# Neutrino mixing as a source of dark energy

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We show that the vacuum condensate due to neutrino mixing in quantum field theory (QFT) contributes to the dark energy budget of the universe which gives rise to the accelerated behavior of cosmic flow. The explanation of the dark energy budget might thus not require to search for exotic candidates (e.g. scalar particles), which, up to now, have not been detected. Although we do not solve the momentum cut-off arbitrariness problem, we point out that some natural choices of the cut-off are possible in our treatment, which are consistent with the energy scale implicit in the QFT mixing formalism and with the observed cosmic accelerated behavior interpreted as dark energy.

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Data coming from cosmic microwave background radiation (CMBR) [1, 2], large scale structure [3, 4] and type Ia supernovae [5], used as standard candles, independently support the picture that the today observed universe can be consistently described as an accelerating Hubble fluid where the contribution of dark energy component to the total matter-energy density is $\Omega_\Lambda \simeq 0.7$. The big challenge is then the one of explaining such a bulk of dark energy component.

On the other hand, in recent years great attention has been devoted to the neutrino mixing phenomenon. The theoretical investigation of the neutrino mixing, firstly proposed by Pontecorvo [6], has been pursed in depth [7, 8, 9, 10, 11, 12], and more recently the issue of the construction of the flavored space of states has been settled in the framework of the quantum field theory (QFT) formalism [13, 14, 15, 16, 17, 18, 19] with the discovery of the unitary inequivalence between the flavored vacuum and the massive neutrino vacuum [13, 14], the associated finding of the neutrino-antineutrino pair condensate contributing to the vacuum energy [20] and the new oscillation formulas [14, 15, 16, 17]. The recent experimental achievements proving neutrino oscillations [21, 22] and the progresses in the QFT theoretical understanding [19] of the neutrino mixing thus provide a challenging and promising path beyond the Standard Model of electroweak interaction for elementary particles.

In this paper we show that these two most interesting issues are intimately bound together in such a way that one of them, namely the neutrino mixing phenomenon, appears to provide a contribution, till now unsuspected, to the vacuum dark energy component. One of the conclusions coming from our discussion may thus be that there is no further need to search for exotic candidates (e.g. scalar particles) for the dark energy component, which, as a matter of fact, have not been detected up to now. The structure of the flavor vacuum and its unitary inequivalence to the vacuum for the massive neutrinos play a significant rôle in obtaining the result we report below.

In the simplest explanation, the so called $\Lambda CDM$ model, the cosmological constant contributes for almost 70% to the total matter-energy density budget. The standard theory of cosmological constant is based on the fact that the vacuum zero point energy cannot violate the Lorentz invariance of the vacuum and therefore the corresponding energy-momentum tensor density has the form $\mathcal{T}_{\mu\nu}^{vac} = \langle 0|\mathcal{T}_{\mu\nu}|0\rangle = \rho_\Lambda g_{\mu\nu}$, where $\rho_\Lambda$ is a constant, i.e. a Lorentz scalar quantity. In the traditional picture the vacuum itself can be thought as a perfect fluid, source of the Einstein field equations and one derives [23] the equation of state $p_\Lambda = w\rho_\Lambda$, with $p_\Lambda$ denoting the vacuum pressure and the adiabatic index $w$ equals $-1$. As well known [24], one of the central pillars of Lorentz invariant local QFT is the very same definition of the vacuum state according to which it is the zero eigenvalue eigenstate of the normal ordered energy, momentum and angular momentum operators. Therefore, excluding by normal ordering zero-point contributions, any non-vanishing vacuum expectation value of one of these operators signals the breakdown of Lorentz invariance, since the vacuum would be dependent on space and/or time. The Lorentz invariance vacuum therefore implies $\mathcal{T}_{\mu\nu}^{vac} = \langle 0| : \mathcal{T}_{\mu\nu} : |0\rangle = 0$, (as usual normal ordering is denoted by the colon : ... :).

