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How long could we live?

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Abstract

We investigate model independent upper bounds on total proton lifetime in the context of grand unified theories with the Standard Model matter content. We find them to be $\tau_p \leqslant 1.5^{+0.5}_{-0.3} \times 10^{39} \frac{(M_X/10^{16}\ \text{GeV})^4}{\alpha^2_{\text{GUT}}} (0.003\ \text{GeV}^3/\alpha)^2$ years and $\tau_p \leqslant 7.1^{+0.0}_{-0.0} \times 10^{36} \frac{(M_X/10^{16}\ \text{GeV})^4}{\alpha^2_{\text{GUT}}} (0.003\ \text{GeV}^3/\alpha)^2$ years in the Majorana and Dirac neutrino case, respectively. These bounds, in conjunction with experimental limits, put lower limit on the mass M_X of gauge bosons responsible for the proton and bound-neutron decay processes. For central values of relevant input parameters we obtain $M_X \geqslant 4.3 \times 10^{14} \sqrt{\alpha_{\text{GUT}}}$ GeV. Our result implies that a large class of non-supersymmetric grand unified models, with typical values $\alpha_{\text{GUT}} \sim 1/39$, still satisfies experimental constraints on proton lifetime. Our result is independent on any CP violating phase and the only significant source of uncertainty is associated with imprecise knowledge of α—the nucleon decay matrix element.

1. Introduction

Grand unified theories [1–4] (GUTs) are the most appealing extensions of the Standard Model (SM) of strong and electroweak interactions. Being founded on the ideas of force and matter unification they always generate two predictions regardless of their exact realization; one is the gauge coupling unification and the other is the proton decay. Of the two it is the latter that offers the *only* unambiguous way to test GUTs [5]. However, despite systematic experimental search it has not been observed so far [6–8]. Even if it is observed, a clear test of GUT might prove difficult due to inherent model dependence of all relevant proton decay contributions [9–11]. Regardless of that, it is worth asking whether we can expect the test of the GUT idea through proton decay experiments with certainty.

There are several generic contributions to nucleon decay in GUTs. (For an incomplete list of various

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studies on proton decay constraints on different unifying theories see [15–23].) In the non-supersymmetric case the most important ones are the Higgs and gauge $d = 6$ contributions. In supersymmetric theories there are two more contributions that generically predict too rapid proton decay. These are the $d = 4$ and $d = 5$ contributions. Of course, the four contributions we mention do not encompass all the possibilities. (For example, presence of extra Higgs representations such as **15** or **10** in an $SU(5)$ GUT can lead to additional contributions through mixing of appropriate components of these representations with the triplet partners of the usual Higgs doublets [24,25]. It is also possible to have sizable contributions without any reference to the GUTs if the theory is supersymmetric [26].) But, they are certainly the most generic ones.

It may come as a surprise that despite their multiplicity and diverse origins all of these contributions can in principle be completely suppressed or forbidden except the gauge $d = 6$ ones. For example, the so-called matter parity forbids the dangerous $d = 4$ contributions and there are numerous different ways to efficiently suppress the $d = 5$ operators and Higgs $d = 6$ operators in realistic scenarios. (For discussion on suppression of $d = 5$ operators see for example [12–14].) In essence, the most promising tests of GUTs can be done through the gauge $d = 6$ contributions.

The idea of using the gauge $d = 6$ dominated branching ratios for the two-body nucleon decays to distinguish between different GUT models of fermion mass has been introduced in the pioneering work of De Rujula, Georgi and Glashow [27]. Their idea has been revisited and elaborated on more recently. Namely, it has been shown that it is possible to make clear test of any GUT with symmetric Yukawa couplings through the nucleon decay channels into antineutrinos [28]. Similar conclusions [29] also hold in the context of flipped $SU(5)$ [27,30–32]. There, the clear test requires symmetric Yukawa couplings in the down-quark sector only. (Flipped $SU(5)$ is to be considered a true GUT in the case of further embedding in $SO(10)$.)

