Tetrahedron:

Model Predictions for Neutrino Oscillations

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Neutrino Mass beyond the SM

- SM: effective low energy theory with non-renormalizable terms
- new physics effects suppressed by powers of small parameter $\frac{M_W}{M}$
- neutrino masses generated by dim-5 operators

$$\frac{\lambda_{ij}}{M} H H L_i L_j \quad \Rightarrow \quad m_\nu = \lambda_{ij} \frac{v^2}{M}$$

λ_{ij} are dimensionless couplings; M is some high scale

- m_ν small: non-renormalizable terms (M is high)

lowest higher dimensional operator that probes high scale physics

- total lepton number and family lepton numbers broken
  - ➡ lepton mixing and CP violation expected
  - ➡ $\mu \to e \gamma$; $\tau \to \mu \gamma$; $\tau \to e \gamma$ decays ; μ - e conversion

See: Talk by A. de Gouvea

Current Status of Oscillation Parameters

- oscillation probability: $$P(\nu_a \to \nu_b) = \left|\langle \nu_b | \nu,\ t\rangle\right|^2 \simeq \sin^2 2\theta\ \sin^2\left(\frac{\Delta m^2}{4E} L\right)$$
- 3 neutrinos global analysis: [solar+KamLAND+CHOOZ+atmospheric +K2K+Minos]

Maltoni, Schwetz, Tortola, Valle (updated Sep 2007)

$$\sin^2\theta_{23} = 0.5\ (0.38 - 0.64), \quad \sin^2\theta_{13} = 0\ (< 0.028) \quad \sin^2\theta_{12} = 0.30\ (0.25 - 0.34)$$

$$\Delta m^2_{23} = (2.38^{+0.2}_{-0.16}) \times 10^{-3}\ \text{eV}^2, \quad \Delta m^2_{12} = (8.1 \pm 0.6) \times 10^{-5}\ \text{eV}^2$$

- indication of non-zero θ_{13}:

$$\sin^2\theta_{13} = 0.016 \pm 0.010\ (1\sigma)$$

Fogli, Lisi, Marrone, Palazzo, Rotunno, June 2008

- Tri-bimaximal Neutrino Mixing:

$$\sin^2\theta_{\text{atm, TBM}} = 1/2 \qquad \sin\theta_{13,\text{TBM}} = 0.$$

$$\sin^2\theta_{\odot,\text{TBM}} = 1/3 \qquad \tan^2\theta_{\odot,\text{TBM}} = 1/2$$

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\tan^2\theta_{\odot,\text{exp}} = 0.429$$

new KamLAND result: $\tan\theta^2_{\odot,exp} = 0.47^{+0.06}_{-0.05}$

Discovery phase into precision phase for some oscillation parameters

Neutrino Mass Spectrum

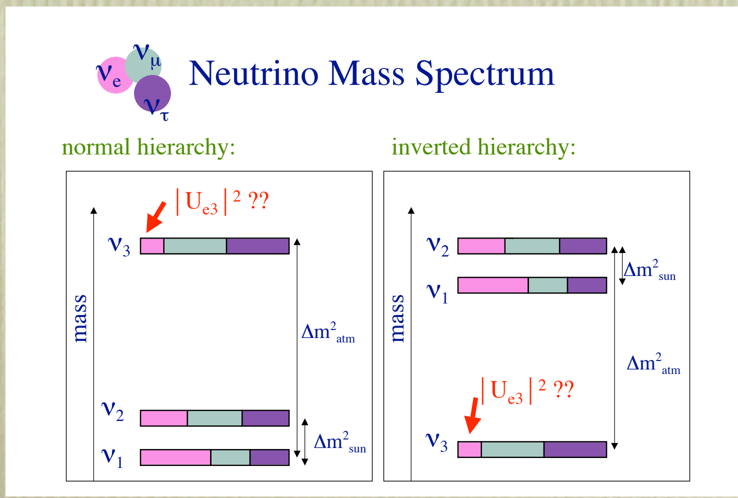
- search for absolute mass scale:
  - end point kinematic of tritium beta decays:

$$\text{Tritium} \rightarrow He^3 + e^- + \overline{\nu}_e$$

Mainz: $m_\nu < 2.2$ eV
KATRIN: increase sensitivity ~ 0.2 eV

  - WMAP + 2dFRGS + Lyα: $\sum(m_{\nu_i}) < (0.7\text{-}1.2)$ eV
  - neutrinoless double beta decay

current bound: $|<m>| < (0.19 - 0.68)$ eV (CUORICINO, Feb 2008)



The known unknowns:

- How small is θ_{13}?
- $\theta_{23} > \pi/4$, $\theta_{23} < \pi/4$, $\theta_{23} = \pi/4$?
- Neutrino mass hierarchy (Δm_{13}^2) ?
- CP violation in neutrino oscillations?

Need for Precision Measurements

- current data post two challenges:
  - why $m_\nu << m_{u,d,l}$
  - why lepton mixing large while quark mixing small
- To answer the first question => Seesaw mechanism: most appealing scenario
- Seesaw: not sufficient to explain the whole mass matrix with mass hierarchy and two large and one small mixing angles
  - ✳ neutrino anarchy: no parametrically small numbers

    [Hall, Murayama, Weiner,'00; Haba, Murayama, '01; de Gouvea, Murayama, '04]
  - ✳ flavor symmetry: there is a structure
    - Possible symmetries show up only in the lepton sector
    - Connection between quark and lepton sectors (GUT symmetry)
- These scenarios have drastically different predictions
- To tell these models apart: Precision measurements important

Flavor Structure

- there are parametrically small numbers
  - $m_2/m_3 << 1$, $\theta_{13} << 1$
- In general, large mixing ⇔ no hierarchy

$$m = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

  - $a >> b, c \Rightarrow \sin^2\theta << 1,\ m_1/m_2 << 1$
  - $a,b,c \sim 1 \Rightarrow \det(m) \sim 1 \Rightarrow \sin^2\theta \sim 1,\ m_1/m_2 \sim 1$
  - $a,b,c \sim 1 \Rightarrow \det(m) << 1 \Rightarrow \sin^2\theta \sim 1,\ m_1/m_2 << 1$

