Neutrino cross sections in few hundred MeV energy region

Jan T. Sobczyk

Institute of Theoretical Physics, University of Wrocław

(in collaboration with A. Ankowski and J. Nowak)

Plan of the talk:

3. **Introduction**

5. **Quasi-elastic scattering off free target (form-factors, axial mass).**

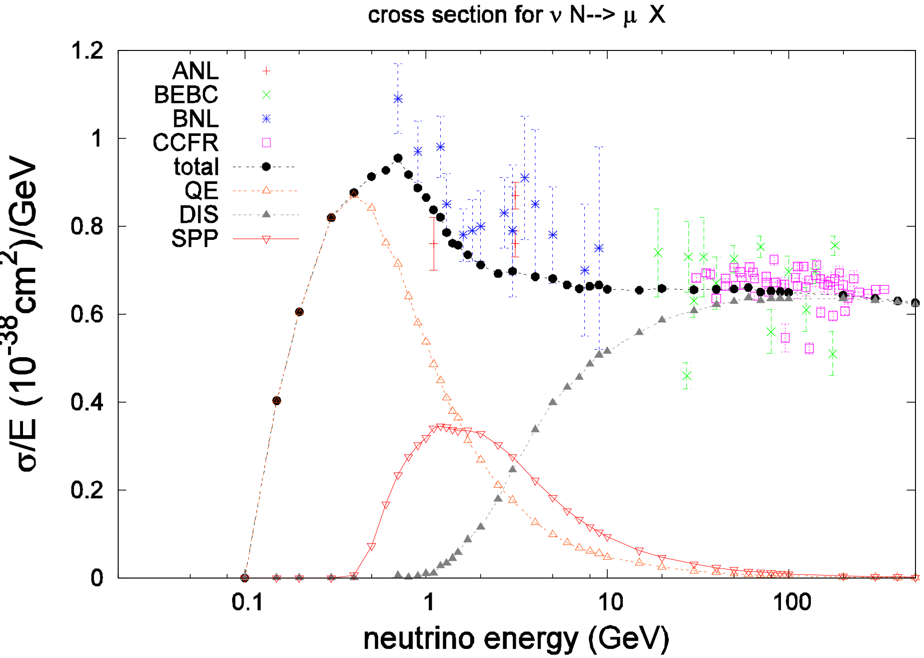
3. **Significance of single pion production.**

9. **Nuclear effects – general remarks.**

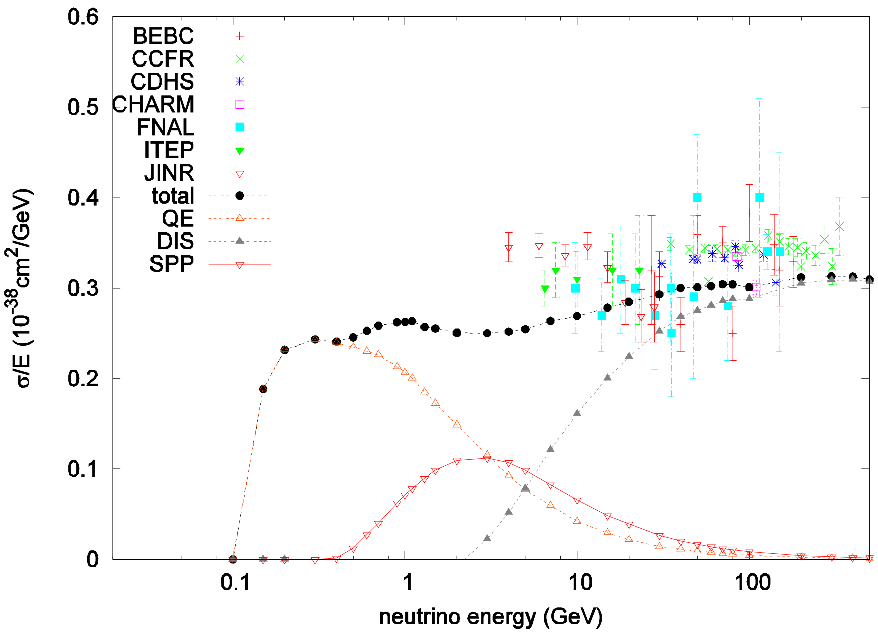
11. **Nuclear effects – numerical results (Fermi gas, spectral function, momentum dependent effective potential).**