But, in general, even the gauge $d = 6$ contributions can be significantly suppressed if not set to zero. For example, one can completely rotate them away in the flipped $SU(5)$ context [33]. The relevant contributions there, which we refer to as the "flipped $SU(5)$ contributions" for obvious reason, represent only one half of all possible gauge $d = 6$ contributions in GUTs. The other half, which we refer to as the "$SU(5)$ contributions" which is due to exchange of proton decay mediating gauge fields present in an $SU(5)$ gauge group cannot be rotated away without a conflict with the measurements on fermion mixing [34]. Nevertheless, it is worth investigating how efficiently one can suppress these contributions, too. Since there are no other gauge $d = 6$ contributions besides the two we mention, this allows us to set an absolute upper bound on nucleon decay lifetimes. Crucial importance of those bounds lies in the fact that they are the only way to know if there is ever hope to test the idea of grand unification with certainty through proton decay experiments. Even if these bounds turn out to be beyond the experimental reach they set correct lower limit on M_{GUT} through an absolute lower bound on the mass of the nucleon decay mediating gauge bosons. In other words, they are the bounds that can tell us which GUT scenarios are a priori ruled out by experimental data. In what follows we concentrate on GUTs with the SM matter content, i.e., the three generation case, due to their phenomenological relevance.

2. Looking for an upper bound on the total proton lifetime

To establish an upper bound on the total proton lifetime we first critically analyze all possible gauge $d = 6$ operators contributing to proton decay. Again, we concentrate solely on these contributions since all other contributions can be set to zero.

Proton lifetime induced by superheavy gauge boson exchange can be written as follows

$$\tau_p = C M_X^4 \alpha_{\mathrm{GUT}}^{-2} m_p^{-5}, \tag{1}$$

where C is a coefficient which contains all information about the flavor structure of the theory. M_X is the mass of the superheavy gauge bosons. $\alpha_{\mathrm{GUT}} = g_{\mathrm{GUT}}^2/4\pi$, where g_{GUT} is the coupling defined at the GUT scale (the scale of gauge unification). To find a true upper bound on the total lifetime we need to find the maximal value for the C coefficient. Then, for a given value of M_X and α_{GUT} we can bound the GUT scenario prediction for the nucleon lifetime.

The relevant gauge $d = 6$ operators contributing to the decay of the proton, in the physical basis [28], are:

$$O(e^C_\alpha, d_\beta) = c(e^C_\alpha, d_\beta)\epsilon_{ijk}\overline{u^C_i}\gamma^\mu u_j \overline{e^C_\alpha}\gamma_\mu d_{k\beta}, \tag{2a}$$

$$O(e_\alpha, d^C_\beta) = c(e_\alpha, d^C_\beta)\epsilon_{ijk}\overline{u^C_i}\gamma^\mu u_j \overline{d^C_{k\beta}}\gamma_\mu e_\alpha, \tag{2b}$$

$$O(\nu_l, d_\alpha, d^C_\beta) = c(\nu_l, d_\alpha, d^C_\beta)\epsilon_{ijk}\overline{u^C_i}\gamma^\mu d_{j\alpha} \overline{d^C_{k\beta}}\gamma_\mu \nu_l, \tag{2c}$$

$$O(\nu^C_l, d_\alpha, d^C_\beta) = c(\nu^C_l, d_\alpha, d^C_\beta)\epsilon_{ijk}\overline{d^C_{i\beta}}\gamma^\mu u_j \overline{\nu^C_l}\gamma_\mu d_{k\alpha}, \tag{2d}$$

where the relevant coefficients are given by

$$c(e^C_\alpha, d_\beta) = k_1^2\left[V_1^{11}V_2^{\alpha\beta} + (V_1V_{UD})^{1\beta}(V_2V_{UD}^\dagger)^{\alpha 1}\right], \tag{3a}$$

$$c(e_\alpha, d^C_\beta) = k_1^2 V_1^{11}V_3^{\beta\alpha} + k_2^2(V_4V_{UD}^\dagger)^{\beta 1}(V_1V_{UD}V_4^\dagger V_3)^{1\alpha}, \tag{3b}$$

$$c(\nu_l, d_\alpha, d^C_\beta) = k_1^2(V_1V_{UD})^{1\alpha}(V_3V_{EN})^{\beta l} + k_2^2 V_4^{\beta\alpha}(V_1V_{UD}V_4^\dagger V_3 V_{EN})^{1l}, \quad \alpha = 1 \text{ or } \beta = 1, \tag{3c}$$

$$c(\nu^C_l, d_\alpha, d^C_\beta) = k_2^2\left[(V_4V_{UD}^\dagger)^{\beta 1}(U_{EN}^\dagger V_2)^{l\alpha} + V_4^{\beta\alpha}(U_{EN}^\dagger V_2 V_{UD}^\dagger)^{l1}\right], \quad \alpha = 1 \text{ or } \beta = 1. \tag{3d}$$