Texture	Hierarchy	$\lvert U_{e3}\rvert$	$\lvert\cos 2\theta_{23}\rvert$ (n.s.)	$\lvert\cos 2\theta_{23}\rvert$	Solar Angle
$\frac{\sqrt{\Delta m^2_{13}}}{2}\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix}$	Normal	$\sqrt{\frac{\Delta m^2_{12}}{\Delta m^2_{13}}}$	O(1)	$\sqrt{\frac{\Delta m^2_{12}}{\Delta m^2_{13}}}$	O(1)
$\sqrt{\Delta m^2_{13}}\begin{pmatrix}1&0&0\\0&\frac{1}{2}&-\frac{1}{2}\\0&-\frac{1}{2}&\frac{1}{2}\end{pmatrix}$	Inverted	$\frac{\Delta m^2_{12}}{\lvert\Delta m^2_{13}\rvert}$	–	$\frac{\Delta m^2_{12}}{\lvert\Delta m^2_{13}\rvert}$	O(1)
$\frac{\sqrt{\Delta m^2_{13}}}{\sqrt{2}}\begin{pmatrix}0&1&1\\1&0&0\\1&0&0\end{pmatrix}$	Inverted	$\frac{\Delta m^2_{12}}{\lvert\Delta m^2_{13}\rvert}$	O(1)	$\frac{\Delta m^2_{12}}{\lvert\Delta m^2_{13}\rvert}$	$\lvert\cos 2\theta_{12}\rvert \sim \frac{\Delta m^2_{12}}{\lvert\Delta m^2_{13}\rvert}$
$\sqrt{\Delta m^2_{13}}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}$	Normal[a]	> 0.1	O(1)	–	O(1)

Altarelli, Feruglio, Masina, 02; Hall, Murayama, Weiner; Sato, Yanagida; Barbieri et al; ...

Leptonic μ - τ Family Symmetry

- two possibilities

$$\begin{pmatrix} \bullet & \bullet & \bullet \\ \bullet & 1 & 1 \\ \bullet & 1 & 1 \end{pmatrix} \Rightarrow \text{normal hierarchy} \qquad \begin{pmatrix} \bullet & 1 & 1 \\ 1 & \bullet & \bullet \\ 1 & \bullet & \bullet \end{pmatrix} \Rightarrow \text{inverted hierarchy}$$

(I) normal hierarchy

$$\begin{pmatrix} \bullet & \bullet & \bullet \\ \bullet & 1 & 1 \\ \bullet & 1 & 1 \end{pmatrix} \longrightarrow \boxed{\Delta m^2_{21} = 0, \quad \theta_{13} = 0, \quad \theta_{23} = \pi/4 \qquad \theta_{12} = 0}$$

to have $\Delta m_{12}^2 \neq 0$ in μ - τ symmetric limit:

$$M_\nu = \frac{\sqrt{\Delta m^2_{atm}}}{2} \begin{pmatrix} c\varepsilon & d\varepsilon & d\varepsilon \\ d\varepsilon & 1+\varepsilon & -1 \\ d\varepsilon & -1 & 1+\varepsilon \end{pmatrix} \longrightarrow$$

$$\boxed{\theta_{13} = 0, \quad \theta_{23} = \pi/4}$$

$$\tan 2\theta_{12} \cong \frac{2\sqrt{2}d}{(1-c)}$$

$$\varepsilon = 4\sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}} \frac{1}{1+c+\sqrt{(c-1)^2+8d^2}}$$

E. Ma ('02)

Leptonic μ - τ Family Symmetry

- breaking of μ - τ symmetry

$$M_\nu = \frac{\sqrt{\Delta m^2_{atm}}}{2}\begin{pmatrix} c\varepsilon & d\varepsilon & b\varepsilon \\ d\varepsilon & 1+a\varepsilon & -1 \\ b\varepsilon & -1 & 1+\varepsilon \end{pmatrix}$$

- breaking in the e-sector: $a = 1, \quad b \neq d$
- breaking in the μ - τ sector: $a \neq 1, \quad b = d$

R. N. Mohapatra ('04)

Symmetry breaking	θ_{13}	$\theta_{23} - \pi/4$
none	0	0
μ-τ sector only	$\sim \Delta m^2_{12} / \Delta m^2_{31}$	$\leq 8^o$ $\sim \sqrt{\Delta m^2_{12}/\Delta m^2_{31}}$
e-sector only	$\sim \sqrt{\Delta m^2_{12} / \Delta m^2_{31}}$	$\leq 4^o$ $\sim \Delta m^2_{12}/\Delta m^2_{31}$
dynamical	$\sim \sqrt{\Delta m^2_{12} / \Delta m^2_{31}}$	large

Leptonic L_e - L_μ - L_τ Family Symmetry

- (II) Inverted hierarchy case: enhanced (L_e - L_μ - L_τ) symmetry

$$\begin{pmatrix} \bullet & 1 & 1 \\ 1 & \bullet & \bullet \\ 1 & \bullet & \bullet \end{pmatrix} \longrightarrow M_\nu = \sqrt{\Delta m^2_{atm}} \begin{pmatrix} z & \sin\theta & \cos\theta \\ \sin\theta & y & d \\ \cos\theta & d & x \end{pmatrix}, \quad x, y, d << 1$$

Barbieri, Hall, Smith, Strumia, Weiner ('98)

- exact (L_e - L_μ - L_τ) limit:

$$M_\nu = \sqrt{\Delta m^2_{atm}} \begin{pmatrix} \bullet & \sin\theta & \cos\theta \\ \sin\theta & \bullet & \bullet \\ \cos\theta & \bullet & \bullet \end{pmatrix} \longrightarrow \Delta m^2_{sol} = 0, \quad \theta_{12} = \frac{\pi}{4}, \quad \sin^2 2\theta_{atm} = \sin^2 2\theta$$

$$\theta_{13} = 0$$

- soft breaking of (L_e - L_μ - L_τ) symmetry:

R, N, Mohapatra ('04)

$$x, y, d \neq 0 \longrightarrow \sin^2 2\theta_{sol} \cong 1 - \left(\frac{\Delta m^2_{sol}}{4 \Delta m^2_{atm}} - z \right)^2$$

correlations not as strong as in normal hierarchy case

- breaking of μ-τ symmetry:

$$(i) \;\; \cos\theta = \sin\theta = \frac{1}{\sqrt{2}}, \; x \neq y: \quad \theta_{13} = \frac{1}{2}(x - y), \quad \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} = 2(x + y + z + d)$$

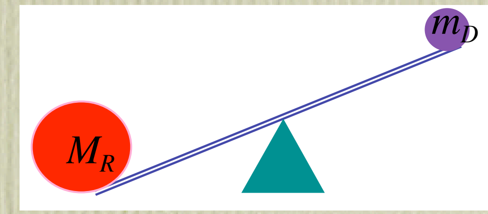
$$(ii) \;\; \cos\theta \neq \sin\theta, \; x = y: \quad \theta_{13} \cong -d \cos 2\theta_{23}$$

Seesaw Mechanism

Minkowski, 1977; Gell-mann, Ramond, Slansky, 1981;
Yanagida, 1979; Mohapatra, Senjanovic, 1981