5. **Conclusions.**

Total neutrino – nucleon cross sections



neutrino



anti-neutrino

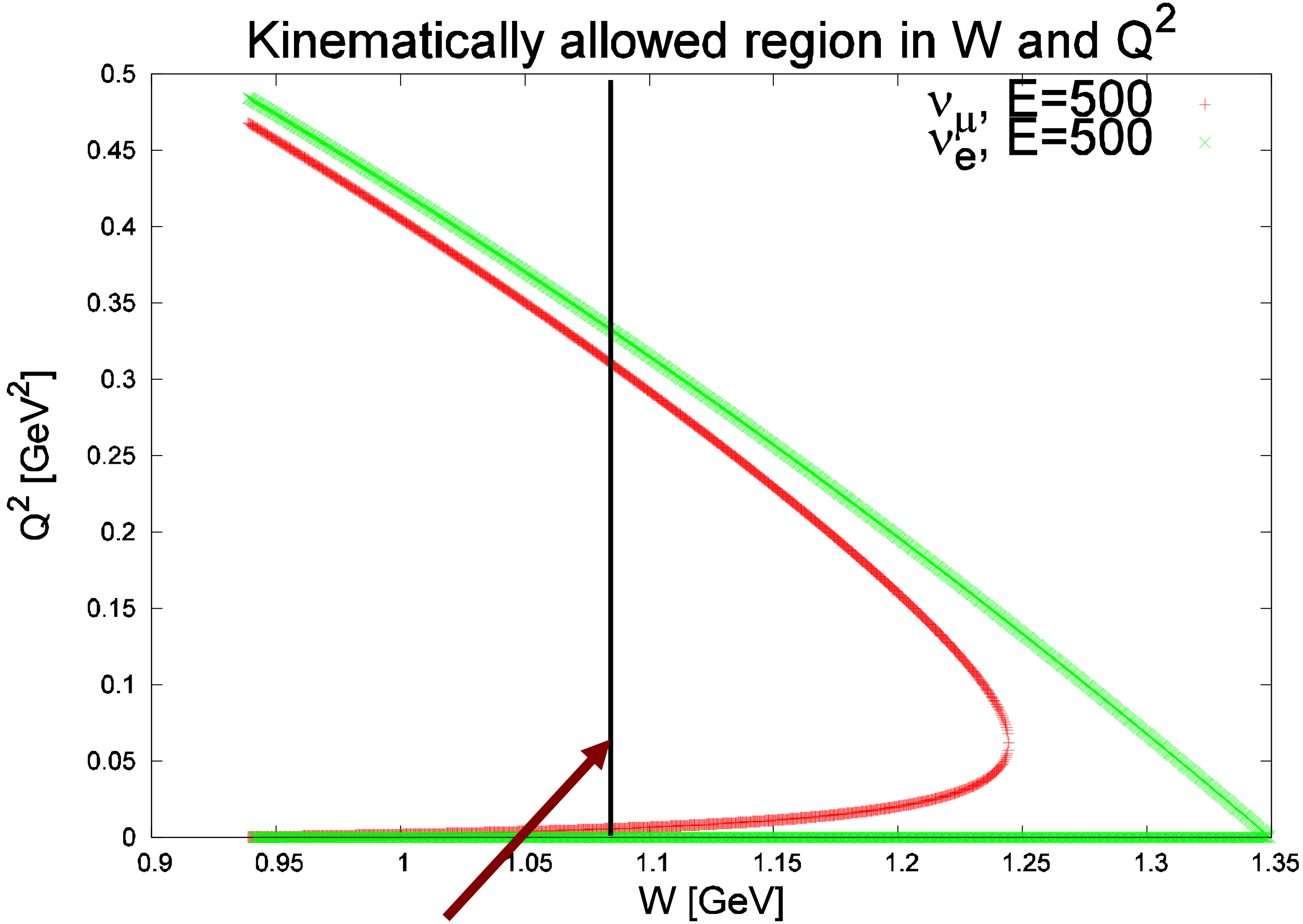
We distinguish:

- quasi-elastic
- single pion production („RES region", e.g. W<=2 GeV)
- more inelastic („DIS region")

Focus on few hundred MeV neutrino energies: quasi-elastic region.

Plots from Wrocław MC generator

Kinematics



Threshold for the pion production

Quasi-elastic reaction - theory

$$\nu + n \rightarrow l^- + p$$

$$\bar{\nu} + p \rightarrow l^+ + n$$

$$\Gamma_\mu = \gamma_\mu F_1(Q^2) + i\sigma_{\mu\nu} q^\nu \frac{F_2(Q^2)}{2M} + \gamma_\mu \gamma_5 F_A(Q^2) + \gamma_5 q_\mu \frac{F_P(Q^2)}{M}$$

CVC – use electromagnetic data

PCAC

$$F_P(Q^2) = \frac{2M^2 F_A(Q^2)}{m_\pi^2 + Q^2}$$

We need the axial form-factor; the standard dipole form

$$F_A(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}$$

g_A =1.26 from neutron decay;
M_A a free parameter (the only one)

The value of axial mass is obtained from experimental data.

Quasi-elastic reaction - theory

$Q^2 << (M_W)^2$

$$\sigma = \frac{M^2 G_F^2 \cos^2\theta_C}{8\pi {E_\nu}^2} \int dq^2 \left[A(q^2) - B(q^2)\frac{(s-u)}{M^2} + C(q^2)\frac{(s-u)^2}{M^4} \right],$$

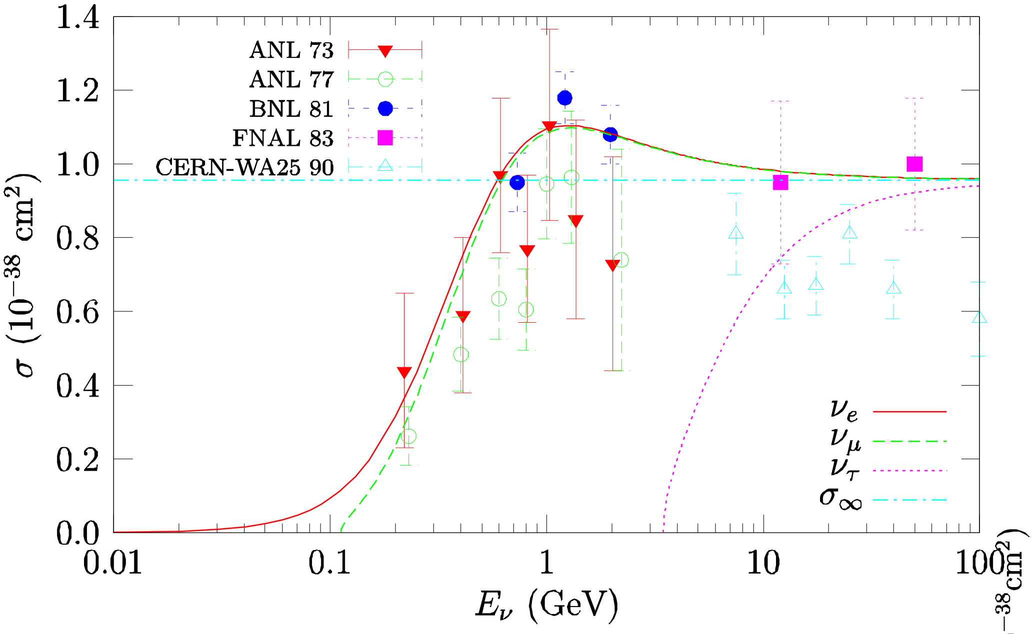
where: $s = (k+p)^2, \qquad u = (k' - p)^2$

