Proton decay in minimal supersymmetric SU(5)

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We systematically study proton decay in the minimal supersymmetric SU(5) grand unified theory. We find that although the available parameter space of soft masses and mixings is quite constrained, the theory is still in accord with experiment.

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I. INTRODUCTION

It has been known for more than ten years that low-energy supersymmetry (SUSY) is tailor made for grand unification: with the desert assumption the gauge couplings of the supersymmetric standard model unify at the single scale $M_{\mathrm{GUT}} \approx 10^{16}$ GeV. Actually, this was foreseen some 20 years ago [1–4]. However, it was noticed almost immediately that supersymmetric grand unified theories (GUTs) [5,6] carry a potential catastrophe of new $d=5$ contributions to the proton decay. This has been studied on and off for the past 20 years (see, for example, [7,8]) with the culminating conclusion [9] that the minimal supersymmetric SU(5) theory is actually ruled out precisely due to the $d=5$ proton decay. To us, ruling out the minimal theory is almost a death blow to the idea of grand unification. It is hard enough to verify the predictions of the minimal GUT; the extended versions of the theory unfortunately stop being predictive. For example, the beauty of matter unification and the naturalness of the see-saw mechanism [10] make a minimal SUSY SO(10) theory [11] more appealing. However, this is a typical example of what we are saying: the theory connects different mass scales, but does not predict them.

In view of the above, it is of extreme importance to be completely sure that the minimal SUSY GUT is ruled out. This has prompted us to reinvestigate this issue in detail. According to us, any rumor of the death of the theory is somewhat premature. More precisely, we study proton decay with arbitrary soft masses and fermion and sfermion mixings and find out the following: the model parameter space is quite constrained but not yet in contradiction with experiment. In other words, the improved measurements of proton decay will provide information about the nature of supersymmetry breaking (i.e., the soft masses) and the fermionic mass textures. This is the sector of the theory completely orthogonal to grand unification and therefore we advocate the point of view that proton decay is not yet a good test of the generic properties of grand unification (here we mean obviously the dimension 5 aspect of it). We should stress here that the so-called decoupling regime seems to be both necessary and sufficient to save the theory from being ruled out.

In short, although we follow [9] in accepting the decoupling of the first two generations of sfermions, we cannot agree on this not being enough. The point is that we know nothing about individual fermion and sfermion mixings. Thus, proton decay simply limits these parameters and, admittedly, the restrictions are quite severe. In all honesty, it is hard to imagine a simple scenario of SUSY breaking which could be in accord with our constraints. However, a phenomenological study must always be separated from theoretical bias and, phenomenologically speaking, the theory is still alive.

II. THE MINIMAL SUPERSYMMETRIC SU(5)

Before starting any discussion of proton decay, one must address the subtle issue of defining a minimal SU(5) theory. Obviously, a reasonable definition should be based on choosing a minimal Higgs sector which contains an adjoint 24 and a pair of 5 and $\overline{5}$ representations.

We will ultimately show that even this theory (as incomplete as it is) is not in conflict with the proton decay experiment. In order to be as general as possible, we perform our calculations for arbitrary values of the parameters of the theory.

In minimal SU(5) we can most generally write (in the renormalizable limit) for the relevant terms in the superpotential of the Higgs and Yukawa sectors

$$W_H = \frac{m_\Sigma}{2}\mathrm{Tr}\Sigma^2 + \frac{\lambda}{3}\mathrm{Tr}\Sigma^3 + \eta \overline{5}_H \Sigma 5_H + m_H \overline{5}_H 5_H, \tag{1}$$

$$W_Y = 5_H 10^T Y^U 10 + \overline{5}_H 10^T Y^D \overline{5}, \tag{2}$$

where Σ is the SU(5) adjoint, 5_H and $\overline{5}_H$ are the Higgs fundamental and antifundamental superfield representations, the 10 and $\overline{5}$ refer to the three generations of matter superfields, and Y's are 3×3 Yukawa matrices.