- Introduce right-handed neutrinos, which are SM gauge singlets [predicted in many GUTs, e.g. SO(10)]
- integrating out RH neutrinos: effective mass matrix

$$\begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

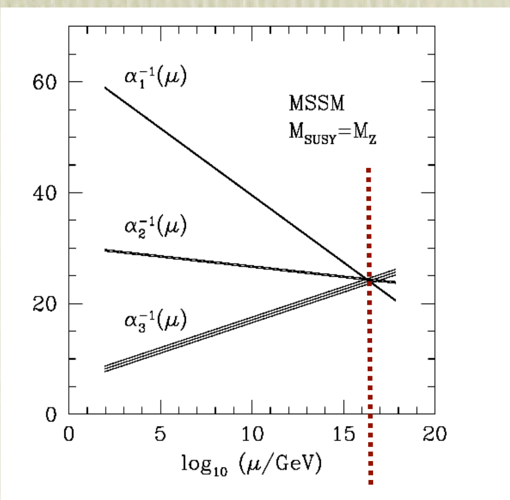


light neutrino mass: $m_\nu \sim \frac{m_D}{M_R} m_D \quad \gg m_D$

heavy neutrino mass: $M \sim M_R$

$$m_\nu \sim \sqrt{\Delta m^2_{atm}} \sim 0.05 \text{ eV}, \; m_D \sim m_t \sim 172 \text{ GeV}$$

$$\Rightarrow M_R \sim 10^{15} \text{GeV} \sim M_{GUT}$$



$M_{GUT} \sim 10^{16}$ GeV

SO(10) GUT

- RH neutrino accommodated in the model

$$16 = \overline{5} + 10 + 1$$

$1 \rightarrow \nu_R$

$16 = (3, 2, 1/6) \sim \begin{bmatrix} u & u & u \\ d & d & d \end{bmatrix}$

$+ (3^*, 1, -2/3) \sim (u^c \; u^c \; u^c)$

$+ (3^*, 1, 1/3) \sim (d^c \; d^c \; d^c)$

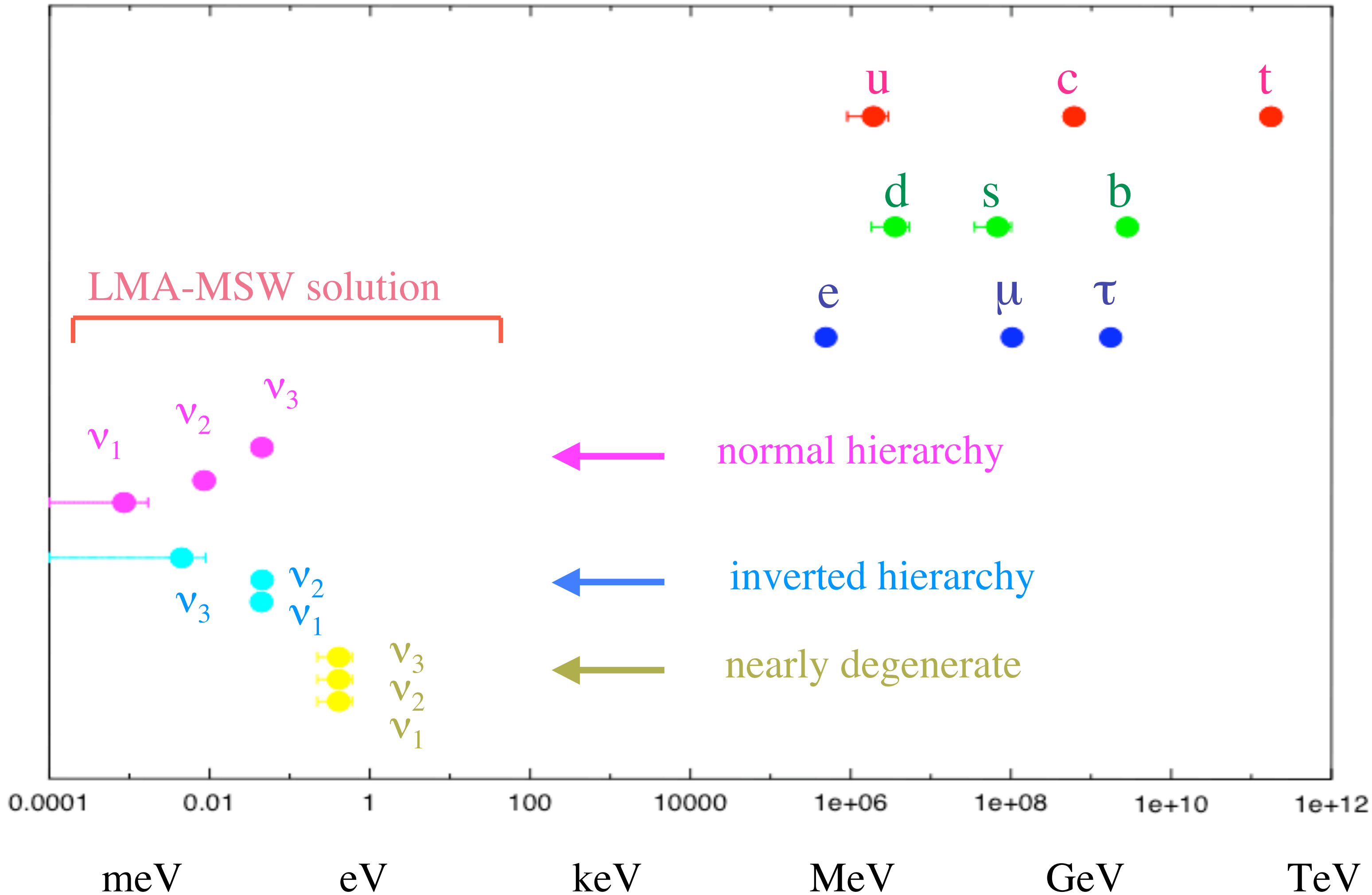
$+ (1, 2, -1/2) \sim \begin{bmatrix} \nu \\ e \end{bmatrix}$

$+ (1, 1, 1) \sim e^c$

$+ (1, 1, 0) \sim \nu^c$

- Natural for seesaw: offer both ingredients, i.e. RH neutrino & heavy scale neutrino oscillation strongly support SO(10)!!
- Quark & Leptons reside in the same GUT multiplets
- One set of Yukawa coupling for a given GUT multiplet
  - ➡ SO(10) relates quarks and leptons (intra-family relations)
  - ➡ reduce # of parameters in Yukawa sector

Mass spectrum of elementary particles



CKM Matrix ⟺ PMNS Matrix

- Quark mixings are small

$$V_{CKM} \sim \begin{pmatrix} 0.9745-0.9757 & 0.219-0.224 & 0.002-0.005 \\ 0.218-0.224 & 0.9736-0.9750 & 0.036-0.046 \\ 0.004-0.014 & 0.034-0.046 & 0.9989-0.9993 \end{pmatrix}$$

- Lepton mixings are large

$$U_{MNS} \sim \begin{pmatrix} 0.79-0.86 & 0.50-0.61 & 0.0-0.16 \\ 0.24-0.52 & 0.44-0.69 & 0.63-0.79 \\ 0.26-0.52 & 0.47-0.71 & 0.60-0.77 \end{pmatrix}$$

- How to realize this when quarks and leptons are unified??