Usually, in the jargon one roughly expresses the Lorentz invariant characterization of the QFT vacuum, by saying that preserving the Lorentz invariance requires to exclude that kinematical terms in the energy-momentum tensor may contribute to the vacuum expectation values.

Suppose that, as said above, the contribution of the (zero point) vacuum energy density is taken to be equivalent to that of the cosmological constant $\Lambda$, which is expressed by $\rho_\Lambda = \Lambda/(8\pi G)$. Then, however, it turns out that the vacuum expectation value of the energy-momentum tensor is divergent, both for bosonic and fermionic fields, and this shortcoming can be addressed as the *cosmological constant problem*. By choosing to regularize the energy-momentum tensor by an ultraviolet cut-off at Planck scale, one gets a huge value for the vacuum energy density $\rho_{vac} \simeq c^5/G^2\hbar \sim 10^{76} GeV^4$ which is 123 orders of magnitude larger than the currently observed $\rho_\Lambda \simeq 10^{-47} GeV^4$. Also using a quantum chromo-dynamics (QCD) cut-off [25] the problem is not solved since $\rho_\Lambda^{QCD} \sim 10^{-3} GeV^4$ is still enormous with respect to the actual observed value.

Furthermore, there is another aspect which has to be taken into account: observations point out that cosmic flow is "today" accelerating while it was not so at intermediate redshift $z$ (e.g. $1 < z < 10$). This situation gave rise to structure formation during the matter dominated era [26, 27]. This is an indication of the fact that any realistic cosmological model should roughly undergo four phases: an early accelerated phase (inflation), intermediate decelerated phases (radiation and matter dominated) and a final, today observed, accelerated phase. Obviously, the dynamical evolution (time dependence) of the cosmological constant through the different phases, breaks the Lorentz invariance of the vacuum and one has to face the problem of the dynamics for the vacuum energy in order to match the observations. In this case, therefore, we are not properly dealing with cosmological constant. Rather, we have to take into account some form of *dark energy* which *evolves* from early epochs inducing the today observed acceleration. Such a dynamical evolution of the dark energy, namely time dependence of the energy vacuum expectation value, violates the Lorentz invariance.

In the literature there are many proposals to achieve cosmological models justifying such a dark energy component, ranging from quintessence [28], to braneworld [29], to extended theories of gravity [30]. These approaches essentially consist in adding new ingredients to the dynamics (e.g. scalar fields), or in modifying cosmological equations (e.g. introducing higher order curvature terms in the effective gravitational action).

In this letter, we suggest that new exotic ingredients might be not actually needed to explain the observed dynamics. As already mentioned, it is possible to show indeed that, due to the condensate of neutrino-antineutrino pairs, the vacuum expectation value of the energy-momentum tensor naturally provides a contribution to the dark energy $\rho_{vac}^{mix}$, which in the early universe satisfies the strong energy condition (SEC) $\rho_{vac}^{mix} + 3p_{vac}^{mix} \geq 0$, and at present epoch behaves approximatively as a cosmological constant. Here $p_{vac}^{mix}$ is the vacuum pressure induced by the neutrino mixing. Under such a new perspective, the energy content of the vacuum condensate could be substantially interpreted as dynamically evolving dark energy capable of consistently reproducing the observed dark energy budget.

The main features of the QFT formalism for the neutrino mixing are summarized as follows. For the sake of simplicity, we restrict ourselves to the two flavor case [13]. Extension to three flavors can be found in Ref. [17]. The relation between the Dirac flavored neutrino fields $\nu_e(x)$, $\nu_\mu(x)$ and the Dirac massive neutrino fields $\nu_1(x)$, $\nu_2(x)$ is given by

$$\begin{pmatrix} \nu_e(x) \\ \nu_\mu(x) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1(x) \\ \nu_2(x) \end{pmatrix} \tag{1}$$

