section:



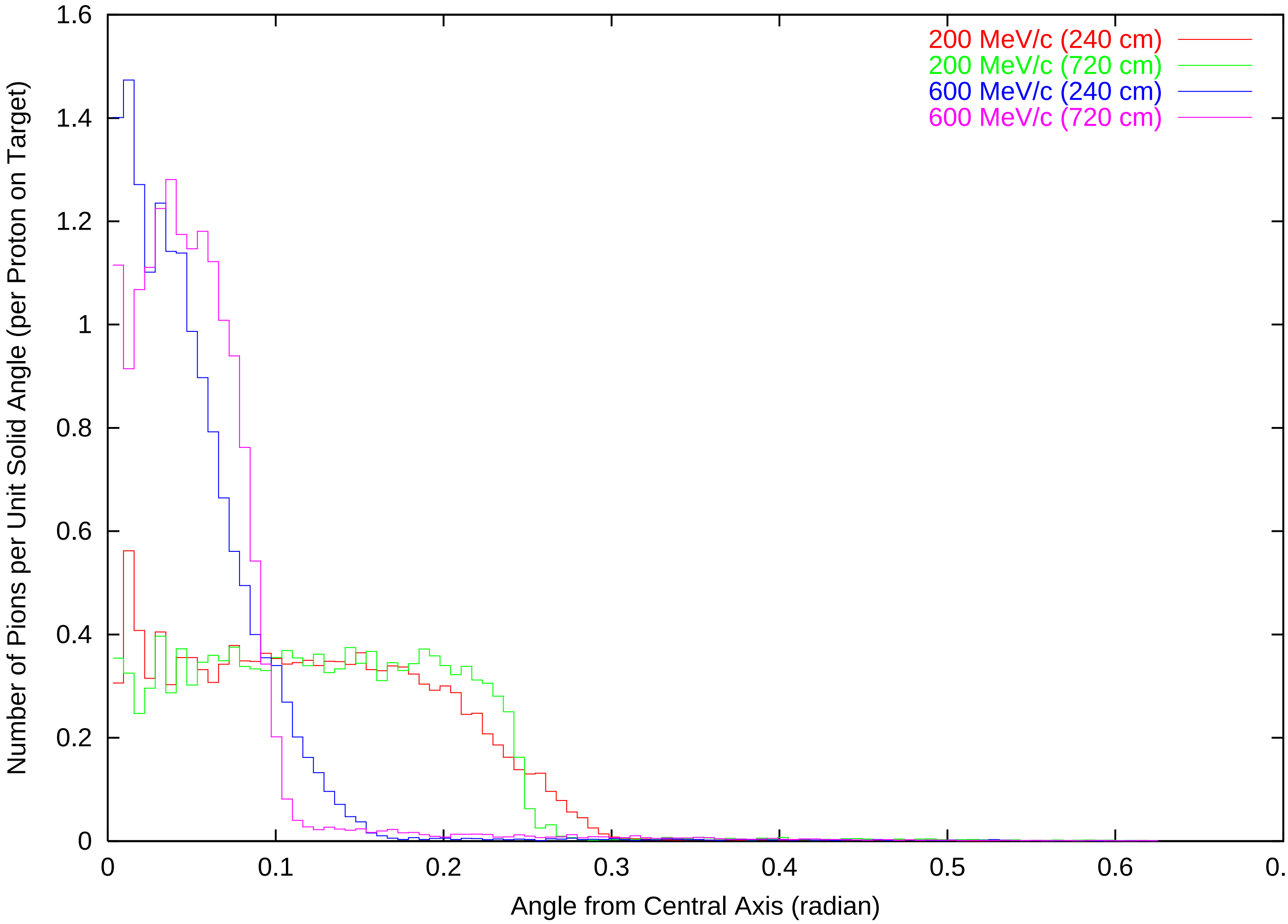
...Scaled momentum distributions in long section:



...Scaled radial distributions:



...Scaled angular distributions:



...Final Muon Yields for $p = 100 - 350$ MeV/c:

	SHORT (240 cm)	LONG (720 cm)
μ^+/P	0.163	0.181
μ^-/P	0.154	0.170
$\mu^\pm/P$	0.317	0.352

(SHORT section equivalent to FS1 geometry)

...RMS emittance for various $p \pm 20$ MeV/c:

	SHORT (240 cm)	LONG (720 cm)
100 MeV/c	1.78π cm rad	1.76π cm rad
200 MeV/c	1.36π cm rad	1.38π cm rad
300 MeV/c	$.941\pi$ cm rad	$.911\pi$ cm rad

- Further improvements...

...In the decay channel:

$$(p_T)_{MAX} \sim 28 \text{ MeV/c}$$

...During pion decay:

$$\begin{aligned} \langle p_T \rangle &\sim \frac{1}{2} m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \langle \sin^2 \theta \rangle \\ &= \frac{1}{4} m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \\ &\approx 15 \text{ MeV/c} \end{aligned}$$

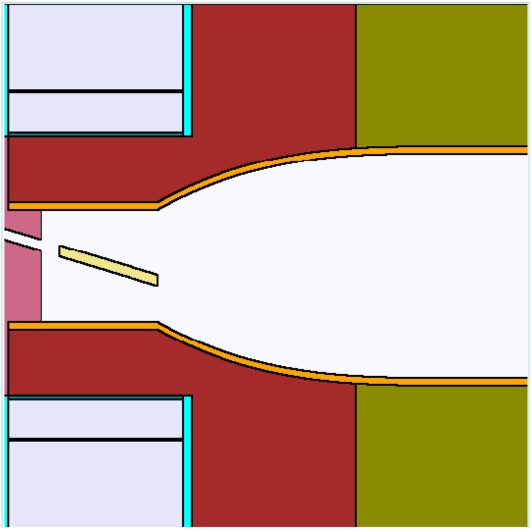
...Increase $(p_T)_{MAX} \rightarrow$ Increase B

...In a 5 T decay channel:

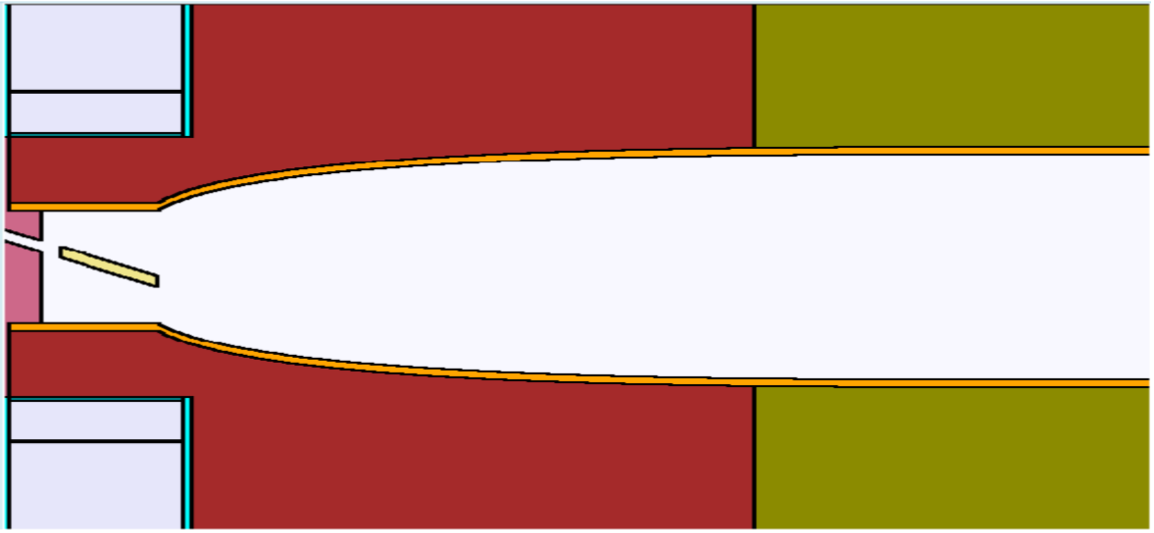
$$(p_T)_{MAX} \sim 56 \text{ MeV/c}$$

with a 30 cm ($R = 15$ cm) aperture

...A “short” initial section (240 cm):

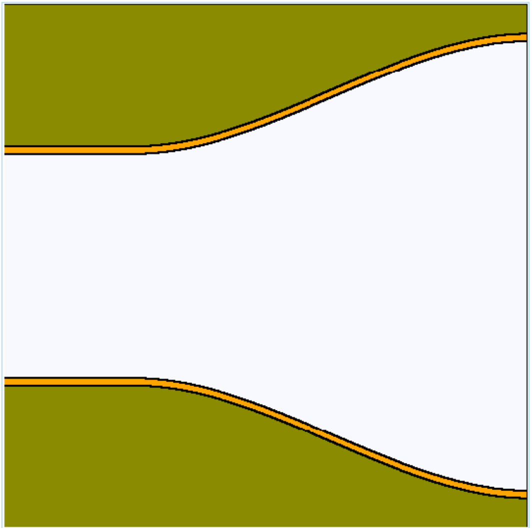


...Or a “long” initial section (720 cm):



...Follow with a 47 m decay channel, with
a field strength of 5 T

...End with another adiabatic matching section,
matching from 5 T to 1.25 T:



...Called a DUET configuration

...Final Muon Yields for $p = 100 - 350$ MeV/c:

	SHORT (240 cm)	LONG (720 cm)	DUET SHORT (240 cm)	DUET LONG (720 cm)
μ^+/P	0.163	0.181	0.192	0.193
μ^-/P	0.154	0.170	0.183	0.185
$\mu^\pm/P$	0.317	0.352	0.375	0.378

...RMS emittance for various $p \pm 20$ MeV/c:

	SHORT (240 cm)	LONG (720 cm)	DUET SHORT (240 cm)	DUET LONG (720 cm)
100 MeV/c	1.78π cm rad	1.76π cm rad	1.42π cm rad	1.30π cm rad
200 MeV/c	1.36π cm rad	1.38π cm rad	1.30π cm rad	1.30π cm rad
300 MeV/c	$.941\pi$ cm rad	$.911\pi$ cm rad	$.936\pi$ cm rad	$.908\pi$ cm rad

- Conclusions:

  ...Losses mostly due to pion decay

  ...Increasing decay channel field strength significantly increases yields

  ...Adiabaticity effects are large for low field strength channels

  ...Phase space density depends:

    - ◇ Weakly on adiabatic effects
    - ◇ Strongly on channel field strength (for low p, but not for high p)

  ...Improved carbon target yields by 18% over FS1 results (for both μ^+ and μ^-)