- family symmetries $\rightarrow$ flavor structure
- two sources of large neutrino mixing $U_{MNS} = U^{\dagger}_{e,L} U_{\nu,L}$

  either $U_{e,L}$

  or $U_{\nu,L}$ (RH neutrino sector)

Models Based on SUSY SO(10)

- large neutrino mixing from neutrino sector

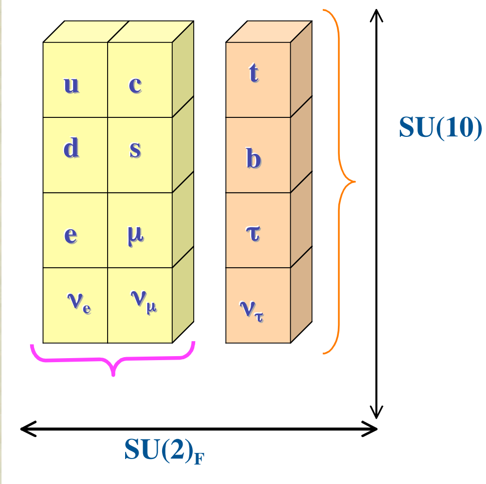
$$U_{MNS} = U_{e,L}^{+} U_{\nu,L}$$

SO(10) GUT + SU(2) family symmetry

Barbieri, Hall, Raby, Romanino; ...

$SO(10) \rightarrow SU(4) \times SU(2)_L \times SU(2)_R$

$\rightarrow SU(3) \times SU(2)_L \times U(1)_Y$



- symmetric mass matrices:

M.-C.C & K.T. Mahanthappa

Up-type quarks ⇔ Dirac neutrinos

Down-type quarks ⇔ charged leptons

$$\text{seesaw} \quad \Rightarrow \quad M_\nu \sim \begin{pmatrix} 0 & 0 & * \\ 0 & 1 & 1 \\ * & 1 & 1 \end{pmatrix}$$

12 parameters accommodate 22 fermion masses, mixing angles and CP phases in both quark and lepton sectors

- prediction for θ_{13}:

$$\sin\theta_{13} \sim \left(\frac{\Delta m^2_{sun}}{\Delta m^2_{atm}}\right)^{1/2} \sim O(0.1) \Rightarrow \text{LMA}$$

Models Based on SUSY SO(10)

Albright & Barr

- large neutrino mixing from charged lepton sector

$$U_{MNS} = U_{e,L}^{\dagger} U_{\nu,L}$$

- lopsided mass matrices:

SO(10) → SU(5)
→ SU(3) x SU(2)$_L$ x U(1)$_Y$

$$M_d^T = M_e \sim \begin{pmatrix} * & * & * \\ * & * & 1 \\ * & * & 1 \end{pmatrix}$$

- large mixing in $U_{e,L}$
  - large mixing in $U_{d,R}$ (effects in B physics)
  - large $\mu \longrightarrow e + \gamma$ rate
- prediction for θ_{13}: can be small; $\sin\theta_{13} \sim 0.05$

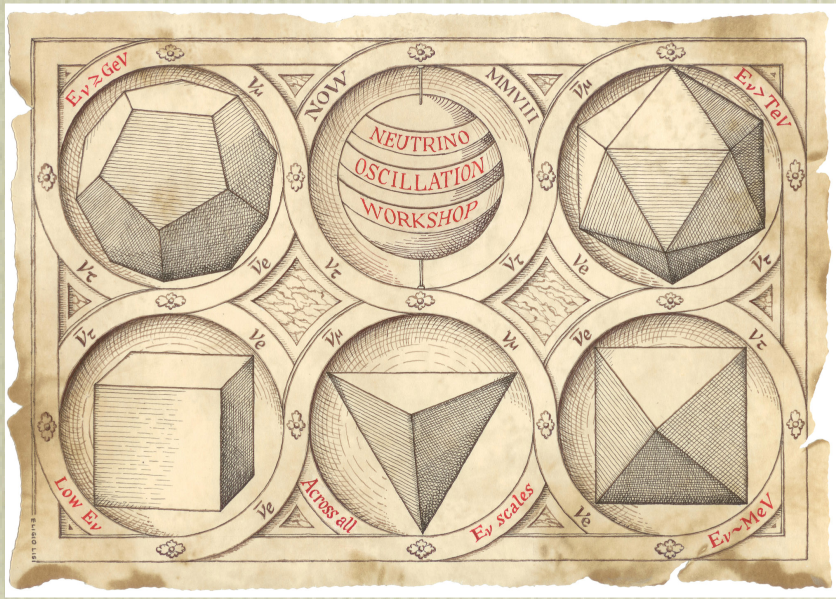
Perfect Geometric Solids & Family Symmetries

solid	faces	vert.	Plato	Hindu	sym.
tetrahedron	4	4	fire	Agni	A_4
octahedron	8	6	air	Vayu	S_4
cube	6	8	earth	Prithvi	S_4
icosahedron	20	12	water	Jal	A_5
dodecahedron	12	20	quintessence	Akasha	A_5

From E. Ma, talk at WHEPP-9, Bangalore

A5

A5



S4

S4

A4

Tri-bimaximal Neutrino Mixing

- neutrino oscillation parameters   Maltoni, Schwetz, Tortola, Valle (updated Sep 2007)

$$\sin^2\theta_{23} = 0.5\ (0.38 - 0.64), \quad \sin^2\theta_{13} = 0\ (< 0.028) \qquad \sin^2\theta_{12} = 0.30\ (0.25 - 0.34)$$

$$\tan^2\theta_{\odot,\mathrm{exp}} = 0.429$$

- tri-bimaximal neutrino mixing   Harrison, Perkins, Scott, 1999

$$U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\sin^2\theta_{\mathrm{atm,\,TBM}} = 1/2 \qquad \sin\theta_{13,\mathrm{TBM}} = 0.$$

$$\sin^2\theta_{\odot,\mathrm{TBM}} = 1/3$$

$$\tan^2\theta_{\odot,\mathrm{TBM}} = 1/2$$

- new KamLAND result: $\tan\theta^2_{\odot,exp} = 0.47^{+0.06}_{-0.05}$
- indication for non-zero θ13: $\sin^2\theta_{13} = 0.016 \pm 0.010\ (1\sigma)$

Fogli, Lisi, Marrone, Palazzo, Rotunno, June 2008

Parametrizing deviations from TBM ⇒ Talk by Werner Rodejohan

Tri-bimaximal Neutrino Mixing

- Neutrino mass matrices:

$$M = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix} \longrightarrow \sin^2 2\theta_{23} = 1 \qquad \theta_{13} = 0$$

solar mixing angle NOT fixed

  - S_3 Mohapatra, Nasri, Yu, 2006; ...
  - D_4 Grimus, Lavoura, 2003; ...
  - μ-τ symmetry Fukuyama, Nishiura, '97; Mohapatra, Nussinov, '99; Ma, Raidal, '01; ...