$$\begin{aligned} A(q^2) = \frac{m_l^2 - q^2}{4M^2} \Big[& |F_A|^2 \left(4 - \frac{q^2}{M^2}\right) - |F_V^1|^2 \left(4 + \frac{q^2}{M^2}\right) \\ & - \frac{q^2}{M^2} |\xi F_V^2|^2 \left(1 + \frac{q^2}{4M^2}\right) - \frac{4q^2}{M^2} \Re\left(F_V^1 (\xi F_V^2)^*\right) \\ & - \frac{m_l^2}{M^2} \left(|F_V^1 + \xi F_V^2|^2 + |F_A|^2 + 4\Re(F_A F_P^*) + \frac{q^2}{M^2} |F_P|^2 \right) \Big], \end{aligned}$$

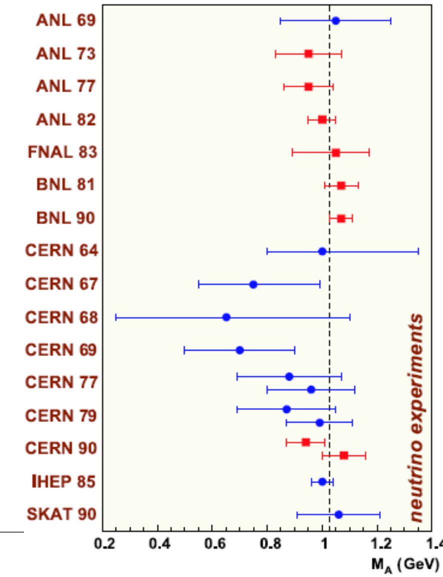
$$B(q^2) = -\frac{q^2}{M^2} \Re\left((F_V^1 + \xi F_V^2) F_A^* \right),$$

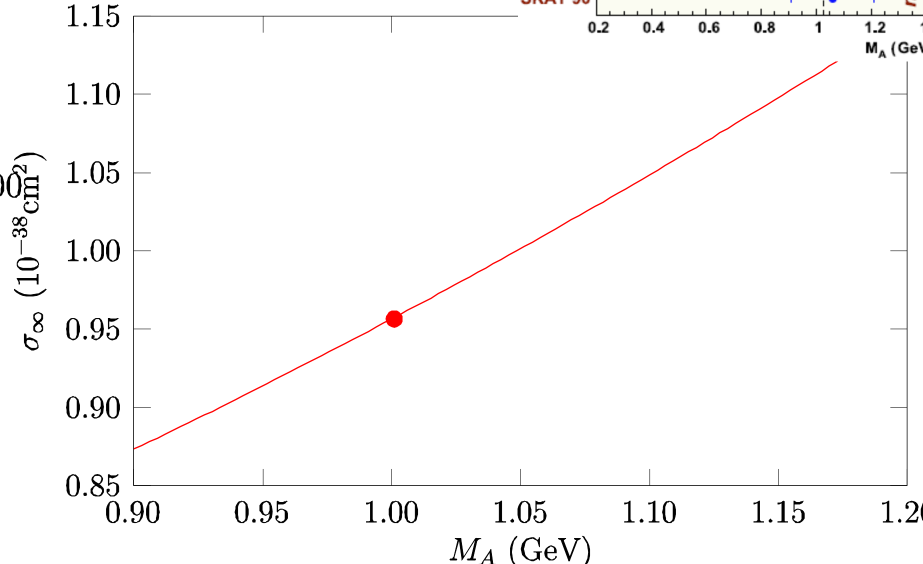
$$C(q^2) = \frac{1}{4} \left(|F_V^1|^2 - \frac{q^2}{4M^2} |\xi F_V^2|^2 + |F_A|^2 \right).$$