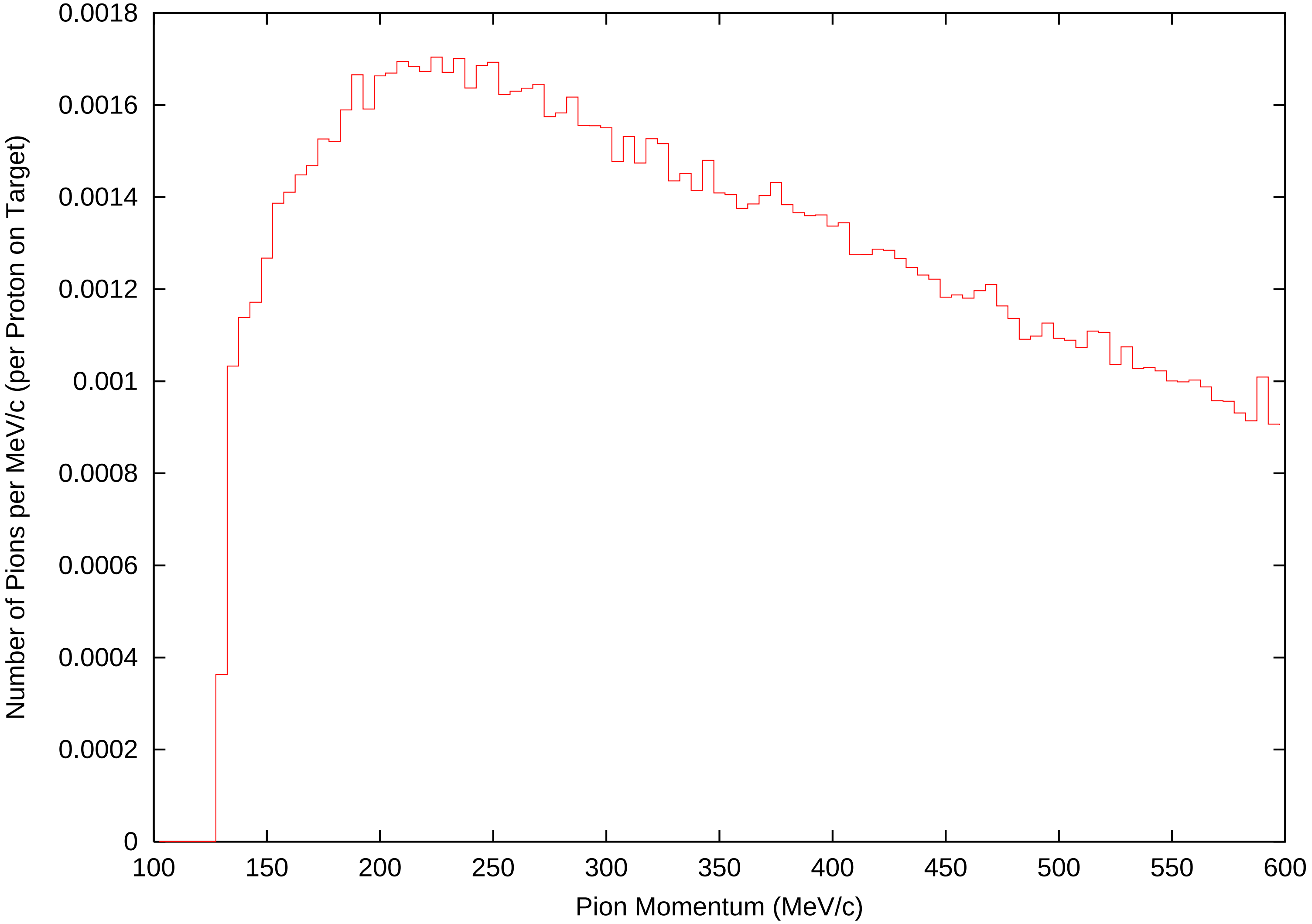
- Future work:

  …“Orange”-type capture magnet
  (Only captures one sign)

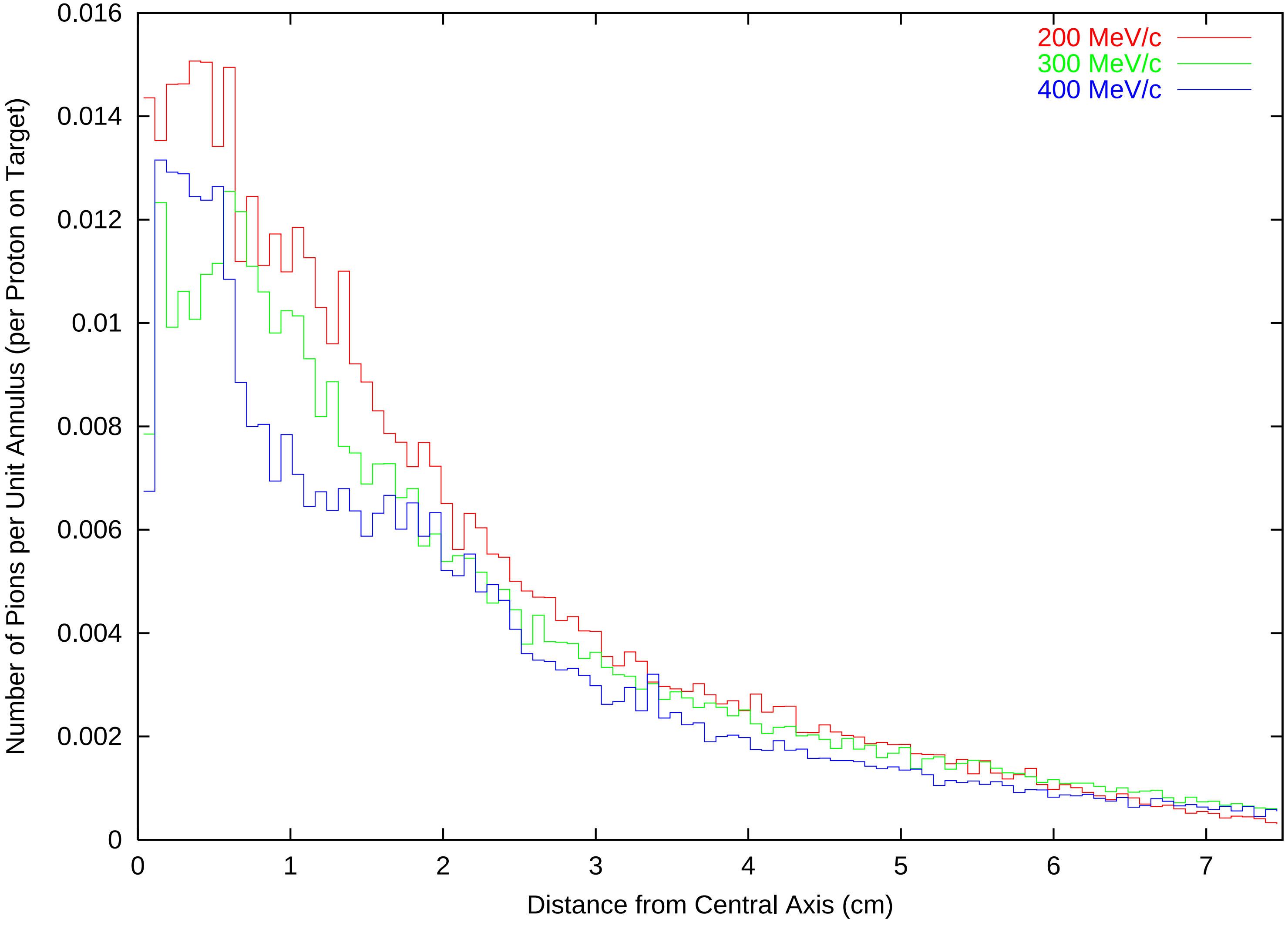
  …Compare to horn & funnel designs
  (Only captures one sign, though)