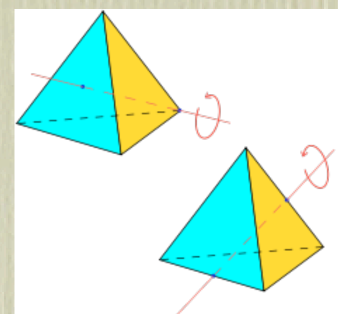
- if $A+B = C + D \longrightarrow \tan^2\theta_{12} = 1/2$ TBM pattern
  - A_4 Ma, '04; Altarelli, Feruglio, '06; .....
  - $Z_3 \times Z_7$ Luhn, Nasri, Ramond, 2007

[Other discrete groups: Hagedorn, Lindner, Plentinger; Chen, Frigerio, Ma; and many others...]

recent claim: S_4 unique group for TBM [C.S. Lam, 2008]

Non-abelian Finite Family Symmetry

- TBM mixing matrix: can be realized in finite group family symmetry based on A4   Ma & Rajasekaran, '01
- even permutations of 4 objects
  - (1234) → (4321)
  - (1234) → (2314)
- invariance group of Tetrahedron



- orbifold compactification: Altarelli, Feruglio, '06

  6D → 4D on T2/Z2

- four in-equivalent representations: 1, 1′, 1″, 3
- Tri-bimaximal mixing arise: Ma, '04; Altarelli, Feruglio, '06; .....
  - three families of lepton doublets ~ 3
  - RH charged leptons ~ 1, 1′, 1″

Non-abelian Finite Family Symmetry

- fermion charge assignments:

$$\begin{pmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{pmatrix}_L \sim 3, \quad e_R \sim 1, \quad \mu_R \sim 1'', \quad \tau_R \sim 1' \qquad \xi \sim 3, \quad \eta \sim 1 \qquad \langle \xi \rangle = \xi_0 \Lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

- SM Higgs ~ singlet under $^{(d)}$T
- operator for neutrino masses: $\frac{HHLL}{M}\left(\frac{\langle\xi\rangle}{\Lambda} + \frac{\langle\eta\rangle}{\Lambda}\right)$
- TBM neutrino mixing from A4 CG coefficients

$$M_\nu = \frac{\lambda v^2}{M_x} \begin{pmatrix} 2\xi_0 + u & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & u - \xi_0 \\ -\xi_0 & u - \xi_0 & 2\xi_0 \end{pmatrix}$$

Form diagonalizable!

$$V_\nu^{\mathrm{T}} M_\nu V_\nu = \mathrm{diag}(u + 3\xi_0,\, u,\, -u + 3\xi_0) \frac{v_u^2}{M_x}$$

$$V_\nu = U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

-- no adjustable parameters
-- neutrino mixing from CG coefficients!

- charged lepton mass matrix: diagonal $\langle \phi \rangle = \phi_0 \Lambda \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$
- no quark CKM mixing!!

The Double Tetrahedral $^{(d)}T$ Symmetry

- consider double covering of A_4
- Classified as a candidate family symmetry that can arise from Type-II B String theories

Frampton, Kaphart, 1995, 2001

- can account for quark sector:

Carr, Frampton, 2007;
Feruglio, Hedgedorn, Lin, Merlo, 2007

exist in A_4: $1,\ 1',\ 1'',\ 3$ → TBM for neutrinos

not in A_4: $2,\ 2',\ 2''$ → 2 +1 assignments for quarks

- Combined with GUT: $^{(d)}T \times SU(5)$ GUT

M.-C.C & K.T. Mahanthappa
Phys. Lett. B652, 34 (2007)

  - only 9 operators allowed: highly predictive model

SU(5) x (d)T Model

M.-C.C & K.T. Mahanthappa
Phys. Lett. B652, 34 (2007)

- CKM mixing matrix

$$M_u = \begin{pmatrix} i\phi_0'^3 & \frac{1-i}{2}\phi_0'^3 & 0 \\ \frac{1-i}{2}\phi_0'^3 & \phi_0'^3 + (1-\frac{i}{2})\phi_0^2 & y'\psi_0\zeta_0 \\ 0 & y'\psi_0\zeta_0 & 1 \end{pmatrix} y_t v_u$$

$y'\psi_0\zeta_0$ → V_{cb}

$$M_d = \begin{pmatrix} 0 & (1+i)\phi_0\psi_0' & 0 \\ -(1-i)\phi_0\psi_0' & \psi_0 N_0 & 0 \\ \phi_0\psi_0' & \phi_0\psi_0' & \zeta_0 \end{pmatrix} y_b v_d \phi_0,$$

$\phi_0\psi_0'$ → V_{ub}

$$\theta_c \simeq \left|\sqrt{m_d/m_s} - e^{i\alpha}\sqrt{m_u/m_c}\right| \sim \sqrt{m_d/m_s},$$

Georgi-Jarlskog relations ⇒ $V_d \neq I$

SU(5) ⇒ $M_d = (M_e)^T$

⇒ corrections to TBM related to θ_c

- MNS matrix:

$$M_e = \begin{pmatrix} 0 & -(1-i)\phi_0\psi_0' & \phi_0\psi_0' \\ (1+i)\phi_0\psi_0' & -3\psi_0 N_0 & \phi_0\psi_0' \\ 0 & 0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0$$

⇒

$$\theta_{12}^e \simeq \sqrt{\frac{m_e}{m_\mu}} \simeq \frac{1}{3}\sqrt{\frac{m_d}{m_s}} \sim \frac{1}{3}\theta_c$$

$$U_{\rm MNS} = V_{e,L}^\dagger U_{\rm TBM} = \begin{pmatrix} 1 & -\theta_c/3 & * \\ \theta_c/3 & 1 & * \\ * & * & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,\rm TBM} - \frac{1}{2}\theta_c\cos\beta$$

$\cos\beta$ → leptonic CPV

$$\theta_{13} \simeq \theta_c/3\sqrt{2}$$

GJ relations in SU(5) ⇒ new QLC relation!