In the supersymmetric standard model language, the Yukawa sector can be rewritten as

$$W_Y = HQ^T Y_U u^c + \overline{H} Q^T Y_D d^c + \overline{H} e^{cT} Y_E L + TQ^T \underline{A} Q + T u^{cT} \underline{B} e^c + \overline{T} Q^T \underline{C} L + \overline{T} u^{cT} \underline{D} d^c, \tag{3}$$

where except for the heavy triplets T and $\overline{T}$ the rest are the minimal supersymmetric standard model (MSSM) superfields in the usual notation. The generation matrices $Y_{U,D,E}$ and $\underline{A}$, $\underline{B}$, $\underline{C}$, and $\underline{D}$ can in principle be arbitrary. In the minimal SU(5) defined above, one finds the usual relations $\underline{A} = \underline{B} = Y_U = Y_U^T$ and $\underline{C} = \underline{D} = Y_D = Y_E$ at the GUT scale. The above definition of minimality implies no new structure at all energies up to $M_{\rm Pl}$. On the other hand, the lepton–down-quark relations can be easily corrected by higher-dimensional operators without introducing any new field at $M_{\rm GUT}$. We postpone the discussion of higher-dimensional operators for the summary and outlook.

As we mentioned before, we do not assume any specific values for the soft mass matrices of squarks and sleptons. However, as emphasized clearly in [9], we cannot have all three generations of squarks contribute to the proton decay. The simplest direction to take, as [9] already did, is to assume the so-called decoupling limit for the sfermions: the first two generations have a mass of order 10 TeV, thus effectively decoupling from the rest, while the third is of order 1 TeV [12–14]. This is still in accord with naturalness constraints, and the limits from flavor violation in neutral current phenomena suggest small mixings with the first two generations of fermions. We will see later that it is possible to make the proton decay be in agreement with experiment, again for some combinations of such mixings being small.

With this in mind, we allow the mass diagonalization matrices to be different for particles and sparticles. For the fermions we have

$$U^T Y_U U_c = Y_U^d,$$

$$D^T Y_D D_c = Y_D^d,$$

$$E_c^T Y_E E = Y_E^d, \tag{4}$$

where X (X_c) is the unitary matrix that rotates the fermion x (x^c) from the flavor to the mass basis. The only combination we know from low-energy experiments is $U^\dagger D = V_{\rm CKM}$ (and a similar one in the lepton sector, $N^\dagger E = V_l$, the leptonic mixing matrix).

Similarly, the unitary matrices $\tilde{X}(\tilde{X}_c)$ rotate the bosons $\tilde{x}(\tilde{x}^c)$ from the flavor to the mass states. Once $SU(2)_L$ is spontaneously broken, there is also in general a nonzero mixing between the bosonic states $\tilde{X}$ and $(\tilde{X}^c)^*$: their relative importance is proportional to $m_W / m_{\tilde{f}}$, which is, for our choice of the squark and slepton masses, not bigger than 1/10. We assume this to be small enough to consider it as a perturbation.

The calculation itself is tedious but straightforward, and thus we leave the details for the Appendix. We simply turn to the systematic analysis of the possible solutions which keep the proton stable enough.

III. WHY PROTON DECAY DOES NOT RULE OUT MINIMAL SU(5)

In this central section of our paper (the only one you should read if you just wish to get to our main point), we stick to the very minimal SUSY SU(5) theory. In other words, we assume the conditions discussed above (valid when $M_{\rm Pl} \to \infty$) in the theory with only 5 and $\overline{5}$ light Higgs representations:

$$\underline{A} = \underline{B} = Y_U = Y_U^T, \tag{5}$$

$$\underline{C} = \underline{D} = Y_D = Y_E, \tag{6}$$

where, of course, these conditions are valid at the unification scale. A quick glance at the Appendix shows that the longevity of the proton can be achieved by, say, the following conditions at 1 GeV:

$$(\tilde{U}^\dagger D)_{31,32} \approx 0, \tag{7}$$

$$(\tilde{D}^\dagger D)_{31,32} \approx 0, \tag{8}$$

$$(\tilde{U}_c^T Y_U^T D)_{31,32} \approx 0, \tag{9}$$

$$(\tilde{N}^T \underline{C}^T D)_{31,32} (\tilde{U}^T \underline{A} D)_{32,31} \approx 0, \tag{10}$$

$$(\tilde{E}_c^\dagger E_c)_{31,32} \approx 0, \tag{11}$$

$$(\tilde{D}_c^\dagger D_c)_{31,32} \approx 0, \tag{12}$$

$$(\tilde{E}^\dagger E)_{31,32} \approx 0, \tag{13}$$

$$(\tilde{N}^\dagger E)_{31,32} \approx 0. \tag{14}$$

If one wishes to quantify these conditions, one cannot take Eqs. (5), (6) at face value, but instead must compute the departure due to the running from $M_{\rm GUT}$ to 1 GeV. It makes no sense to do this here; after all, this is just a prototype example and it can surely be satisfied at any scale.