being $\theta$ the mixing angle. The mixing transformation (1) can be written as $\nu_\sigma(x) \equiv G_\theta^{-1}(t)\ \nu_i(x)\ G_\theta(t)$, where $(\sigma, i) = (e,1), (\mu,2)$, and $G_\theta(t)$ is the transformation generator. The flavor annihilation operators are defined as $\alpha^r_{\mathbf{k},\sigma}(t) \equiv G_\theta^{-1}(t)\ \alpha^r_{\mathbf{k},i}\ G_\theta(t)$ and $\beta^r_{-\mathbf{k},\sigma}(t) \equiv G_\theta^{-1}(t)\ \beta^r_{-\mathbf{k},i}\ G_\theta(t)$. They annihilate the flavor vacuum $|0(t)\rangle_{e,\mu} \equiv G_\theta^{-1}(t)\ |0\rangle_{1,2}$, where $|0\rangle_{1,2}$ is the vacuum annihilated by $\alpha^r_{\mathbf{k},i}$ and $\beta^r_{-\mathbf{k},i}$.

The crucial point of our discussion is that $|0(t)\rangle_{e,\mu}$, which is the physical vacuum where neutrino oscillations are experimentally observed, is [13] a (coherent) condensate of $\alpha_{\mathbf{k},i}$ ($\beta_{\mathbf{k},i}$) neutrinos (antineutrinos):

$${}_{e,\mu}\langle 0|\alpha^{r\dagger}_{\mathbf{k},i}\alpha^r_{\mathbf{k},i}|0\rangle_{e,\mu} = {}_{e,\mu}\langle 0|\beta^{r\dagger}_{\mathbf{k},i}\beta^r_{\mathbf{k},i}|0\rangle_{e,\mu} = \sin^2\theta\ |V_\mathbf{k}|^2, \tag{2}$$

where $i = 1,2$, the reference frame $\mathbf{k} = (0,0,|\mathbf{k}|)$ has been adopted for convenience, $V_\mathbf{k}$ is the Bogoliubov coefficient entering the mixing transformation (see for example Refs. [13, 17]) and $|0\rangle_{e,\mu}$ denotes $|0(t)\rangle_{e,\mu}$ at a conventionally chosen time $t = 0$. As a consequence of its condensate structure the physical vacuum $|0(t)\rangle_{e,\mu}$ turns out to be unitary inequivalent to $|0\rangle_{1,2}$ [13]. For brevity, we omit here to reproduce the explicit expression of $V_\mathbf{k}$ which can be found, e.g., in Refs. [13, 17, 19]. We only recall that $V_\mathbf{k}$ is zero for $m_1 = m_2$, it has a maximum at $|\mathbf{k}| = \sqrt{m_1 m_2}$ and, for $|\mathbf{k}| \gg \sqrt{m_1 m_2}$, it goes like $|V_\mathbf{k}|^2 \simeq (m_2 - m_1)^2/(4|\mathbf{k}|^2)$. The oscillation formulas for the flavor charges $Q_{e,\mu}(t)$ are obtained by computing their expectation values in the physical vacuum $|0\rangle_{e,\mu}$ [17, 19].

Let us now calculate the contribution $\rho_{vac}^{mix}$ of the neutrino mixing to the vacuum energy density. We consider the Minkowski metric (therefore we use the notation $\eta^{\mu\nu}$ instead of $g^{\mu\nu}$). The particle mixing and oscillations in curved background will be analyzed in a separate paper. Eq. (2) suggests that the energy content of the physical vacuum gets contributions from the $\alpha_{\mathbf{k},i}$ and $\beta_{-\mathbf{k},i}$ neutrino condensate. Therefore, as customary in such circumstances, we must compute the (0,0) component of the energy-momentum tensor $T_{00} = \int d^3x \mathcal{T}_{00}(x)$ for the fields $\nu_1$ and $\nu_2$,

$$: T^{00}_{(i)} := \sum_r \int d^3\mathbf{k}\, \omega_{k,i} \left( \alpha^{r\dagger}_{\mathbf{k},i}\alpha^r_{\mathbf{k},i} + \beta^{r\dagger}_{-\mathbf{k},i}\beta^r_{-\mathbf{k},i} \right), \tag{3}$$

with $i = 1,2$ and where $: ... :$ denotes the normal ordering of the $\alpha_{\mathbf{k},i}$ and $\beta_{-\mathbf{k},i}$ operators. Note that $T^{00}_{(i)}$ is time independent.