The mixing matrices $V_1 = U_C^\dagger U$, $V_2 = E_C^\dagger D$, $V_3 = D_C^\dagger E$, $V_4 = D_C^\dagger D$, $V_{UD} = U^\dagger D$, $V_{EN} = E^\dagger N$, and $U_{EN} = E_C^\dagger N_C$. $\alpha, \beta = 1, 2$, $l = 1, 2, 3$, while i, j, and k are the color indices. (Our convention for the diagonalization of the up, down and charged lepton Yukawa matrices is specified by $U_C^T Y_U U = Y_U^{\text{diag}}$, $D_C^T Y_D D = Y_D^{\text{diag}}$, and $E_C^T Y_E E = Y_E^{\text{diag}}$.) The quark mixing is given by $V_{UD} = U^\dagger D = K_1 V_{\text{CKM}} K_2$, where K_1 and K_2 are diagonal matrices containing three and two phases, respectively. The leptonic mixing $V_{EN} = K_3 V_l^D K_4$ in case of Dirac neutrino, or $V_{EN} = K_3 V_l^M$ in the Majorana case. V_l^D and V_l^M are the leptonic mixing matrices at low scale in the Dirac and Majorana case, respectively. The gauge $d = 6$ operators have to be run from the GUT scale down to 1 GeV, i.e., the proton decay scale, and the appropriate amplitude computed in the usual way. (For details, see for example [35].)

In the above expressions $k_1 = g_{\text{GUT}} M^{-1}_{(X,Y)}$, and $k_2 = g_{\text{GUT}} M^{-1}_{(X',Y')}$, where $M_{(X,Y)}$, $M_{(X',Y')} \approx M_{\text{GUT}}$ are the masses of the superheavy gauge bosons. All terms proportional to k_1 are obtained when we integrate out $(X, Y) = (\mathbf{3}, \mathbf{2}, 5/3)$, where X and Y fields have electric charge $4/3$ and $1/3$, respectively. These are the fields appearing in theories based on the $SU(5)$ gauge group. Thus, we call their contributions the "$SU(5)$ contributions". Integrating out $(X', Y') = (\mathbf{3}, \mathbf{2}, -1/3)$ we obtain the terms proportional to k_2. These contributions we refer to as the "flipped $SU(5)$ contributions" since they appear in the flipped $SU(5)$ scenario. The electric charge of Y' is $-2/3$, while X' has the same charge as Y. Again, there are no other gauge contributions in any GUT besides these.

Minimization of the total decay rate represents formidable task since there are in principle 42 unknown parameters. To face the challenge we look for a solution where the "$SU(5)$ contributions" and "flipped $SU(5)$ contributions" are suppressed (minimized) independently. Since we expect that in general the associated gauge bosons and couplings have different values this is also the most natural way to look for the minimal decay rate value. Moreover, the bounds obtained is such a manner will be independent of the underlying gauge symmetry.

The "flipped $SU(5)$ contributions" are set to zero by the following two conditions [33]:

$$V_4^{\beta\alpha} = (D_C^\dagger D)^{\beta\alpha} = 0, \quad \alpha = 1 \text{ or } \beta = 1 \quad \text{(Condition I)},$$

$$(U_C^\dagger E)^{1\alpha} = 0 \quad \text{(Condition II)}.$$

(Condition I cannot be satisfied in the case of symmetric down quark Yukawa couplings.) Therefore, in the presence of all gauge $d = 6$ contributions, in the Majorana neutrino case, there only remain the contributions appearing in $SU(5)$ models. These, however, cannot be set to zero [34] in the case of three generations of matter fields. But, as we now show, they can be significantly suppressed. There are two major scenarios to be considered that defer by the way proton decays:

- There are no decays into the meson-charged antilepton pairs.