In the above equations, we simply mean that all the terms must be small. How small? It is hard to quantify this precisely and, honestly speaking, it seems to us a premature task. Our aim was to demonstrate that the theory is still consistent with data and from the above formulas it is obvious. If (when) proton decay is discovered and the decay modes measured, it may be sensible to see how small the above terms should be. Suffice it to say that a percent suppression of the super Kobayashi-Maskowa (KM) results should be enough [9]. This means that on the average each vertex should be suppressed by a factor of 1/3 or so with respect to the minimal supergravity predictions. It is very difficult to say more: in fact, one may be tempted to estimate that, for example, the combinations on the left-hand sides of the above equations need to be at least 10^{-2} the same combinations in super KM. However this is not automatically necessary or enough. The fact is that we have a nonlinear system, since the total decay in a specified mode is proportional to the square of a sum of single diagrams, each of them proportional to the product of four unknown mixings. Some of these mixings contribute to different diagrams, and some depend on others, so the task of constraining them numerically seems exaggerated in view of our complete ignorance of all these parameters. What we can say for sure is that if each of the diagrams in the Appendix is suppressed by a factor of 1/100 with respect to the minimal supergravity predictions, proton decay is not too fast and minimal supersymmetric SU(5) is not ruled out.

Notice that all the terms can be made to vanish by a judicious choice of squark and slepton mixing matrices. In other words, at this point the proton decay limits provide information on the properties of sfermions and *not* on the structure of the unified theory.

Notice further that the so-called super KM basis, in which the mixing angles of fermions and sfermions are equal, does not work for the proton decay, since Eqs. (7), (9), (10), and (14) are not satisfied. If you believe in super-KM, you would conclude that the theory is ruled out. It is obvious though, from our work, that this is not true in general.

Notice even further that all the relations (7)–(14) do not require the extreme minimality conditions (5)–(6). More precisely, one can opt for the improvement of the fermion mass relations and still save the proton.

One might worry that the above constraints for the sfermion and fermion mixing matrices could be in contradiction with the experimental bounds on the flavor violation low-energy processes. Fortunately, this is not true. Namely, the same conditions (7)–(14) suffice to render neutral current flavor violation inoffensive (of course, the decoupling is necessary for this to be true).

The analysis in the Appendix has been done with the assumption of no left-right sfermion, neutralino, or chargino mixing. As we explained at the end of the previous section, this mixing can be included in a perturbative way: one can show that, up to two mass insertions, the same constraints (7)–(14) kill all the contributions to nucleon decay. This is enough to increase the nucleon lifetime above the experimental limit, since each mixing multiplies the diagram by at least 1/10.

Up to now we have discussed only the $d=5$ nucleon decay. What about a generic $d=6$ contribution of gauge bosons relevant for both ordinary and SUSY GUTs? In the very minimal case, $Y_D = Y_E$ and $Y_U = Y_U^T$, this is completely determined by the CKM matrix [15]. However, as soon one abandons this unrealistic situation, this is not true anymore and the individual up and down quark and lepton mixings enter the game and proton decay is not as determined as before [16,17].

IV. SUMMARY AND OUTLOOK

We hope to have convinced the reader that the supersymmetric SU(5) theory even in its very minimal version is still alive and still in accord with the nucleon decay limits. All that is required is simply small mixing angles among squarks (sleptons) and/or quarks (leptons), on top of the decoupling hypothesis, which sees the first two generations of sfermions pushed to the 10 TeV region.

Does this mean that the proton decay experiments really probe the sfermion and fermion mixing matrices? More precisely, are there any other uncertainties involved in this game? At first glance, the answer is no. After all, we have carefully defined the minimal theory and found the predictions discussed above. However, two points can still be raised.

(i) *Triplet-octet splitting (higher dimensional operators in the Higgs sector).* In order to appreciate this point, let us discuss the origin of the problem in question. If one assumes that the heavy particles in the adjoint superfield Σ (the color octet and the weak triplet) have masses equal to $M_{\rm GUT}$, the gauge couplings unify at $\approx M_{\rm GUT} \approx 10^{16}$ GeV. In this case, the masses of heavy triplets T and $\overline{T}$ are smaller than $\approx 3.6 \times 10^{15}$ GeV [9]. A factor of around 20 increase of triplet masses according to [9] is sufficient to satisfy all the experimental constraints.