We remark that we have ${}_{e,\mu}\langle 0|\ :\ T^{00}_{(i)}\ :\ |0\rangle_{e,\mu} = {}_{e,\mu}\langle 0(t)|\ :\ T^{00}_{(i)}\ :\ |0(t)\rangle_{e,\mu}$, for any $t$, within the QFT formalism for neutrino mixing. The contribution $\rho_{vac}^{mix}$ of the neutrino mixing to the vacuum energy density is thus obtained:

$$\frac{1}{V}\ {}_{e,\mu}\langle 0| \sum_i : T^{00}_{(i)} : |0\rangle_{e,\mu} = \rho_{vac}^{mix}\ \eta^{00}\ . \tag{4}$$

By using Eq.(2), we then have

$$\rho_{vac}^{mix} = \frac{2}{\pi} \sin^2\theta \int_0^K dk\, k^2 (\omega_{k,1} + \omega_{k,2}) |V_{\mathbf{k}}|^2, \qquad (5)$$

where the choice of the cut-off $K$ will be discussed below.

Similarly, the expectation value of $T^{jj}_{(i)}$ in the vacuum $|0\rangle_{e,\mu}$ gives the contribution $p_{vac}^{mix}$ of the neutrino mixing to the vacuum pressure:

$$\frac{1}{V}\ {}_{e,\mu}\langle 0| \sum_i : T^{jj}_{(i)} : |0\rangle_{e,\mu} = p_{vac}^{mix}\ \eta^{jj}\ , \qquad (6)$$

where no summation on the index $j$ is intended. Being, for each diagonal component,

$$: T^{jj}_{(i)} := \sum_r \int d^3\mathbf{k}\, \frac{k^j k^j}{\omega_{k,i}} \left( \alpha^{r\dagger}_{\mathbf{k},i} \alpha^r_{\mathbf{k},i} + \beta^{r\dagger}_{-\mathbf{k},i} \beta^r_{-\mathbf{k},i} \right), \quad (7)$$

(no summation on repeated indices), in the case of the isotropy of the momenta: $k^1 = k^2 = k^3$, $3(k^j)^2 = k^2$, we have $T^{11} = T^{22} = T^{33}$ and the following equation holds

$$p_{vac}^{mix} \ = \ -\frac{2}{3\pi} \sin^2\theta \int_0^K dk k^4 \left[ \frac{1}{\omega_{k,1}} + \frac{1}{\omega_{k,2}} \right] |V_{\mathbf{k}}|^2. \, (8)$$

Eqs.(5) and (8) show that Lorentz invariance is broken and $\rho_{vac}^{mix} \neq -p_{vac}^{mix}$ for any value of the masses $m_1$ and $m_2$ and independently of the choice of the cut-off. We observe that $w \simeq -1/3$ when the cut-off is chosen to be $K \gg m_1, m_2$, cf. Eqs.(5) and (8) and the discussion below for the choice of $K$.