Quark-Lepton Complementarity

lepton mixing

parameter	Best-fit value	3σ range
θ_{12}	33.2^o	$28.7^o - 38.1^o$
θ_{23}	45^o	$35.7^o - 55.6^o$
θ_{13}	2.6^o	$0 - 12.5^o$

quark mixing

parameter	Best-fit value	3σ range
θ_c	12.88^o	$12.75^o - 13.01^o$
θ^q_{23}	2.36^o	$2.25^o - 2.48^o$
θ^q_{13}	0.21^o	$0.17^o - 0.25^o$

$$\theta_{12} + \theta_c = 45^o$$

Raidal, '04; Smirnov & Minakata, '04

quark-lepton complementarity relation

↕

quark-lepton unification?

more generally:

$$\theta_{12} + \theta_C \left(\frac{1}{\sqrt{2}} + \frac{\theta_C}{4} \right) \simeq \frac{\pi}{4}$$

k

See: Talk by Walter Winter

Plentinger, Seidl, Winter, 08; Frampton, Matsuzaki, 08; King 05; King Antusch, 05

RG effects: $\Delta\theta_c \sim \theta_c^4$

MSSM: normal hierarchy $\Delta\theta_{12} < 0.1°$

Schmidt & Smirnov, '06

Motivate measurements of neutrino mixing angles to at least the accuracy of the measured quark mixing angles

Neutrino Mass Sum Rule

- sum rule among three neutrino masses: $m_1 - m_3 = 2m_2$
- including CP violation:

$$\begin{aligned} m_1 &= u_0 + 3\xi_0 e^{i\theta} \\ m_2 &= u_0 \\ m_3 &= -u_0 + 3\xi_0 e^{i\theta} \end{aligned}$$

$$\begin{aligned} \Delta m^2_{atm} &\equiv |m_3|^2 - |m_1|^2 = -12 u_0 \xi_0 \cos\theta \\ \Delta m^2_{\odot} &\equiv |m_2|^2 - |m_1|^2 = -9\xi_0^2 - 6 u_0 \xi_0 \cos\theta \end{aligned}$$

- leads to sum rule

$$\Delta m^2_{\odot} = -9\xi_0^2 + \frac{1}{2}\Delta m^2_{atm} \longrightarrow \Delta m^2_{atm} > 0$$

normal hierarchy predicted!!

- constraint on Majorana phases:

$$0 > \cos\theta > -\frac{3}{2}\frac{\xi_0}{u_0}$$

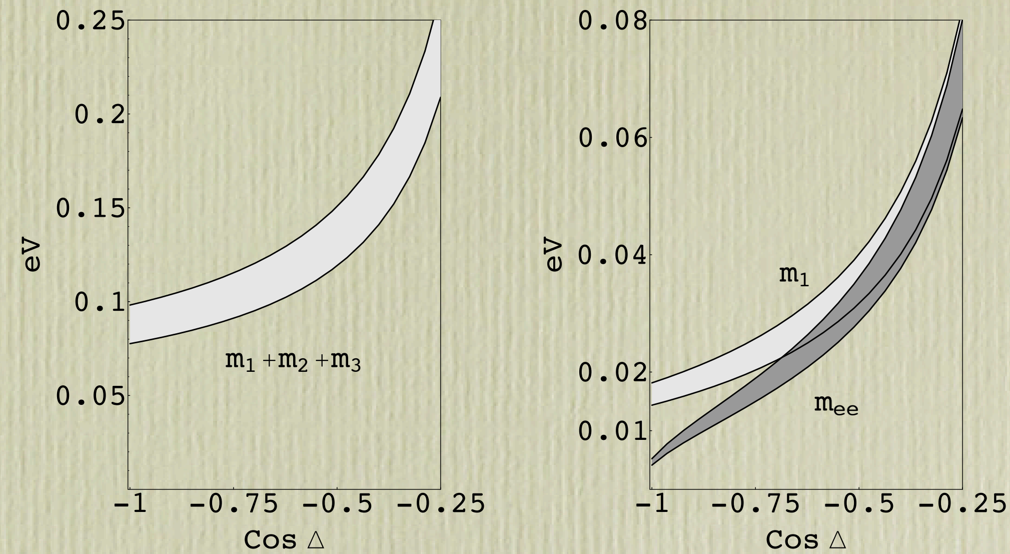
- neutrino-less double beta decay:

$$\begin{aligned} \xi_0 &= \frac{1}{3}\sqrt{(\frac{1}{2} - r)\Delta m^2_{atm}} \\ u_0 &= -\frac{1}{4\cos\theta}\sqrt{\frac{\Delta m^2_{atm}}{(\frac{1}{2} - r)}} \end{aligned}$$

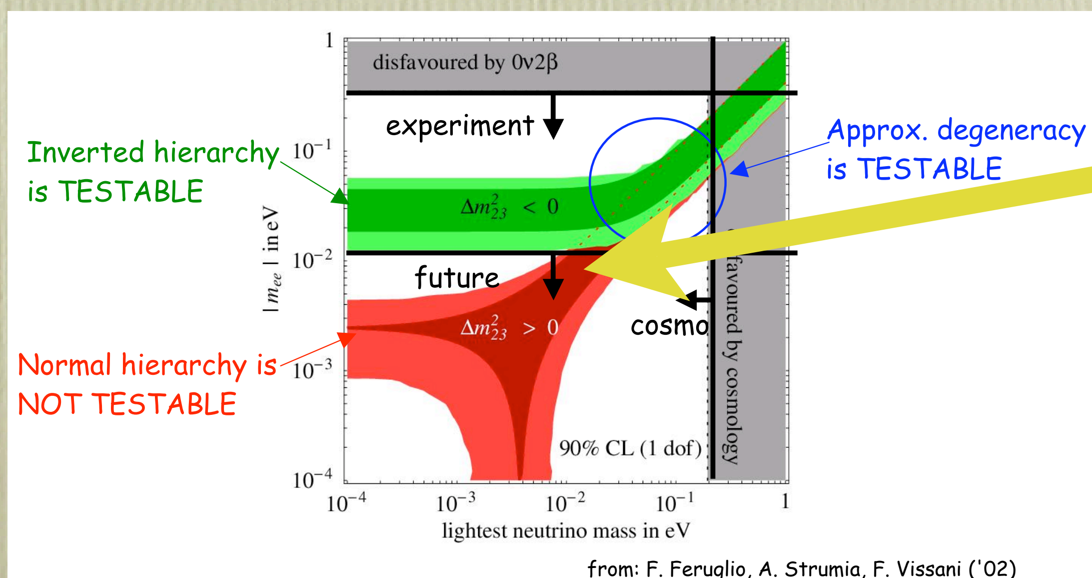
$$r \equiv \Delta m^2_{\odot} / \Delta m^2_{atm}$$

$$|\langle m_{ee}\rangle|^2 = \left[-\frac{1+4r}{9} + \frac{1}{8(1-2r)\cos^2\theta}\right]\Delta m^2_{atm}$$