A simple possibility that allows this is to increase $M_{\rm GUT}$ itself by a similar factor of 20 or so. This turns out to be easily satisfied by simply splitting the octet and triplet masses in Σ and allowing them to be smaller than $M_{\rm GUT}$ [18].

Imagine, for example, that the octets and triplets are light enough, so that their masses originate from dimension-4 Planck-scale-induced terms in the superpotential, i.e., assume that the renormalizable cubic term in the superpotential (1) is negligible. In that case, $m_3 = 4 m_8$ [19], which at the one-loop level increases the proton decay mediating Higgs triplet masses by about a factor of 30.

(ii) *Improving the Yukawa sector with higher-dimensional operators.* In the minimal SU(5) theory and in the limit $M_{\rm Pl} \to \infty$, the proton decay mediating Higgs triplet couplings is set by SU(5) symmetry, since they must be equal to the ordinary doublet couplings (5), (6). These relations can be, in the spirit of [20], changed by the nonrenormalizable $1/M_{\rm Pl}$ suppressed operators [21–23]. This induces unfortunately additional uncertainty in the constraints for the sfermion and fermion mixings.

In other words, to us the nucleon decay not only cannot rule out the structure of the theory, but even in the case of observation it would not easily provide enough information about sfermion and fermion individual mixings. In any case, we see no reason whatsoever why one should search for modifications of the theory at the GUT scale or below for the sake of proton decay. If you need to do model building, do not look here for an excuse.

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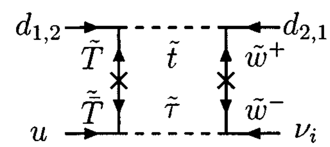
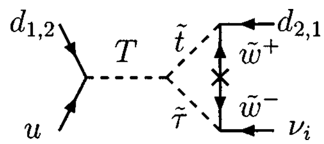
APPENDIX

In this appendix, we present the complete set of diagrams responsible for $d=5$ nucleon decay in the minimal supersymmetric SU(5) theory. In our notation, T and $\overline{T}$-stand for heavy Higgs triplets, $\tilde{T}$ and $\tilde{\overline{T}}$ denote their fermionic partners, $\tilde{w}^{\pm}$ stands for W-inos, $\tilde{h}_{+,0}$ and $\tilde{\overline{h}}_{-,0}$ are light Higgsinos, and $\tilde{V}_0$ stand for neutral gauginos.

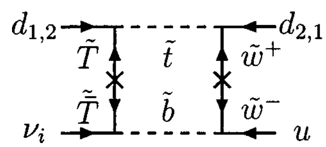
(i) $\underline{p\to(K^+,\pi^+,\rho^+,K^{*+})\overline{\nu_i}}$, $\underline{n\to(\pi^0,\rho^0,\eta,\omega,K^0,K^{*0})\overline{\nu_i}}$ $(i=1,2,3)$



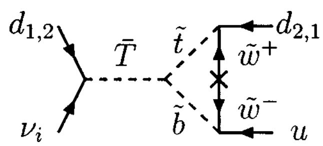
$$\propto \quad (D^T\underline{A}\tilde{U})_{13,23}(\tilde{U}^\dagger D)_{32,31}(N^T\tilde{E}^*)_{i3}(\tilde{E}^T\underline{C}^T U)_{31} \tag{A1}$$



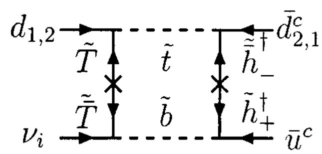
$$\propto \quad (D^T\underline{A}U)_{11,21}(N^T\tilde{E}^*)_{i3}(\tilde{E}^T\underline{C}^T\tilde{U})_{33}(\tilde{U}^\dagger D)_{32,31} \tag{A2}$$



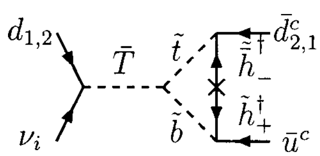
$$\propto \quad (D^T\underline{A}\tilde{U})_{13,23}(\tilde{U}^\dagger D)_{32,31}(U^T\tilde{D}^*)_{13}(\tilde{D}^T\underline{C}N)_{3i} \tag{A3}$$



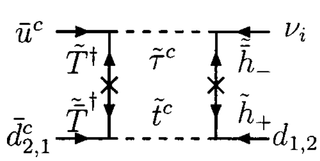
$$\propto \quad (D^T\underline{C}N)_{1i,2i}(U^T\tilde{D}^*)_{13}(\tilde{D}^T\underline{A}\tilde{U})_{33}(\tilde{U}^\dagger D)_{32,31} \tag{A4}$$