  …“Real” solenoid designs & fields

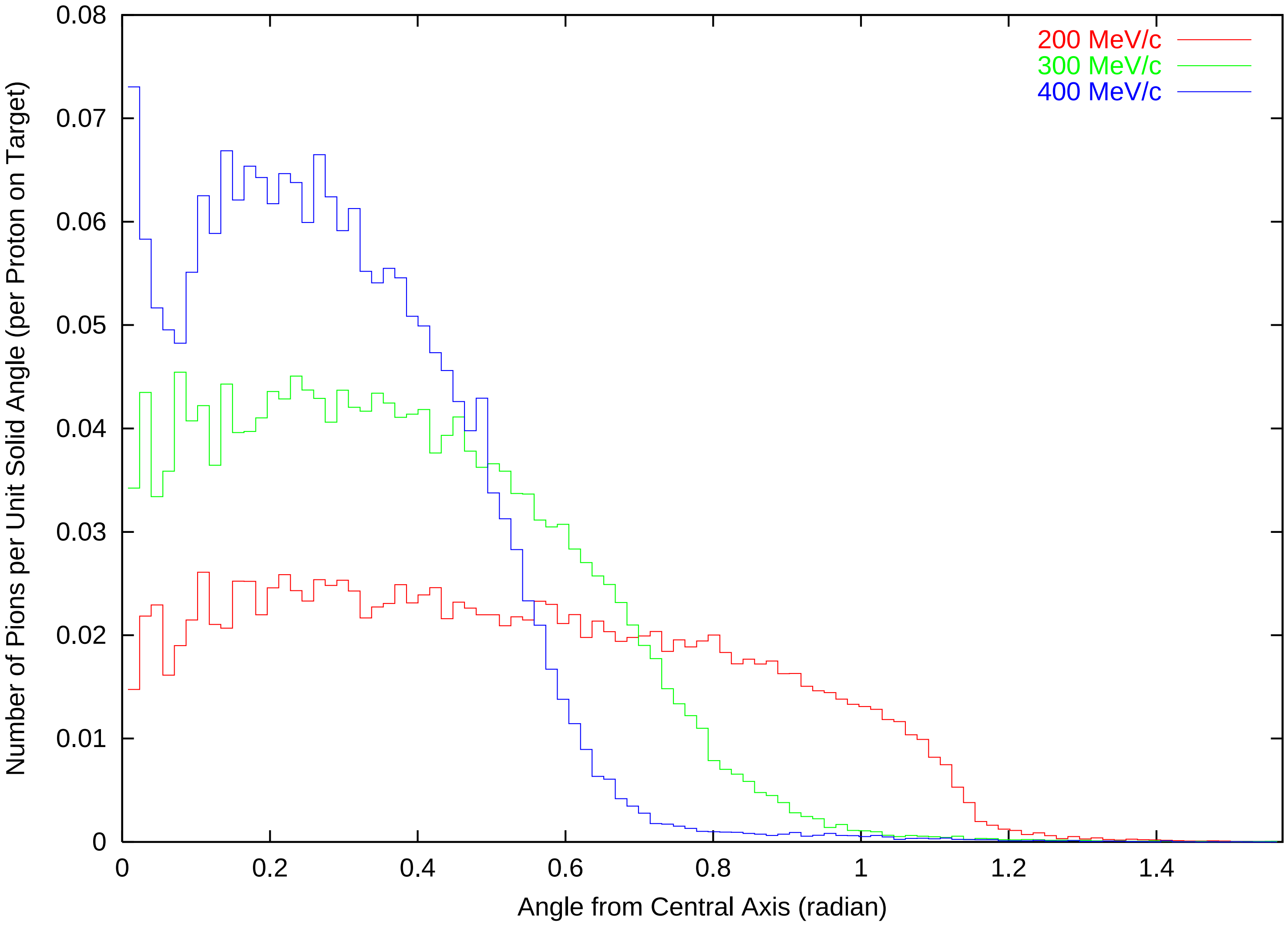
Initial momentum distribution:



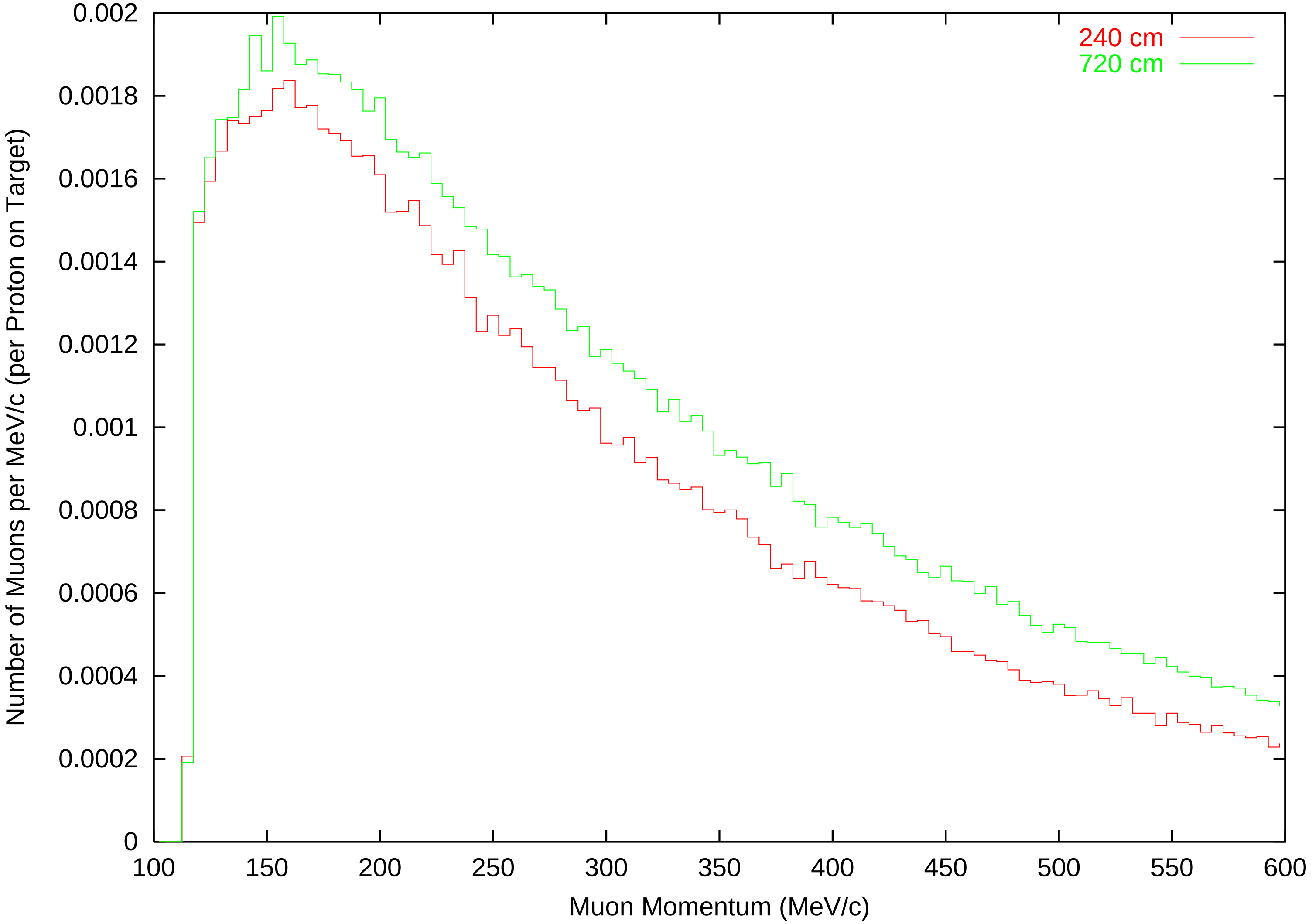
Initial radial distributions:



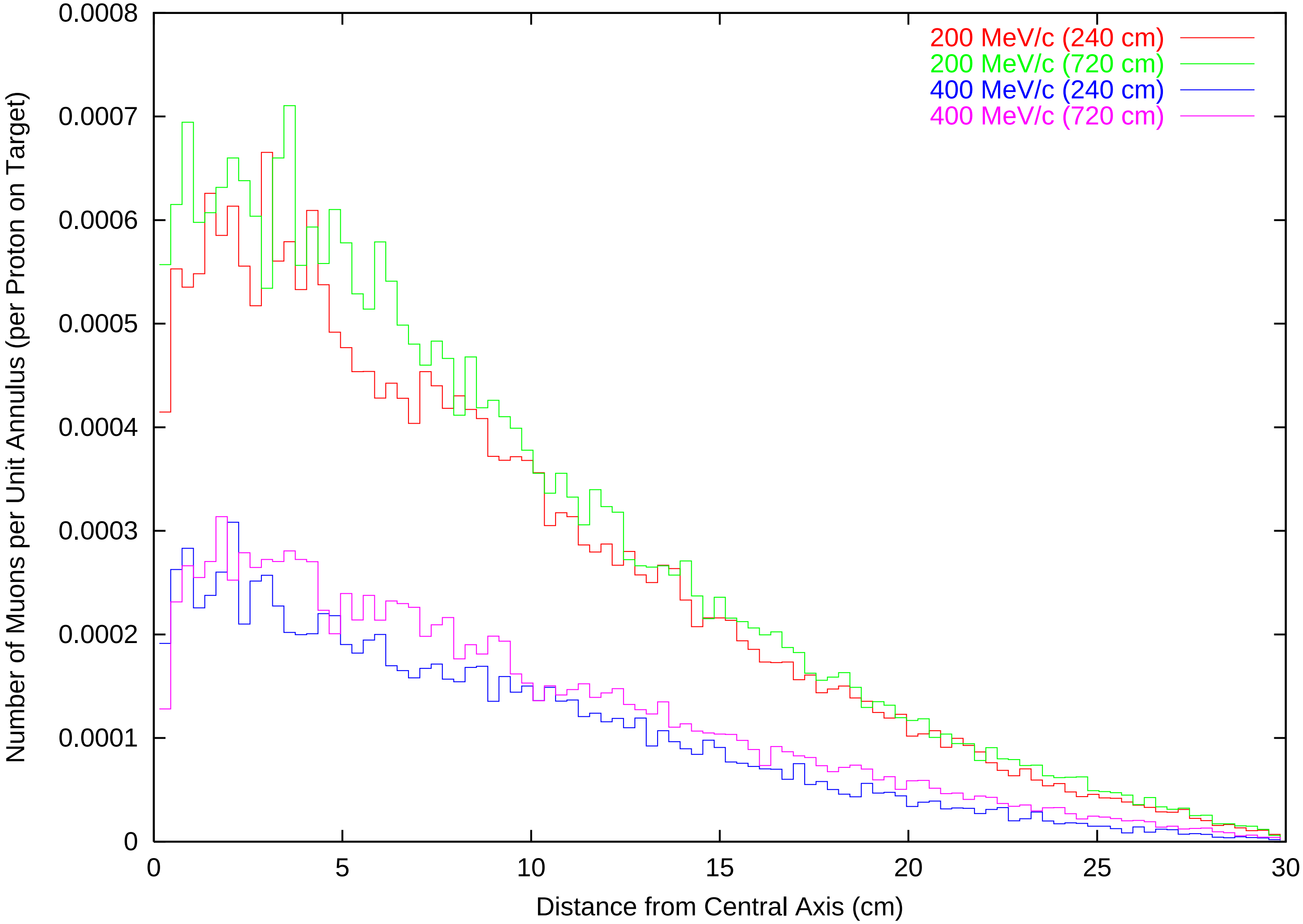
Initial angular distributions:



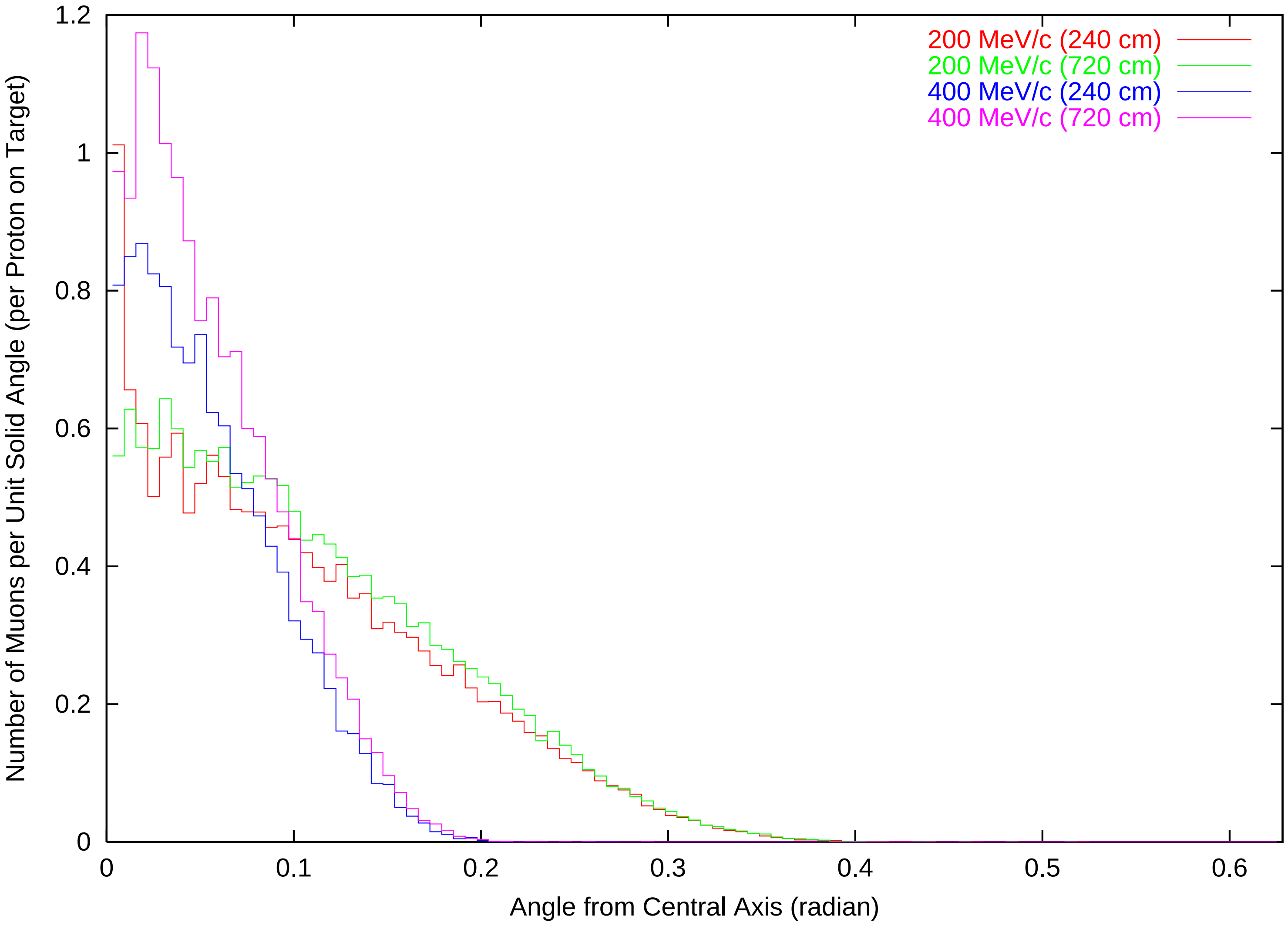
Final momentum distributions:



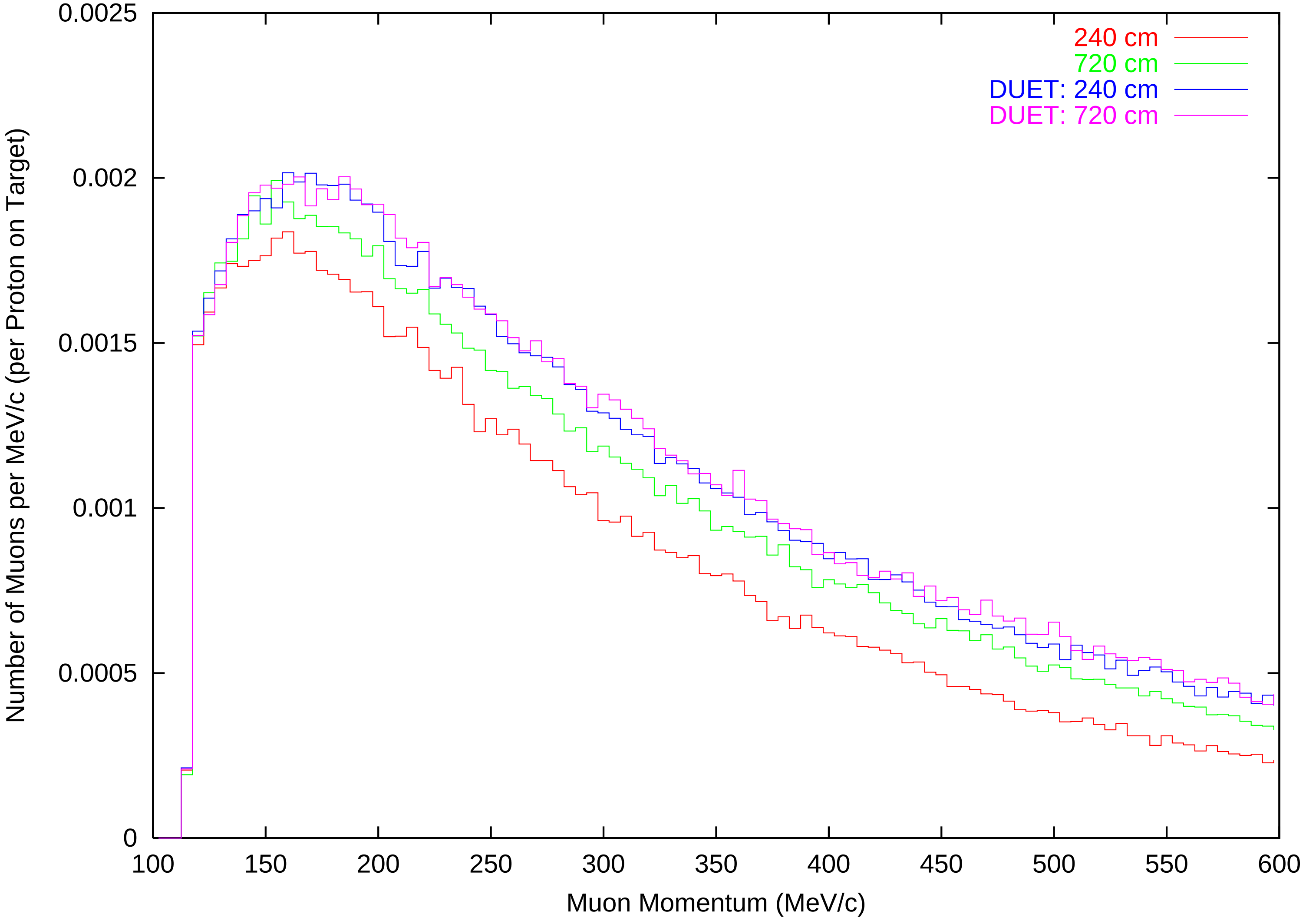
Final radial distributions:



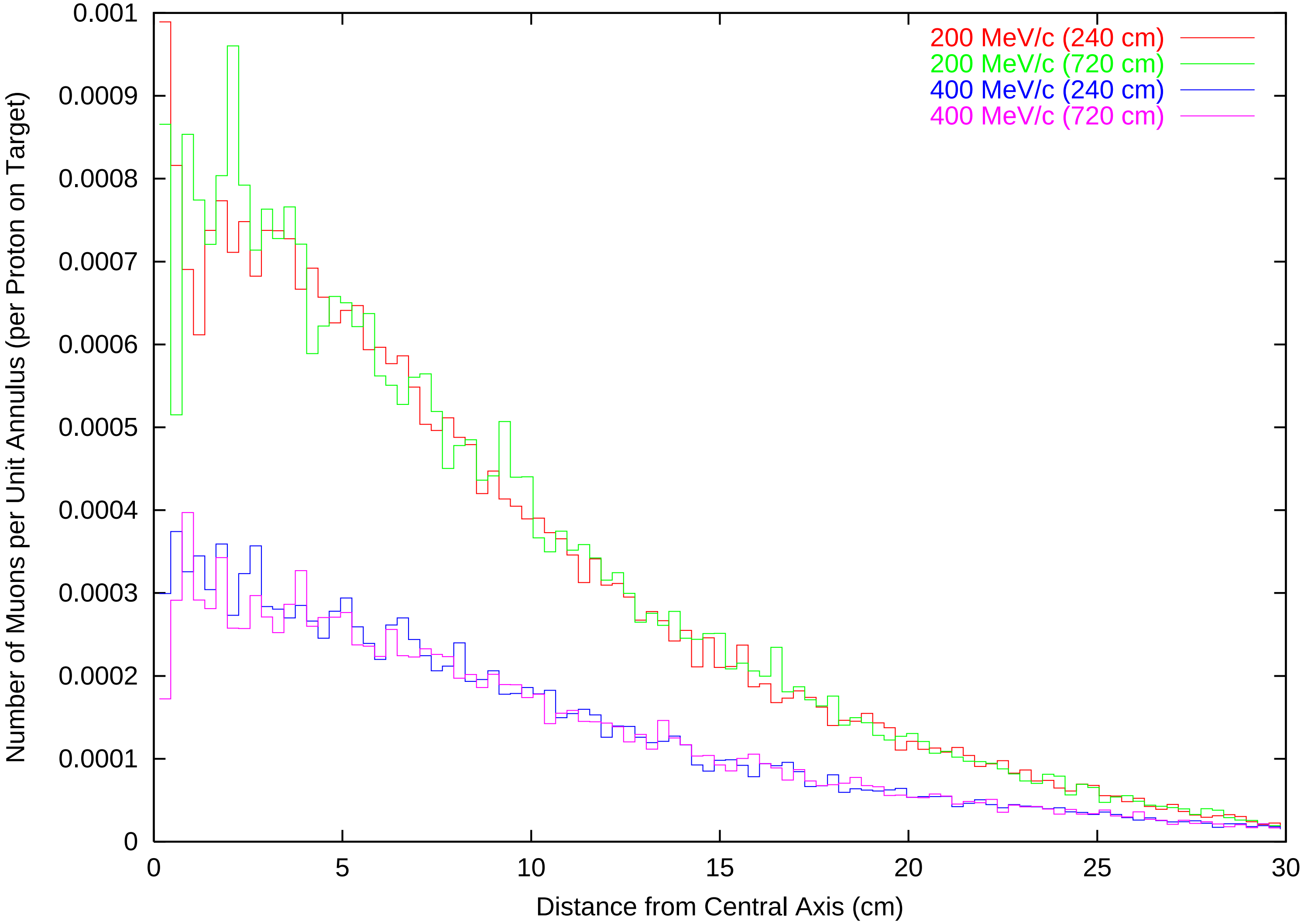
Final angular distributions:



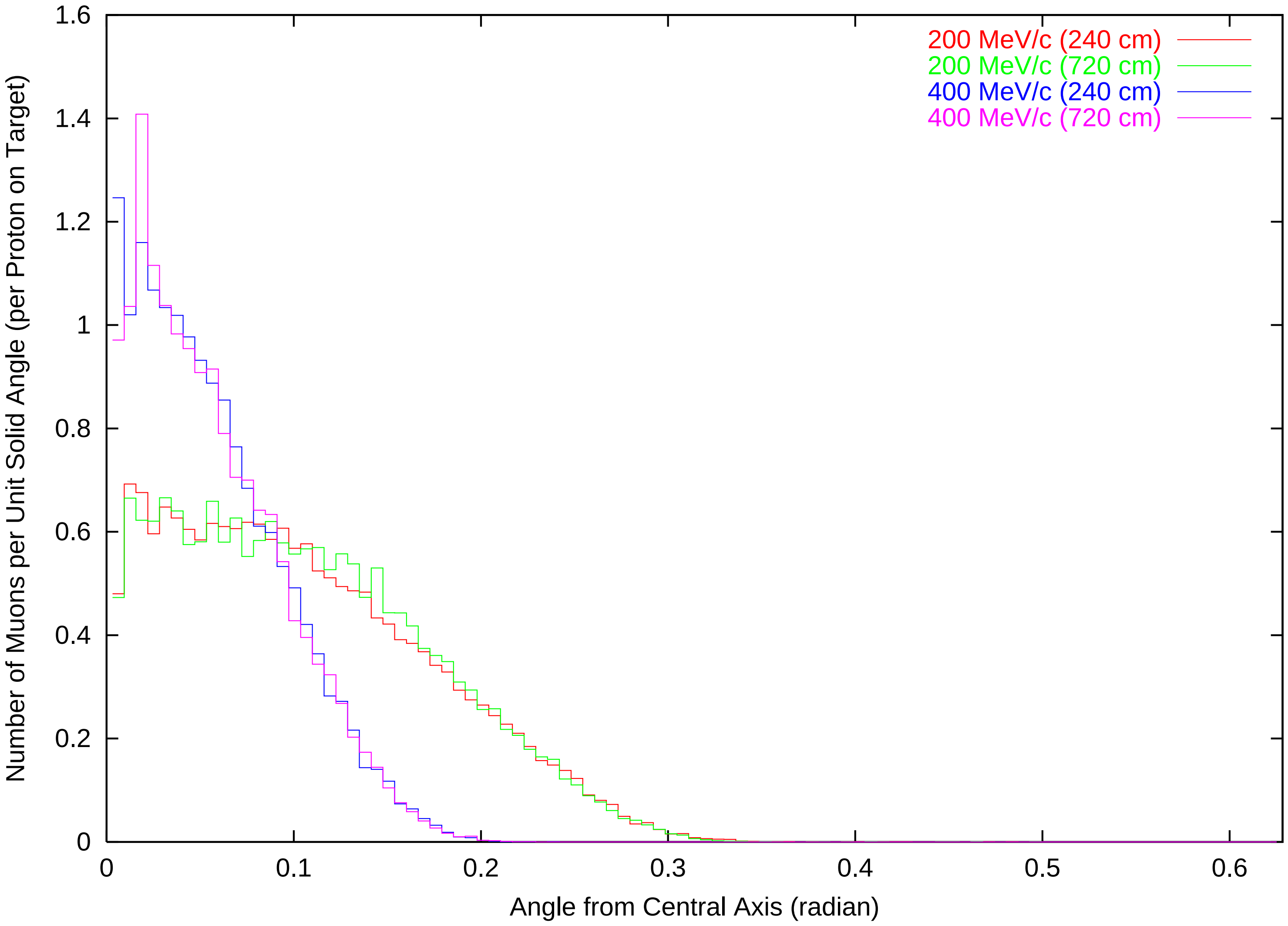
Final DUET momentum distributions:



Final DUET radial distributions:



Final DUET angular distributions:



Final Muon Yields ($p = 100 - 350$ MeV/c):

			DUET	
	SHORT (240 cm)	LONG (720 cm)	SHORT (240 cm)	LONG (720 cm)
μ^+/P	0.1630	0.1814	0.1922	0.1931
μ^-/P	0.1535	0.1701	0.1827	0.1851
$\mu^\pm/P$	0.3165	0.3515	0.3749	0.3781

Mean & RMS Emittance ($p \pm 20$ MeV/c):

			DUET	
	SHORT (240 cm)	LONG (720 cm)	SHORT (240 cm)	LONG (720 cm)
100 MeV/c	1.159π cm rad 1.775π cm rad	1.133π cm rad 1.757π cm rad	$.9493\pi$ cm rad 1.418π cm rad	$.8176\pi$ cm rad 1.300π cm rad
150 MeV/c	1.077π cm rad 1.666π cm rad	1.072π cm rad 1.642π cm rad	$.9379\pi$ cm rad 1.425π cm rad	$.9279\pi$ cm rad 1.411π cm rad
200 MeV/c	$.8719\pi$ cm rad 1.361π cm rad	$.9003\pi$ cm rad 1.384π cm rad	$.8438\pi$ cm rad 1.298π cm rad	$.8529\pi$ cm rad 1.300π cm rad
250 MeV/c	$.7457\pi$ cm rad 1.144π cm rad	$.7163\pi$ cm rad 1.122π cm rad	$.6982\pi$ cm rad 1.078π cm rad	$.6983\pi$ cm rad 1.082π cm rad
300 MeV/c	$.6071\pi$ cm rad $.9414\pi$ cm rad	$.5657\pi$ cm rad $.9114\pi$ cm rad	$.6096\pi$ cm rad $.9363\pi$ cm rad	$.5905\pi$ cm rad $.9083\pi$ cm rad
350 MeV/c	$.5232\pi$ cm rad $.8065\pi$ cm rad	$.5323\pi$ cm rad $.8202\pi$ cm rad	$.5186\pi$ cm rad $.8142\pi$ cm rad	$.5019\pi$ cm rad $.7909\pi$ cm rad