Models with Tri-bimaximal Neutrino Mixing



For A4: Altarelli et al, 2006



prediction in
A_4 and (d)T models

LFV in A_4 ⇒

Talk by Luca Merlo

TBM $\longleftrightarrow$ Leptogenesis

See: Talk by M. Plumacher

- TBM mixing arises from underlying broken discrete symmetries (A_4, Z_7 x Z_3) through type-I seesaw — E. Jenkins, A. Manohar, 2008
  - ➡ exact TBM mixing

    $\sin\theta_{13} = 0 \;\Rightarrow\; J_{CP}^{lep} \propto \sin\theta_{13} = 0$

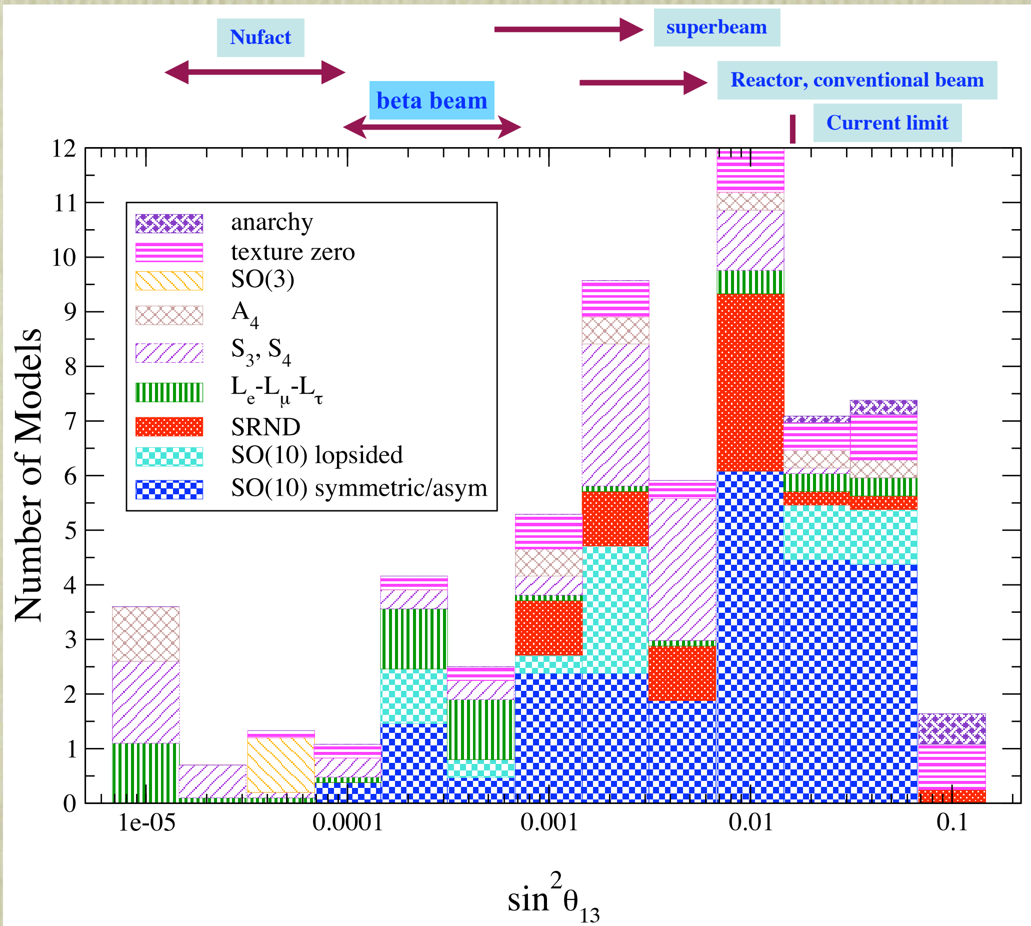
    CP violation through Majorana phases: $\alpha_{21},\ \alpha_{31}$
  - ➡ no leptogenesis as $Im(y_D y_D^\dagger) = 0$
  - ➡ true even when flavor effects included
  - corrections to TBM pattern due to high dim operators

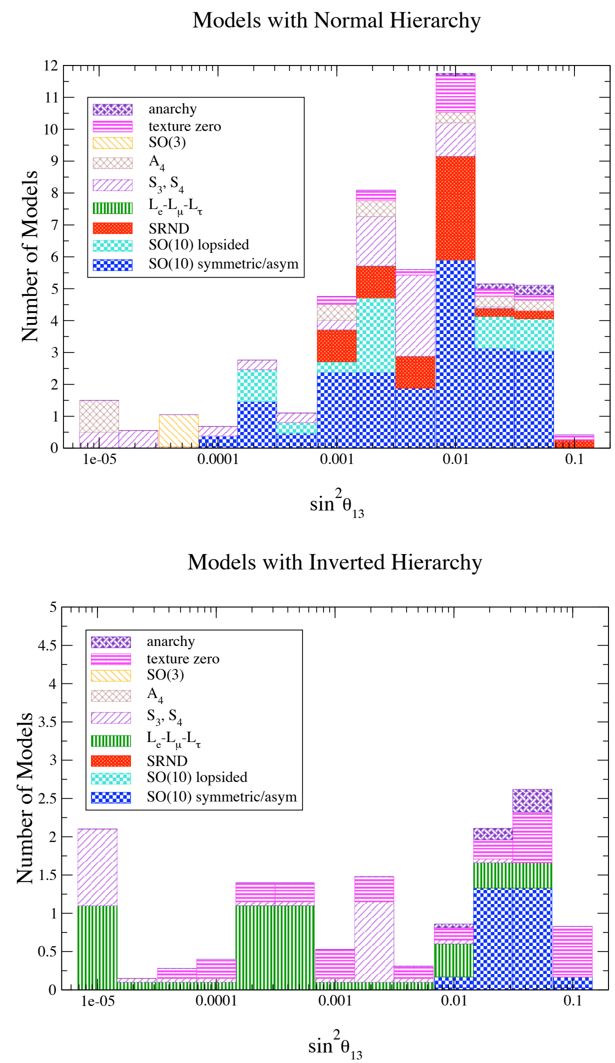
    small symmetry breaking parameter $\eta \ll 1$:

    $\sin\theta_{13} \sim \eta \sim 10^{-2},\ \epsilon \sim 10^{-6}$ can be generated
- type-II seesaw contribution in S_3 — R.N. Mohapatra, H.B. Yu, 2006
  - exact TBM limit: $\varepsilon_2^{II} \simeq -\frac{3}{8\pi}\frac{m_1 M_2 \sin\varphi_1}{v^2 \sin^2\beta}$ $\quad \varphi_1$: one of the Majorana phases