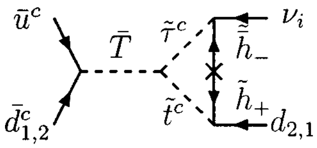
$$\propto \quad (D^T\underline{A}\tilde{U})_{13,23}(\tilde{U}^\dagger Y_D^* D_c^*)_{32,31}(U_c^\dagger Y_U^\dagger\tilde{D}^*)_{13}(\tilde{D}^T\underline{C}N)_{3i} \tag{A5}$$



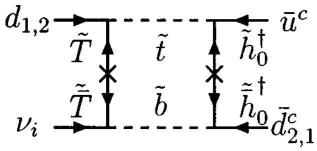
$$\propto \quad (D^T\underline{C}N)_{1i,2i}(U_c^\dagger Y_U^\dagger\tilde{D}^*)_{13}(\tilde{D}^T\underline{A}\tilde{U})_{33}(\tilde{U}^\dagger Y_D^* D_c^*)_{32,31} \tag{A6}$$



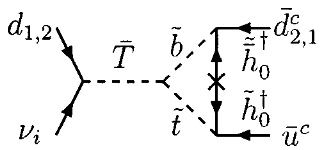
$$\propto \quad (U_c^\dagger\underline{B}^*\tilde{E}_c^*)_{13}(\tilde{E}_c^T Y_E N)_{3i}(D^T Y_U\tilde{U}_c)_{13,23}(\tilde{U}_c^\dagger\underline{D}^* D_c^*)_{32,31} \tag{A7}$$



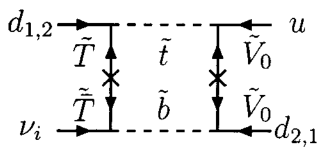
$$\propto \quad (U_c^\dagger \underline{D}^* D_c^*)_{11,12}(D^T Y_U \tilde{U}_c)_{23,13}(\tilde{U}_c^\dagger \underline{B}^* \tilde{E}_c^*)_{33}(\tilde{E}_c^T Y_E N)_{3i} \tag{A8}$$



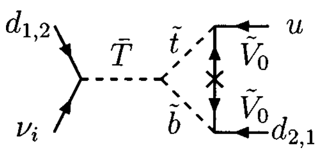
$$\propto \quad (D^T \underline{A}\tilde{U})_{13,23}(\tilde{U}^\dagger Y_U^* U_c^*)_{31}(D_c^\dagger Y_D^\dagger \tilde{D}^*)_{23,13}(\tilde{D}^T \underline{C} N)_{3i} \tag{A9}$$



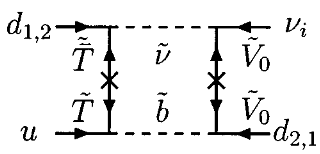
$$\propto \quad (D^T \underline{C} N)_{1i,2i}(U_c^\dagger Y_U^\dagger \tilde{U}^*)_{13}(\tilde{U}^T \underline{A}\tilde{D})_{33}(\tilde{D}^\dagger Y_D^* D_c^*)_{32,31} \tag{A10}$$



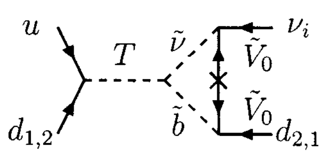
$$\propto \quad (D^T \underline{A}\tilde{U})_{13,23}(\tilde{U}^\dagger U)_{31}(D^T \tilde{D}^*)_{23,13}(\tilde{D}^T \underline{C} N)_{3i} \tag{A11}$$



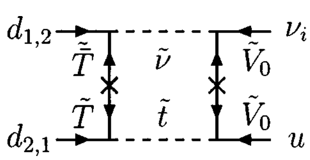
$$\propto \quad (D^T \underline{C} N)_{1i,2i}(D^T \tilde{D}^*)_{23,13}(\tilde{D}^T \underline{A}\tilde{U})_{33}(\tilde{U}^\dagger U)_{31} \tag{A12}$$



$$\propto \quad (D^T \underline{C}\tilde{N})_{13,23}(\tilde{N}^\dagger N)_{3i}(D^T \tilde{D}^*)_{23,13}(\tilde{D}^T \underline{A} U)_{31} \tag{A13}$$