Quasi-elastic reaction



(from Naumov)





Big experimental uncertainty

The limiting value depends on the axial mass

Under assumption of dipole vector form-factors:

$$\sigma_\infty = \frac{G_F^2 \cos^2\theta_C}{6\pi}\Big[M_V^2 + g_A^2 M_A^2 + \frac{2\xi(\xi+2)M_V^4}{(4M^2-M_V^2)^2}(M^2-M_V^2) + \frac{3\xi(\xi+2)M_V^8}{(4M^2-M_V^2)^3}\Big(\frac{4M^2}{4M^2-M_V^2}\ln\frac{4M^2}{M_V^2}-1\Big)\Big].$$

(A. Ankowski)

Quasi-elastic reaction

Dipole electromagnetic form-factors:

$$G_E^V(q^2) = \frac{1}{(1 - q^2/M_V^2)^2}, \qquad G_M^V(q^2) = \frac{1+\xi}{(1 - q^2/M_V^2)^2},$$

$$F_V^1(q^2) = \left(1 - \frac{q^2}{4M^2}\right)^{-1}\left[G_E^V(q^2) - \frac{q^2}{4M^2}G_M^V(q^2)\right],$$

$$\xi F_V^2(q^2) = \left(1 - \frac{q^2}{4M^2}\right)^{-1}\left[-G_E^V(q^2) + G_M^V(q^2)\right],$$

$$1 + \xi = \mu_{proton} - \mu_{neutron} = 2.79 - (-1.91) = 4.7$$

One can find better fits to the existing data, BBBA2005

$$G_E^V(Q^2) = G_{ep}(Q^2) - G_{en}(Q^2),$$
$$G_M^V(Q^2) = G_{mp}(Q^2) - G_{mn}(Q^2)$$

$$G(q^2) = \frac{\sum_{k=0}^{n} a_k \tau^k}{1 + \sum_{k=1}^{n+2} b_k \tau^k}$$

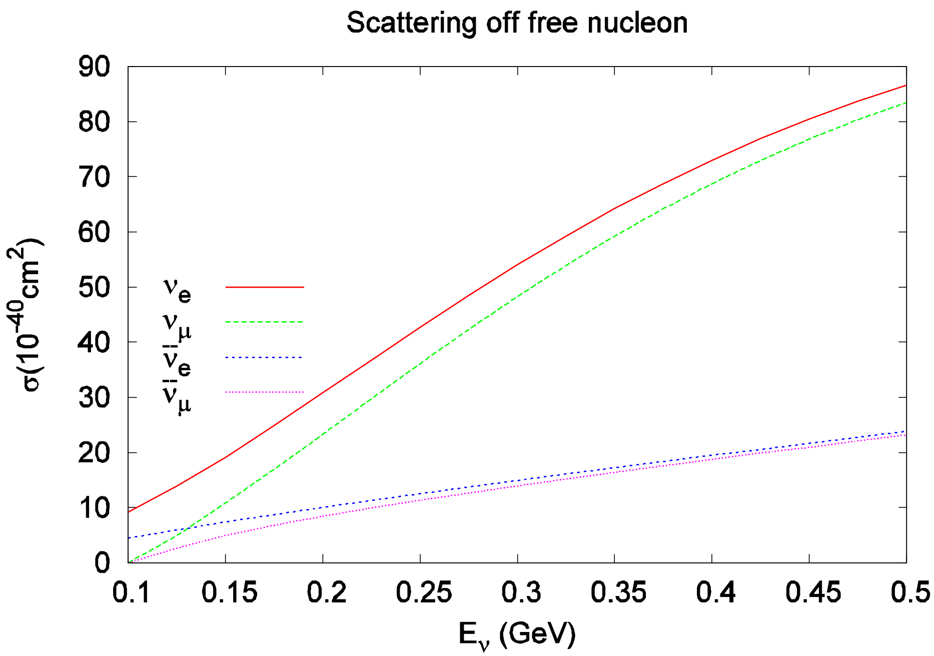
use $a_0=1$ for G_{ep}, G_{mp}, G_{mn}, and $a_0=0$ for G_{en}.