It is worth stressing that the violation of the Lorentz invariance originates from the neutrino-antineutrino condensate structure of the vacuum. Indeed, as it appears from the computation reported above, in the absence of such a condensate, i.e. with $|V_{\mathbf{k}}|^2 = 0$, the vacuum expectation value of *each* of the $(0,0)$ and $(j,j)$ components of the energy-momentum tensor would be zero. We also remark that the non-zero expectation value we obtain is time-independent since, for simplicity, we are considering the Minkowski metric. When the curved background metric is considered, $|V_{\mathbf{k}}|^2$ gets a dependence on time, as we will show in a forthcoming paper. In any case, the contribution to the vacuum expectation value of $T^{\mu\nu}$ is found to be non-vanishing, in the present computation (or in the curved background case), not because the adopted metric is flat (or not), but because of the nontrivial structure of physical vacuum due to the mixing phenomenon (which manifests itself in the non-vanishing of $|V_{\mathbf{k}}|^2$).

The above result holds in the early universe, when the universe curvature radius is comparable with the oscillation length. At the present epoch, in which the breaking of the Lorentz invariance is negligible, the non-vanishing vacuum energy density $\rho_{vac}^{mix}$ compatible with Lorentz invariance cannot come from condensate contributions carrying a non-vanishing $\partial_\mu \sim k_\mu = (\omega_k, k_j)$, as it happens in Eqs.(3) and (7) (see also Eqs.(5) and (8)). This means that it can only be imputed to the lowest energy contribution of the vacuum condensate, approximatively equal to

$$\rho_\Lambda^{mix} = \sum_i m_i \int \frac{d^3x}{(2\pi)^3}\ {}_{e,\mu}\langle 0| : \bar{\nu}_i(x) \nu_i(x) : |0\rangle_{e,\mu}. \quad (9)$$

Consistently with Lorentz invariance, the state equation is now $\rho_\Lambda^{mix} \sim -p_\Lambda^{mix}$, where explicitly

$$\rho_\Lambda^{mix} = \frac{2}{\pi} \sin^2\theta \int_0^K dk\, k^2 \left[ \frac{m_1^2}{\omega_{k,1}} + \frac{m_2^2}{\omega_{k,2}} \right] |V_{\mathbf{k}}|^2. \quad (10)$$

The result (10) shows that, at present epoch, the vacuum condensate, coming from the neutrino mixing, can contribute to the dark energy component of the universe, with a behavior similar to that of the cosmological constant [20].

We observe that, since, at present epoch, the characteristic oscillation length of the neutrino is much smaller than the radius of curvature of the universe, the mixing treatment in the flat space-time, in such an epoch, is a good approximation of that in FRW space-time. More interesting is also the fact that, at present epoch, the space-time dependent condensate contributions, carrying a non-vanishing $k_\mu$, are missing (they do not contribute to the energy-momentum vacuum expectation value). The modes associated to these missing contributions are not long-wave-length modes and therefore they are negligible in the present flat universe, i.e with respect to the scale implied by an infinite curvature radius.

As shown in Ref. [31], in a dense background of neutrinos, as in the case of the early universe during the Big Bang Nucleosyntesis, flavor particle-antiparticle pairs are produced by mixing and oscillations with typical momentum $k \sim \frac{m_1+m_2}{2}$, the average mass of the neutrinos. This introduces us to comment on the problem of the choice of the cut-off $K$ in the integrations in the equations above.

We do not have the solution for such a problem. However, in our approach there is an indication of possible choices suggested by the natural energy scale of the neutrino mixing in the QFT formalism. As already mentioned above, the ultraviolet cut-off at Planck scale, as well as the QCD one, give huge unacceptable values for the today observed vacuum energy density. It is therefore imperative to explore alternative routes. One of the merits of the present approach is indeed to point out that, although the arbitrariness problem is not solved, other possible choices exist which not only are consistent with the intrinsic energy scale of the mixing phenomenon, but also lead to quite acceptable values for the vacuum energy density. We thus arrive at the cut-off choice which is suggested by the natural scale appearing in the QFT formalism of the mixing phenomenon, i.e. we may set $K \simeq \sqrt{m_1 m_2}$ [20]. Another possibility, as suggested in Ref. [32] on similar grounds, is the sum of the two neutrino masses, $K = m_1 + m_2$. Both choices lead to values of $\rho_\Lambda^{mix}$ compatible with the observed value of $\rho_\Lambda$. The latter choice is also quite near to another possibility,

$K \sim \frac{m_1+m_2}{2}$, which could be related to the discussion of Ref. [31], although this is referred to background neutrinos. It is indeed an interesting open question the relation between the (hot) dark matter and the dark energy, namely, from the perspective of the present paper, of the relation between dark matter and the vacuum structure. Such a question will be object of our future study.