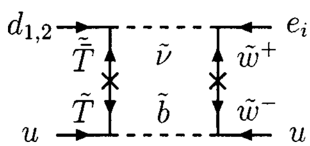


$$\propto \quad (U^T \underline{A} D)_{11,12}(D^T \tilde{D}^*)_{23,13}(\tilde{D}^T \underline{C}\tilde{N})_{33}(\tilde{N}^\dagger N)_{3i} \tag{A14}$$

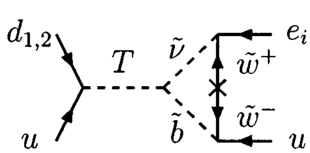


$$\propto \quad (D^T \underline{C}\tilde{N})_{13,23}(\tilde{N}^\dagger N)_{3i}(U^T \tilde{U}^*)_{13}(\tilde{U}^T \underline{A} D)_{32,31} \tag{A15}$$

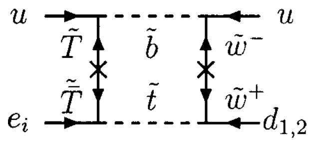
(ii) $\underline{p \to (K^0, \pi^0, \eta, K^{*0}, \rho^0, \omega) e_i^+}$, $\underline{n \to (K^-, \pi^-, K^{*-}, \rho^-) e_i^+}$ $(i=1,2,$ for K^* only $i=1)$



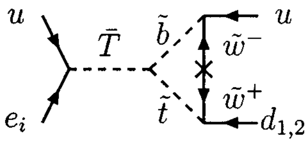
$$\propto \quad (D^T \underline{C}\tilde{N})_{13,23}(\tilde{N}^\dagger E)_{3i}(U^T \tilde{D}^*)_{13}(\tilde{D}^T \underline{A} U)_{31} \tag{A16}$$



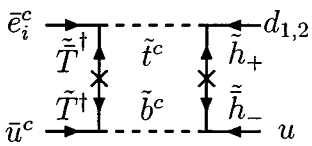
$$\propto \quad (D^T \underline{A} U)_{11,21}(U^T \tilde{D}^*)_{13}(\tilde{D}^T \underline{C}\tilde{N})_{33}(\tilde{N}^\dagger E)_{3i} \tag{A17}$$



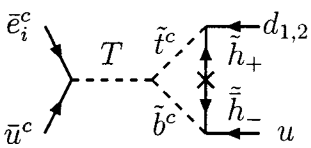
$$\propto \quad (U^T \underline{A} \tilde{D})_{13} (\tilde{D}^\dagger U)_{31} (D^T \tilde{U}^*)_{13,23} (\tilde{U}^T \underline{C} E)_{3i} \tag{A18}$$



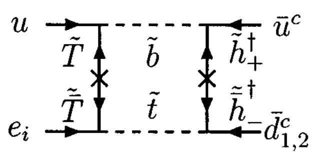
$$\propto \quad (U^T \underline{C} E)_{1i} (D^T \tilde{U}^*)_{13,23} (\tilde{U}^T \underline{A} \tilde{D})_{33} (\tilde{D}^\dagger U)_{31} \tag{A19}$$



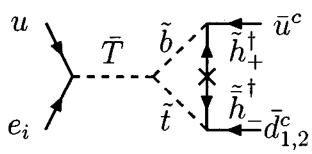
$$\propto \quad (E_c^\dagger \underline{B}^\dagger \tilde{U}_c^*)_{i3} (\tilde{U}_c^T Y_U^T D)_{31,32} (U^T Y_D \tilde{D}_c)_{13} (\tilde{D}_c^\dagger \underline{D}^\dagger U_c^*)_{31} \tag{A20}$$



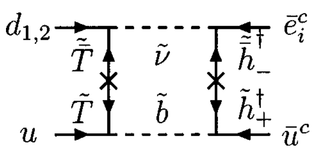
$$\propto \quad (E_c^\dagger \underline{B}^\dagger U_c^*)_{i1} (U^T Y_D \tilde{D}_c)_{13} (\tilde{D}_c^\dagger \underline{D}^\dagger \tilde{U}_c^*)_{33} (\tilde{U}_c^T Y_U^T D)_{31,32} \tag{A21}$$



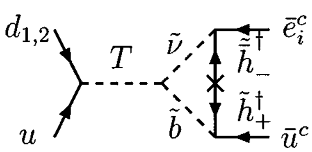
$$\propto \quad (U^T \underline{A} \tilde{D})_{13} (\tilde{D}^\dagger Y_U^* U_c^*)_{31} (D_c^\dagger Y_D^\dagger \tilde{U}^*)_{13,23} (\tilde{U}^T \underline{C} E)_{3i} \tag{A22}$$