All contributions to the decay of the proton into charged antileptons and a meson can be set to zero. Namely, after we implement conditions I and II, we can set to zero Eq. (3b) by choosing

$$V_1^{11} = \left(U_C^\dagger U\right)^{11} = 0 \quad \text{(Condition III)}. \tag{4}$$

(This condition cannot be implemented in the case of symmetric up-quark Yukawa couplings.) On the other hand, Eq. (3a) can be set to zero only if we impose

$$\left(V_2 V_{UD}^\dagger\right)^{\alpha 1} = \left(E_C^\dagger U\right)^{\alpha 1} = 0 \quad \text{(Condition IV)}. \tag{5}$$

Therefore with conditions I–IV there are only decays into antineutrinos and, in the Majorana neutrino case, the only non-zero coefficients are

$$c\left(\nu_l, d_\alpha, d_\beta^C\right) = k_1^2 (V_1 V_{UD})^{1\alpha} (V_3 V_{EN})^{\beta l}. \tag{6}$$

So, indeed, there exists a large class of models for fermion masses where there are no decays into a meson and charged antileptons.

Up to this point all conditions we impose are consistent with the unitarity constraint and experimental data on fermion mixing. (In the $SU(5)$ case we have to impose conditions III and IV only.) We now proceed and investigate the decay channels with antineutrinos. From Eq. (6) we see that it is not possible to set to zero all decays since the factor $(V_1 V_{UD})^{1\alpha}$ can be set to zero for only one value of α in order to satisfy the unitarity constraint. Therefore we have to compare the following two cases:

Case (a) $(V_1 V_{UD})^{11} = 0$ (Condition V).

In this case the chiral Lagrangian technique yields:

$$\Gamma_a\left(p \to \pi^+ \bar\nu_i\right) = 0,$$

$$\Gamma_a\left(p \to K^+ \bar\nu\right) = C(p, K)\left[1 + \frac{m_p}{3m_B}(D + 3F)\right]^2 \times \frac{|V_{\text{CKM}}^{32} V_{\text{CKM}}^{21} - V_{\text{CKM}}^{31} V_{\text{CKM}}^{22}|^2}{|V_{\text{CKM}}^{31}|^2 + |V_{\text{CKM}}^{21}|^2},$$

$$\Gamma_a\left(n \to \pi^0 \bar\nu_i\right) = 0,$$

$$\Gamma_a\left(n \to K^0 \bar\nu\right) = C(n, K)\left[1 + \frac{m_n}{3m_B}(D + 3F)\right]^2 \times \frac{|V_{\text{CKM}}^{32} V_{\text{CKM}}^{21} - V_{\text{CKM}}^{31} V_{\text{CKM}}^{22}|^2}{|V_{\text{CKM}}^{31}|^2 + |V_{\text{CKM}}^{21}|^2},$$

$$\Gamma_a(n \to \eta \bar\nu_i) = 0,$$

where

$$C(a, b) = \frac{(m_a^2 - m_b^2)^2}{8\pi m_a^3 f_\pi^2} A_L^2 |\alpha|^2 k_1^4. \tag{7}$$

Case (b) $(V_1 V_{UD})^{12} = 0$ (Condition VI).

All the decays channels into antineutrinos are non-zero in this case. Associated decay rates are:

$$\Gamma_b\left(p \to \pi^+ \bar\nu\right) = C(p, \pi)[1 + D + F]^2 \times \frac{|V_{\text{CKM}}^{32} V_{\text{CKM}}^{21} - V_{\text{CKM}}^{31} V_{\text{CKM}}^{22}|^2}{|V_{\text{CKM}}^{22}|^2 + |V_{\text{CKM}}^{32}|^2},$$

$$\Gamma_b\left(p \to K^+ \bar\nu\right) = C(p, K)\left[\frac{2m_p}{3m_B} D\right]^2 \times \frac{|V_{\text{CKM}}^{32} V_{\text{CKM}}^{21} - V_{\text{CKM}}^{31} V_{\text{CKM}}^{22}|^2}{|V_{\text{CKM}}^{22}|^2 + |V_{\text{CKM}}^{32}|^2},$$

$$\Gamma_b\left(n \to \pi^0 \bar\nu\right) = C(n, \pi)\frac{[1 + D + F]^2}{2}\Gamma\left(p \to \pi^+ \bar\nu\right),$$

$$\Gamma_b\left(n \to K^0 \bar\nu\right) = C(n, K)\left[1 + \frac{m_n}{3m_B}(D - 3F)\right]^2 \times \frac{|V_{\text{CKM}}^{32} V_{\text{CKM}}^{21} - V_{\text{CKM}}^{31} V_{\text{CKM}}^{22}|^2}{|V_{\text{CKM}}^{22}|^2 + |V_{\text{CKM}}^{32}|^2},$$

$$\Gamma_b(n \to \eta \bar\nu) = C(n, \eta)\frac{[1 + D - 3F]^2}{6} \times \frac{|V_{\text{CKM}}^{32} V_{\text{CKM}}^{21} - V_{\text{CKM}}^{31} V_{\text{CKM}}^{22}|^2}{|V_{\text{CKM}}^{22}|^2 + |V_{\text{CKM}}^{32}|^2}.$$