By using $\sin^2\theta \simeq 0.3$, $m_i$ of order of $10^{-3}eV$, so that $\delta m^2 = m_2^2 - m_1^2 \simeq 8 \times 10^{-5} eV^2$, and one of the above choices for $K$, for example $K \sim \frac{m_1+m_2}{2}$, we obtain $\rho_\Lambda^{mix} \sim 1.1 \times 10^{-47} GeV^4$, which is in agreement with the estimated value of the dark energy. The other two choices lead to values of $\rho_\Lambda^{mix}$ also compatible with the estimated value of $\rho_\Lambda$, i.e. $\rho_\Lambda^{mix} \sim 0.7 \times 10^{-47} GeV^4$ and $\rho_\Lambda^{mix} \sim 5.5 \times 10^{-47} GeV^4$, respectively.

We remark that, unless one works in the present approach, such incredibly small values for the cut-off would be ruled out, since one would think that regularization of quantum effects from the physics beyond the standard model should come at a very high scale, e. g. the Planck scale. However, in the present case such a belief is actually unfounded: it is indeed in conflict with simple facts such as the disagreement of 123 orders of magnitude with the observed dark energy value, as recalled above. On the contrary, being bounded to a flat computational basis, as observed in Refs. [20, 32], the presence of $|V_{\mathbf{k}}|^2$ (with its behavior as a function of the momentum) in the integrations naturally leads to one of the above small cut-off choices. The non-perturbative physics of the neutrino mixing thus points to the relevance of soft momentum (long-wave-length) modes.

In this connection, we also remark that Eqs. (5) and (10) show that the contribution to the dark energy induced from the neutrino mixing of course goes to zero in the no-mixing limit, i.e. when the mixing angle $\theta = 0$ and/or $m_1 = m_2$. However, those equations also show that the contribution depends on the specific QFT nature of the mixing: indeed, it is absent in the quantum mechanical (Pontecorvo) treatment of the mixing, where $V_{\mathbf{k}}$ is anyhow zero. This confirms that the contribution discussed above is a genuine QFT non-perturbative feature and it is thus of different origin with respect to the ordinary vacuum energy contribution of massive spinor fields arising from a radiative correction at some perturbative order [33]. This leads us to believe that a neutrino–antineutrino asymmetry, if any, related with lepton number violation [31], would not affect much our result. We will consider the problem of such an asymmetry in a future work.

In conclusion, it has been shown that the vacuum condensate due to neutrino mixing contributes to the dark energy budget of the universe. We have computed the expectation value of the energy-momentum tensor in the vacuum state where neutrino oscillations are observed and, through a careful choice of the momentum cut-off, we have obtained acceptable values for vacuum energy density. Different behaviors of the vacuum expectation value of the energy-momentum tensor have been discussed referring to different boundary conditions in different universe epochs. The result here obtained allows to decouple the dark energy problem (i.e. observed acceleration) from the cosmological constant one. Such a decoupling is achieved without the need of introducing auxiliary fields or mechanisms, apart neutrino mixing and the arbitrary momentum cut-off choice.