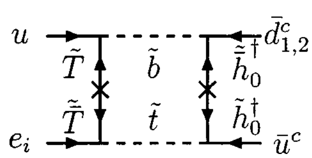
$$\propto \quad (U^T \underline{C} E)_{1i} (D_c^\dagger Y_D^\dagger \tilde{U}^*)_{13,23} (\tilde{U}^T \underline{A} \tilde{D})_{33} (\tilde{D}^\dagger Y_U^* U_c^*)_{31} \tag{A23}$$



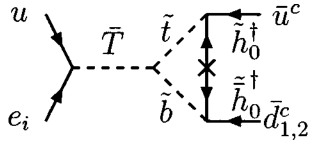
$$\propto \quad (D^T \underline{C} \tilde{N})_{13,23} (\tilde{N}^\dagger Y_E^\dagger E_c^*)_{3i} (U_c^\dagger Y_U^\dagger \tilde{D}^*)_{13} (\tilde{D}^T \underline{A} U)_{31} \tag{A24}$$



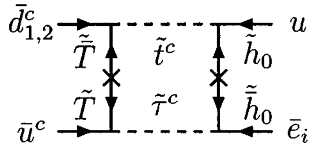
$$\propto \quad (D^T \underline{A} U)_{11,21} (U_c^\dagger Y_U^\dagger \tilde{D}^*)_{13} (\tilde{D}^T \underline{C} \tilde{N})_{33} (\tilde{N}^\dagger Y_E^\dagger E_c^*)_{3i} \tag{A25}$$



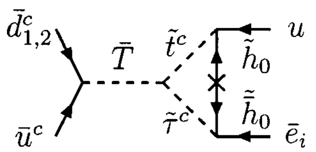
$$\propto \quad (U^T \underline{A} \tilde{D})_{13} (\tilde{D}^\dagger Y_D^* D_c^*)_{31,32} (U_c^\dagger Y_U^\dagger \tilde{U}^*)_{13} (\tilde{U}^T \underline{C} E)_{3i} \tag{A26}$$



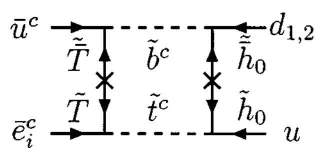
$$\propto \quad (U^T \underline{C} E)_{1i}(D_c^\dagger Y_D^\dagger \tilde{D}^*)_{13,23}(\tilde{D}^T \underline{A}\tilde{U})_{33}(\tilde{U}^\dagger Y_U^* U_c^*)_{31} \tag{A27}$$



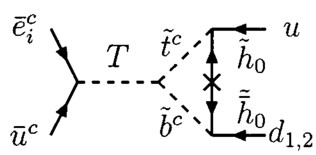
$$\propto \quad (D_c^\dagger \underline{D}^\dagger \tilde{U}_c^*)_{13,23}(\tilde{U}_c^T Y_U^T U)_{31}(E^T Y_E^T \tilde{E}_c)_{i3}(\tilde{E}_c^\dagger \underline{B}^\dagger U_c^*)_{31} \tag{A28}$$



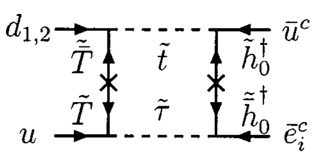
$$\propto \quad (D_c^\dagger \underline{D}^\dagger U_c^*)_{11,21}(E^T Y_E^T \tilde{E}_c)_{i3}(\tilde{E}_c^\dagger \underline{B}^\dagger \tilde{U}_c^*)_{33}(\tilde{U}_c^T Y_U^T U)_{31} \tag{A29}$$



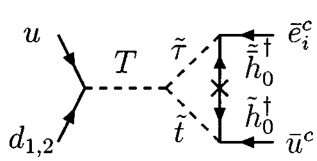
$$\propto \quad (U_c^\dagger \underline{D}^* \tilde{D}_c^*)_{13}(\tilde{D}_c^T Y_D^T D)_{31,32}(U^T Y_U \tilde{U}_c)_{13}(\tilde{U}_c^\dagger \underline{B}^* E_c^*)_{3i} \tag{A30}$$



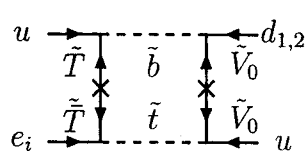
$$\propto \quad (E_c^\dagger \underline{B}^\dagger U_c^*)_{i1}(D^T Y_D \tilde{D}_c)_{13,23}(\tilde{D}_c^\dagger \underline{D}^\dagger \tilde{U}_c^*)_{33}(\tilde{U}_c^T Y_U^T U)_{31} \tag{A31}$$