The nice thing about these results is that they are completely independent of *all* CP violating phases including those of V_{CKM} and V_l and any mixing angles beyond the CKM ones. (This is completely un-

Table 1
Proton lifetimes in years for Majorana and Dirac neutrinos in units of $M_X^4/\alpha_{\text{GUT}}^2$, where the mass of gauge bosons is taken to be 10^{16} GeV

Channel	Majorana		Dirac	
	Case (a)	Case (b)	Case (a)	Case (b)
$p \to \pi^+ \bar{\nu}$	∞	$5.1^{+1.7}_{-1.1} \times 10^{38}$	$5.4^{+1.8}_{-1.2} \times 10^{38}$	$2.6^{+0.9}_{-0.6} \times 10^{38}$
$p \to K^+ \bar{\nu}$	$1.0^{+0.4}_{-0.2} \times 10^{38}$	$2.5^{+0.9}_{-0.6} \times 10^{40}$	$6.8^{+0.0}_{-0.0} \times 10^{36}$	$7.2^{+0.0}_{-0.0} \times 10^{36}$
Total	$1.0^{+0.4}_{-0.2} \times 10^{38}$	$5.0^{+1.7}_{-1.1} \times 10^{38}$	$6.7^{+0.0}_{-0.0} \times 10^{36}$	$7.1^{+0.0}_{-0.0} \times 10^{36}$

Table 2
Lifetimes for bounded neutrons in years for Majorana and Dirac neutrinos in units of $M_X^4/\alpha_{\text{GUT}}^2$, where the mass of gauge bosons is taken to be 10^{16} GeV

Channel	Majorana		Dirac	
	Case (a)	Case (b)	Case (a)	Case (b)
$n \to \pi^0 \bar{\nu}$	∞	$1.0^{+0.3}_{-0.2} \times 10^{39}$	$1.1^{+0.4}_{-0.2} \times 10^{39}$	$5.2^{+1.8}_{-1.2} \times 10^{38}$
$n \to K^0 \bar{\nu}$	$1.1^{+0.4}_{-0.2} \times 10^{38}$	$6.7^{+2.3}_{-1.5} \times 10^{39}$	$1.9^{+0.0}_{-0.0} \times 10^{36}$	$1.9^{+0.0}_{-0.0} \times 10^{36}$
$n \to \eta \bar{\nu}$	∞	$1.5^{+0.5}_{-0.3} \times 10^{41}$	$1.6^{+0.5}_{-0.3} \times 10^{41}$	$7.6^{+2.5}_{-1.7} \times 10^{40}$
Total	$1.1^{+0.4}_{-0.2} \times 10^{38}$	$8.8^{+2.9}_{-2.0} \times 10^{38}$	$1.9^{+0.0}_{-0.0} \times 10^{36}$	$1.9^{+0.0}_{-0.0} \times 10^{36}$

expected since there are in principle 42 different angles and phases that could a priori enter our results.) Also, in the limit $V_{\text{CMK}}^{13} \to 0$ all decay rates vanish as required in the case of three generations of matter fields [34]. To demonstrate these two properties we adopt the so-called "standard" parametrization of V_{CKM} [36–39] that utilizes angles θ_{12}, θ_{23}, θ_{13}, and a phase δ_{13}. (For example, in that parametrization $V_{\text{CKM}}^{13} = e^{-i\delta_{13}} s_{13}$.) The relevant terms read $V_{\text{CKM}}^{32} V_{\text{CKM}}^{21} - V_{\text{CKM}}^{31} V_{\text{CKM}}^{22} = e^{i\delta_{13}} s_{13}$, $|V_{\text{CKM}}^{22}|^2 + |V_{\text{CKM}}^{32}|^2 = c_{12}^2 + s_{12}^2 s_{13}^2$ and $|V_{\text{CKM}}^{31}|^2 + |V_{\text{CKM}}^{21}|^2 = s_{12}^2 + c_{12}^2 s_{13}^2$, where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. Hence, all one needs to know are angles θ_{12} and θ_{13}.