Distinguishing Models

C. Albright & M.-C.C, 2006





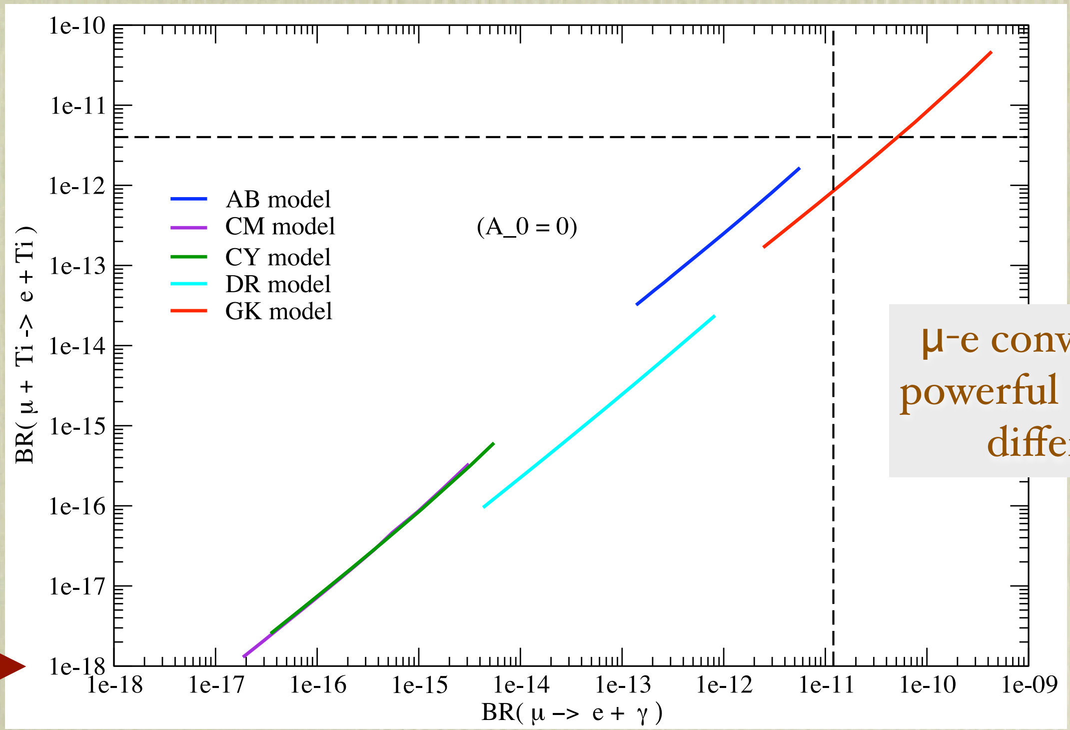
	$\sin^2 2\theta_{13}$	$\sin\theta_{13}$
current limit	10^{-1}	0.16
reactor	10^{-2}	0.05
Conventional beam	10^{-2}	0.05
superbeam	3×10^{-3}	2.7×10^{-2}
Neutrino factory	$(5\text{-}50) \times 10^{-5}$	$(3.5\text{-}11) \times 10^{-3}$

LFV Rare Processes

C. Albright & M.-C.C, 2008

predictions for LFV processes in five viable SUSY SO(10) models:

-- assuming MSUGRA boundary conditions
-- including Dark Matter constraints from WMAP



μ-e conversion could be powerful in distinguishing different models

sensitivity of proposed MECO-type exp

reach at MEG

TeV Scale Seesaw

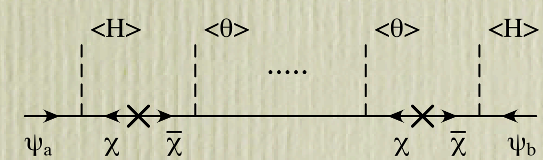
M.-C.C, A. de Gouvea, B. Dobrescu, 2007

- SM x U(1)$_{NA}$ + 3 ν_R: charged under U(1)$_{NA}$ symmetry, broken by <ϕ>
- U(1)$_{NA}$ forbids usual dim-4 Dirac operator and dim-5 Majorana operator

$$m_{LL} \sim \frac{HHLL}{M} \to M \sim 10^{14}\ GeV$$

- neutrino masses generated by very high dimensional operators

$$m_{LL} \sim \left(\frac{\langle\phi\rangle}{M}\right)^p \frac{HHLL}{M} \to M \sim TeV, \quad \text{for large } p \qquad \frac{\langle\phi\rangle}{M} \sim \text{not too small}$$

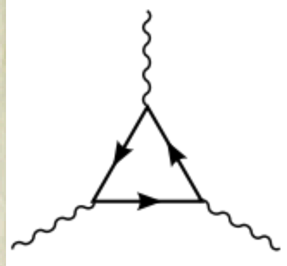


- anomaly cancellations: charge of different families of fermions related => predict flavor mixing
- Through couplings to Z': can probe neutrino sector at colliders

Type III seesaw at LHC ⇒Talk by Roberto Franceschini

Non-anomalous v.s. Anomalous U(1)

- anomaly cancellations: relating charges of different fermions
  - $[U(1)]^3$ condition generally difficult to solve
    - Green-Schwarz mechanism [anomalous U(1)]
    - exotic fields in addition to RH neutrinos



- most models utilized anomalous U(1):
  - earlier claim that U(1) has to be anomalous to be compatible with SU(5) while giving rise to realistic fermion mass and mixing patterns

L.E. Ibanez, G.G. Ross   1994

- non-anomalous U(1) can be compatible with SUSY SU(5) while giving rise to realistic fermion mass and mixing patterns
  - no exotics other than 3 RH neutrinos
  - U(1) also forbids Higgs-mediated proton decay

M.-C.C, D.R.T. Jones, A. Rajaraman, H.B. Yu, 2008

Conclusion

- finite group family symmetry: group theoretical origin for mixing
- Predictions of existing models for θ_{13}: 0 - current bound
- Precision measurements for the θ_{13} and mass hierarchy can tell different scenarios apart:
  - leptonic family symmetry vs GUT
  - inverted hierarchy, small 1-3 mixing => lepton symmetry
  - large 1-3 mixing => inconclusive
- deviation from maximal θ_{23} may tell how symmetry is broken
- May probe other interesting relations: e.g.
  - quark-lepton complementarity: $\theta_{12} + \theta_c = 45^o$
  - new quark-lepton complementarity: $$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,\mathrm{TBM}} - \frac{1}{2}\theta_c\cos\beta = \frac{1}{2} - \frac{1}{2}\theta_c\cos\beta$$
- LFV rare processes can be a robust test

Precision Measurements Indispensable!!