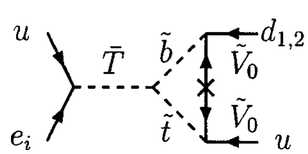
$$\propto \quad (D^T \underline{A}\tilde{U})_{13,23}(\tilde{U}^\dagger Y_U^* U_c^*)_{31}(E_c^\dagger Y_E^* \tilde{E}^*)_{i3}(\tilde{E}^T \underline{C}^T U)_{31} \tag{A32}$$



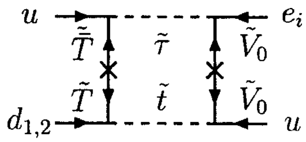
$$\propto \quad (U^T \underline{A} D)_{11,12}(U_c^\dagger Y_U^\dagger \tilde{U}^*)_{13}(\tilde{U}^T \underline{C}\tilde{E})_{33}(\tilde{E}^\dagger Y_E^\dagger E_c^*)_{3i} \tag{A33}$$



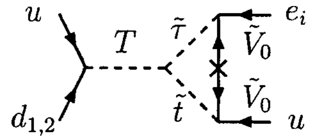
$$\propto \quad (U^T \underline{A}\tilde{D})_{13}(\tilde{D}^\dagger D)_{31,32}(U^T \tilde{U}^*)_{13}(\tilde{U}^T \underline{C} E)_{3i} \tag{A34}$$



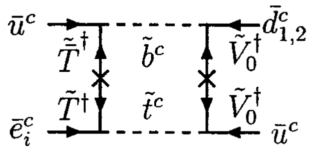
$$\propto \quad (U^T \underline{C} E)_{1i}(U^T \tilde{U}^*)_{13}(\tilde{U}^T \underline{A}\tilde{D})_{33}(\tilde{D}^\dagger D)_{31,32} \tag{A35}$$



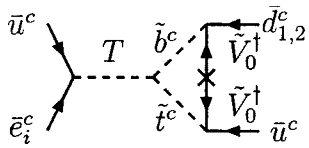
$$\propto \quad (U^T \underline{C}\tilde{E})_{13}(\tilde{E}^\dagger E)_{3i}(U^T\tilde{U}^*)_{13}(\tilde{U}^T \underline{A} D)_{31,32} \tag{A36}$$



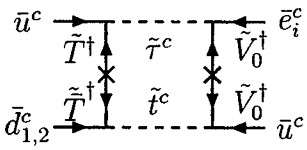
$$\propto \quad (U^T\underline{A}D)_{11,12}(U^T\tilde{U}^*)_{13}(\tilde{U}^T\underline{C}\tilde{E})_{33}(\tilde{E}^\dagger E)_{3i} \tag{A37}$$



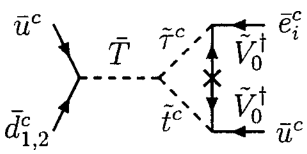
$$\propto \quad (U_c^\dagger \underline{D}^*\tilde{D}_c^*)_{13}(\tilde{D}_c^T D_c^*)_{31,32}(U_c^\dagger\tilde{U}_c)_{13}(\tilde{U}_c^\dagger\underline{B}^*E_c^*)_{3i} \tag{A38}$$



$$\propto \quad (U_c^\dagger\underline{B}^*E_c^*)_{1i}(U_c^\dagger\tilde{U}_c)_{13}(\tilde{U}_c^\dagger\underline{D}^*\tilde{D}_c^*)_{33}(\tilde{D}_c^T D_c^*)_{31,32} \tag{A39}$$



$$\propto \quad (U_c^\dagger\underline{B}^*\tilde{E}_c^*)_{13}(\tilde{E}_c^T E_c^*)_{3i}(U_c^\dagger\tilde{U}_c)_{13}(\tilde{U}_c^\dagger\underline{D}^*D_c^*)_{31,32} \tag{A40}$$



$$\propto \quad (U_c^\dagger\underline{D}^*\tilde{D}_c^*)_{11,12}(U_c^\dagger\tilde{U}_c)_{13}(\tilde{U}_c^\dagger\underline{B}^*\tilde{E}_c^*)_{33}(\tilde{E}_c^T E_c^*)_{3i} \tag{A41}$$

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