We present numerical values of all relevant two body decay lifetimes for proton and bounded neutron decays in Tables 1 and 2, respectively. Clearly, it is Case (b) that gives the lowest total decay rate in the Majorana neutrino case. (We also include the Dirac neutrino case for completeness.) Lifetimes are given in units of $M_X^4/\alpha_{\text{GUT}}^2$, where the gauge boson mass is taken to be 10^{16} GeV. To generate these values we use $m_p = 938.3$ MeV, $D = 0.81$, $F = 0.44$, $m_B = 1150$ MeV, $f_\pi = 139$ MeV, $A_L = 1.43$, and the most conservative value $\alpha = 0.003$ GeV^3 [40]. Indicated uncertainties reflect the errors in measurement of angles θ_{12} and θ_{13} only. These are well known and their sines are: $s_{12} = 0.2243 \pm 0.0016$, and $s_{13} = 0.0037 \pm 0.0005$ [41]. Note that the most poorly known parameter is actually α; the most recent QCD lattice calculations [42,43] indicate that its value could be three times bigger than the value we use. If that result persists it would reduce the lifetime bounds we present by a factor of ten.

- There are no decays into the meson–antineutrino pair in the Majorana neutrino case.

Let us show that it is also possible to set to zero all nucleon decay channels into a meson and antineutrinos. After conditions I and II, we could impose $(V_1 V_{UD})^{1\alpha} = 0$ (Condition VII) instead of $V_1^{11} = 0$. (Again, these two equalities are exclusive in the case $V_{\text{CKM}}^{13} \neq 0$.) Therefore, in the Majorana neutrino case, there are no decays into antineutrinos (see Eq. (3c)). In this case the property that the gauge contributions vanish as $|V_{\text{CKM}}^{13}| \to 0$ is obvious since $|V_1^{11}| = |V_{\text{CKM}}^{13}|$. We have to further investigate all possible values of $V_2^{\beta\alpha}$ and $V_3^{\beta\alpha}$. Now, we can choose $V_2^{\beta\alpha} = 0$ and $V_3^{\beta\alpha} = 0$, except for the case $\alpha = \beta = 2$ (Condition VIII). In that case there are only decays into a strange mesons and muons. Let us call this Case (c). To understand which case gives us an upper bound on the total proton decay lifetime in the Majorana neutrino case, we compare the predictions coming from the Case (b) and Case (c). The ratio between the relevant decay rates is given by

$$\frac{\Gamma_c(p\to K^0\mu^+)}{\Gamma_b(p\to\pi^+\bar\nu)} = 2\big(c_{12}^2+s_{12}^2s_{13}^2\big)\frac{(m_p^2-m_K^2)^2}{(m_p^2-m_\pi^2)^2}\frac{[1+\frac{m_p}{m_B}(D-F)]^2}{[1+D+F]^2} = 0.33. \quad (8)$$

Thus, the upper bound on the proton lifetime in the case of Majorana neutrinos indeed corresponds to the total lifetime of Case (c). We find it to be

$$\tau_p \leqslant 1.5^{+0.5}_{-0.3}\times 10^{39}\frac{(M_X/10^{16}\ \text{GeV})^4}{\alpha_{\text{GUT}}^2} \times \big(0.003\ \text{GeV}^3/\alpha\big)^2\ \text{years}, \quad (9)$$

where the gauge boson mass is given in units of 10^{16} GeV. We explicitly indicate the dependence of our results on the nucleon decay matrix element. These bounds are applicable to any GUT regardless whether the scenario is supersymmetric or not. If the theory is based on $SU(5)$ the above bounds are obtained by imposing conditions VII and VIII. If the theory contains both $SU(5)$ and flipped $SU(5)$ contributions, in addition to these, one needs to impose conditions I and II.