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[1] P. de Bernardis et al., Nature, **404**, 955 (2000).
[2] D.N. Spergel et al. ApJS, **148**, 175 (2003).
[3] S. Dodelson et al., ApJ, **572**, 140 (2002).
[4] A.S. Szalay et al., ApJ, **591**, 1 (2003).
[5] A.G. Riess et al., ApJ, **607**, 665 (2004).
[6] B. Pontecorvo, Sov. Phys. JETP **6**, 429 (1957) [Zh. Eksp. Teor. Fiz. **33**, 549 (1957)].
[7] S. M. Bilenky and B. Pontecorvo, Phys. Rept. **41**, 225 (1978).
[8] S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. **59**, 671 (1987).
[9] R. N. Mohapatra and P. B. Pal, World Sci. Lect. Notes Phys. **41**, 1 (1991).
[10] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978).
[11] S.P. Mikheyev and A. Yu. Smirnov, Sov. J. Nucl. Phys. **42**, 913 (1985).
[12] C. Giunti, C. W. Kim and U. W. Lee, Phys. Rev. D **44**, 3635 (1991).
[13] M. Blasone and G. Vitiello, Annals Phys. **244**, 283 (1995).
[14] M. Blasone, P.A. Henning and G. Vitiello, Phys. Lett. B **451**, 140 (1999).
[15] K. Fujii, C. Habe and T. Yabuki, Phys. Rev. D **59**, 113003 (1999);
[16] C.R. Ji, Y. Mishchenko, Phys. Rev. D **65**, 096015 (2002).
[17] M. Blasone, A. Capolupo and G. Vitiello, Phys. Rev. D **66**, 025033 (2002).
[18] A. Capolupo, C. R. Ji, Y. Mishchenko and G. Vitiello, Phys. Lett. B **594**, 135 (2004).
M. Blasone, A. Capolupo, O. Romei and G. Vitiello, Phys. Rev. D **63**, 125015 (2001);
A. Capolupo, Ph.D. Thesis [hep-th/0408228].
[19] M. Blasone, A. Capolupo, F. Terranova and G. Vitiello, Phys. Rev. D **72** 013003 (2005).
[20] M. Blasone, A. Capolupo, S. Capozziello, S. Carloni and G. Vitiello, Phys. Lett. A **323**, 182 (2004).
[21] SNO Collaboration, Phys. Rev. Lett. 89, No. 1, 011301 (2002).
[22] K2K collaboration, E. Aliu et al, Phys. Rev. Lett. 94, 081802 (2005).
[23] S.M. Carroll, Living Rev. Rel. **4**, 1 (2001).
P.J.E. Peebles, B. Ratra, Rev. Mod. Phys. **75**, (2003).

V. Sahni, Lect. Notes Phys. **653**, 141 (2004).
[24] C. Itzykson and J. B. Zuber, *Quantum Field Theory*, (McGraw-Hill, New York, 1980).
S. Schweber, *An itroduction Relativistic Quantum Field Theory*, (Harper and Row, 1961).
[25] E.V. Shuryak, *The QCD vacuum, hadrons and the superdense matter*, (World Scientific Publishing, Singapore, 1988).
[26] S. Capozziello, S. Nojiri and S.D. Odintsov, Phys. Lett. B **632**, 597 (2006).
[27] S. Capozziello, S. Nojiri and S.D. Odintsov, Phys. Lett. B **634**, 93 (2006).
[28] V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D **9**, 373 (2000).
[29] C. Deffayet, G. R. Dvali and G. Gabadadze, Phys. Rev. D **65**, 044023 (2002).
[30] R. Kerner, Gen. Rel. Grav. **14**, 453 (1982).
S. Capozziello, V. F. Cardone and A. Troisi, Phys. Rev. D **71**, 043503 (2005).
T.P. Sotiriou, Phys. Rev. D **73**, 063515 (2006).
[31] D. Boyanovsky and C. M. Ho, Phys. Rev. D **69**, 125012 (2004).
[32] G. Barenboim and N. E. Mavromatos, Phys. Rev. D **70**, 093015 (2004).
[33] S. Coleman and E. Weinberg, Phys. Rev. D **7**, 6 (1973).