We plot the proton bounds in the M_X–α_{GUT} plane for the Majorana (Dirac) neutrino case in Figs. 1 (2). Our results, in conjunction with the experimental limits on nucleon lifetime, set an absolute lower bound on mass of superheavy gauge bosons. Since their mass is identified with the unification scale after the threshold corrections are incorporated in the running [44] this also sets the lower bound on the unification scale. Using the most stringent limit on partial proton lifetime ($\tau_p \geqslant 50\times 10^{32}$ years) for the $p\to\pi^0e^+$ channel [41] and setting $\alpha = 0.003\ \text{GeV}^3$ [40], we obtain

$$M_X \geqslant 4.3^{+0.3}_{-0.3}\times 10^{14}\sqrt{\alpha_{\text{GUT}}}\ \text{GeV}, \quad (10)$$

where α_{GUT} *usually* varies from $1/40$ for non-supersymmetric theories to $1/24$ for supersymmetric theories. For example, if we take a non-supersymmetric value $\alpha_{\text{GUT}} = 1/39$, we obtain

$$M_X \geqslant 7\times 10^{13}\ \text{GeV}. \quad (11)$$

Again, this result implies that any non-supersymmetric theory with $\alpha_{\text{GUT}} = 1/39$ is eliminated if its unifying scale is bellow 7.0×10^{13} GeV regardless of the

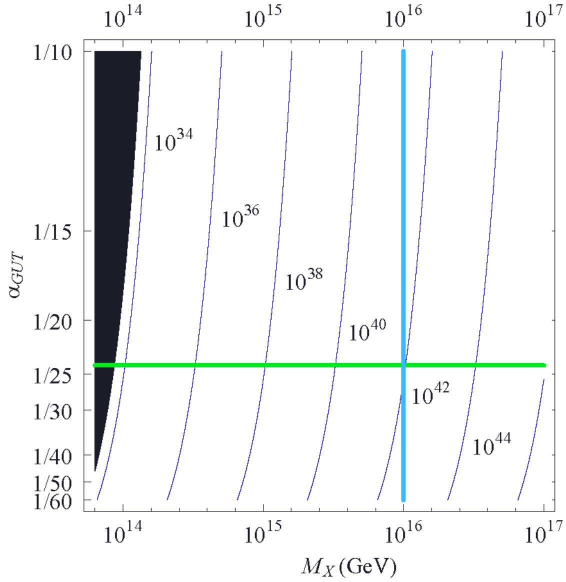

Fig. 1. Isoplot for the upper bounds on the total proton lifetime in years in the Majorana neutrino case in the M_X–α_{GUT} plane. The value of the unifying coupling constant is varied from 1/60 to 1/10. The conventional values for M_X and α_{GUT} in SUSY GUTs are marked in thick lines. Experimentally excluded region is given in black.

exact form of the Yukawa sector of the theory. Note that majority of non-supersymmetric extensions of the Georgi–Glashow $SU(5)$ model yield GUT scale which is slightly above 10^{14} GeV. Hence, as far as the experimental limits on proton decay are concerned, these extensions still represent viable scenarios of models beyond the SM. Region of M_X excluded by the experimental result is also shown in Figs. 1 and 2.

At this point the following two observations are in order:

(1) All three cases (Cases (a)–(c)) yield comparable lifetimes (within a factor of ten) even though they significantly defer in decay pattern predictions;
(2) We use the most stringent experimental limit on partial proton lifetime as if it represents the limit on the total proton lifetime. Even though this is not correct (see discussion in [41]) it certainly yields the most conservative bound on M_X.

One can easily extend our results to a class of orbifold GUT theories [45,46] where all matter fields live on an "unbroken" brane. In essence, to obtain the

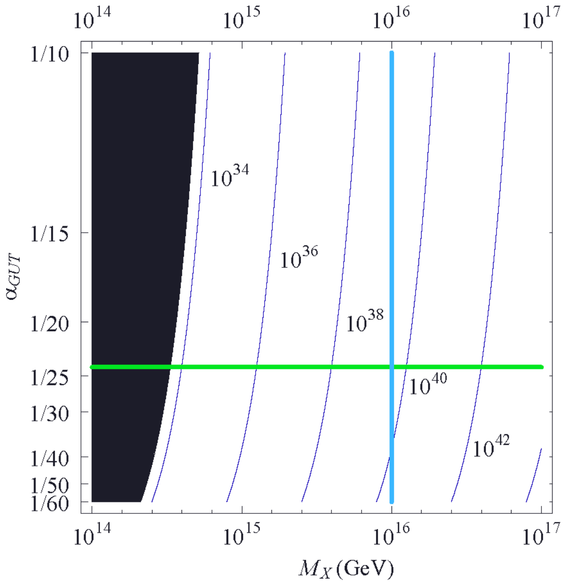

Fig. 2. Isoplot for the upper bounds on the total proton lifetime in years in the Dirac neutrino case in the M_X–α_{GUT} plane. The value of the unifying coupling constant is varied from $1/60$ to $1/10$. The conventional values for M_X and α_{GUT} in SUSY GUTs are marked in thick lines. Experimentally excluded region is given in black.

lower limit on the gauge boson mass in those theories, it suffices to multiply the limit presented in Eq. (11) by $\sqrt{\pi/2}$. (This factor accounts for the fact that the two-body decay of the proton is due to exchange of an entire Kaluza–Klein (KK) tower of states [47] associated with the proton decay mediating gauge boson.) The bound obtained in such a way then corresponds to the limit on the compactification scale of extra dimension(s). (Recall that in orbifold GUTs the gauge bosons responsible for proton decay belong to the KK tower where the lightest gauge boson in the tower has the mass equal to the orbifold compactification scale.) Curiously enough, exact unification of gauge couplings in the five-dimensional $S^1/(Z_2 \times Z_2')$-type orbifold models usually requires the compactification scale to be slightly above 10^{14} GeV [48–50]. This would imply that the orbifold GUT theories with the matter fields all located on the "unbroken" brane could soon be completely ruled out if the proton decay is not observed in the next generation of the proton decay experiments.

In order to complete our analysis let us finally demonstrate the possibility to set to zero the Higgs $d = 6$ and $d = 5$ contributions. The triplets $T = (\mathbf{3}, \mathbf{1}, -2/3)$ and $\bar{T} = (\bar{\mathbf{3}}, \mathbf{1}, 2/3)$ have the following interactions:

$$W_T = \int d^2\theta \{ [\hat{Q}\underline{A}\hat{Q} + \hat{U}^C\underline{C}\hat{E}^C + \hat{D}^C\underline{E}\hat{N}^C]\hat{T} + [\hat{Q}\underline{B}\hat{L} + \hat{U}^C\underline{D}\hat{D}^C]\hat{\bar{T}} \} + \text{h.c.} \tag{12}$$

Choosing $\underline{A}_{ij} = -\underline{A}_{ji}$ and $\underline{D}_{ij} = 0$, except for $i = j = 3$, the Higgs $d = 6$ and $d = 5$ contributions are indeed set to zero. It is also possible to have SUSY scenarios where the $d = 5$ operators are strongly suppressed by particular realization of superparticle spectrum [51]. In any case, even if SUSY is realized at low energies we are sure that the upper bound is coming from the gauge $d = 6$ contributions.

3. Summary

We have investigated the possibility of finding an upper bound on the total nucleon decay lifetime in the context of grand unified theories with the Standard Model matter content. This bound originates from the gauge $d = 6$ contributions, since all other contributions are quite model dependent and can always be suppressed. In the Majorana neutrino case the bound is $\tau_p \leqslant 1.5 \times 10^{39} \frac{(M_X/10^{16}\ \text{GeV})^4}{\alpha_{\mathrm{GUT}}^2} (0.003\ \text{GeV}^3/\alpha)^2$ years, while in the Dirac neutrino case $\tau_p \leqslant 7.1 \times 10^{36} \frac{(M_X/10^{16}\ \text{GeV})^4}{\alpha_{\mathrm{GUT}}^2} (0.003\ \text{GeV}^3/\alpha)^2$ years. These bounds are valid in both supersymmetric and non-supersymmetric scenarios and are grand unifying gauge group independent. Moreover, there is no dependence of our results on CP violating phases nor any angles beyond those of CKM. Our bounds are very useful for two reasons. Firstly, in the context of realistic grand unified theories they indicate whether it is possible to test these theories in their *entire* flavor parameter space with certainty through proton decay experiments. Secondly, they put an absolute lower bound on the mass of proton decay mediating gauge bosons. We obtain $M_X \geqslant 4.3^{+0.3}_{-0.3} \times 10^{14} \sqrt{\alpha_{\mathrm{GUT}}}$ GeV for a reasonable set of input parameters. Since this mass is usually identified with the unifying scale through threshold matching conditions our bounds can be interpreted as the lower bounds on the GUT scale itself. We have also addressed implications our bounds have on the popular class of the so-called "orbifold" models.

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