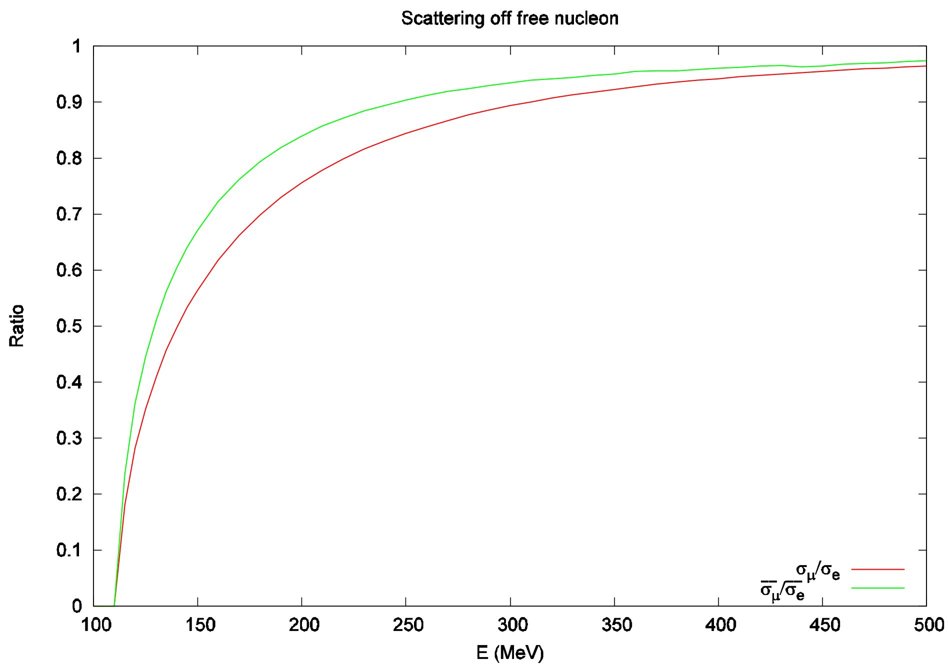
$$\tau = \frac{Q^2}{4M^2}$$

Observable	a_1	a_2	b_1	b_2	b_3	b_4
G_{ep}	-.577E-01 ± 0.165		11.2 ± 0.217	13.6 ± 1.39	33.0 ± 8.95	
G_{mp}	0.150 ± 0.312E-		11.1 ± 0.103	19.6 ± 0.282	7.54 ± 0.967	
G_{en}	1.38 ± 0.313	-0.214 ± 0.506E-	8.51 ± 3.59	59.9 ± 15.3	13.6 ± 3.49	2.57 ± 0.592
G_{mp}	1.82 ± 0.402		14.1 ± 0.597	20.7 ± 2.54	69.7 ± 14.1	

(from R. Bradford talk at NuInt05)

Quasi-elastic reaction





The central objects of the analysis: cross section ratios:

$$\frac{\sigma\ (\nu_\mu)}{\sigma\ (\nu_e)},\quad \frac{\sigma\ (\bar{\nu}_\mu)}{\sigma\ (\bar{\nu}_e)}$$

Quasi-elastic reaction

Do theoretical uncertainties:

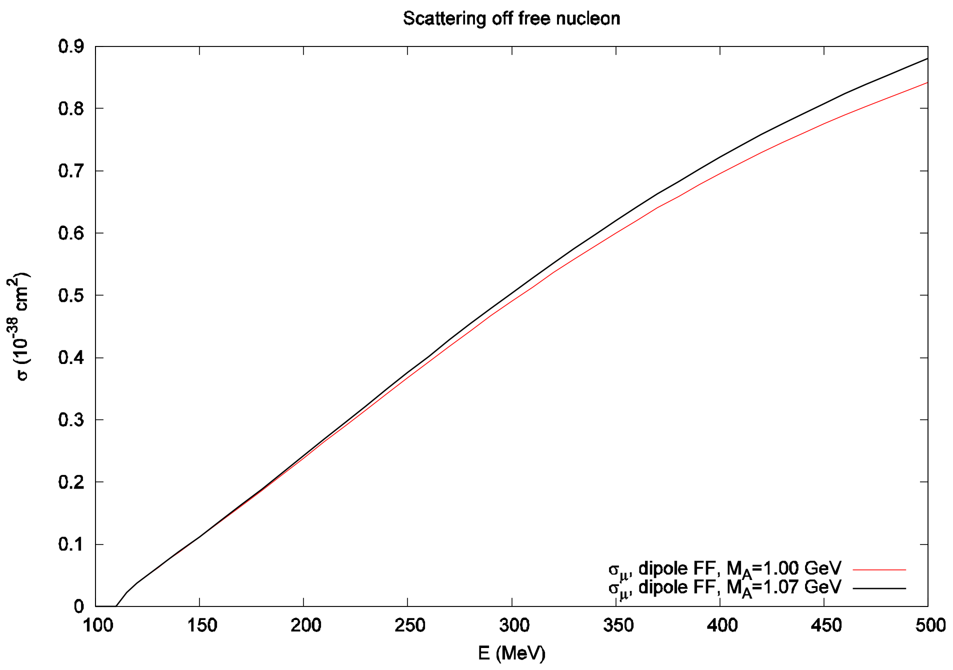
• axial mass

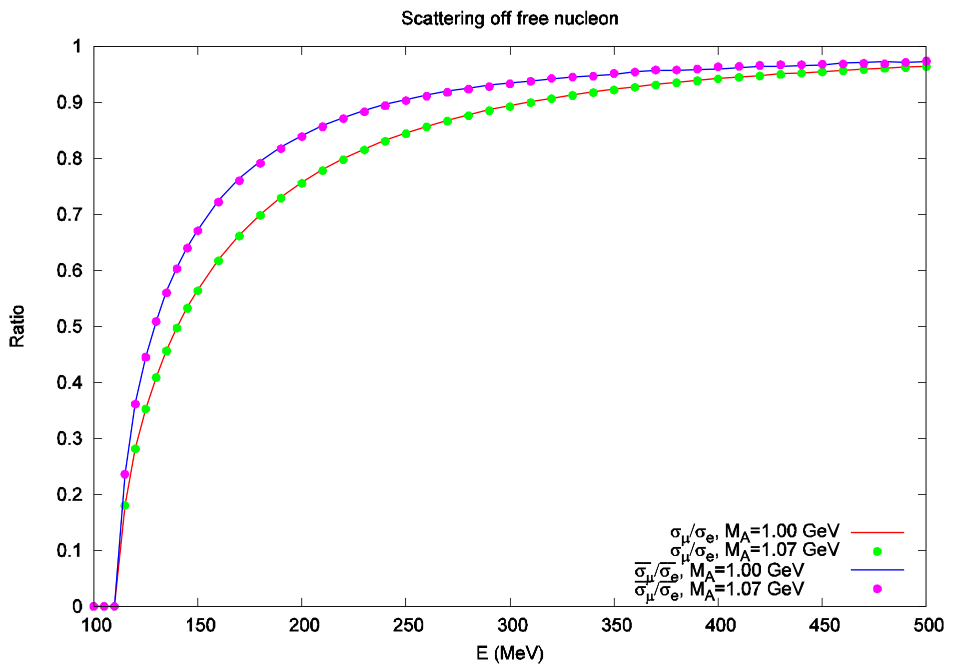
•electromagnetic form-factors

have an impact on cross sections ratios?

Quasi-elastic reaction

Axial mass …

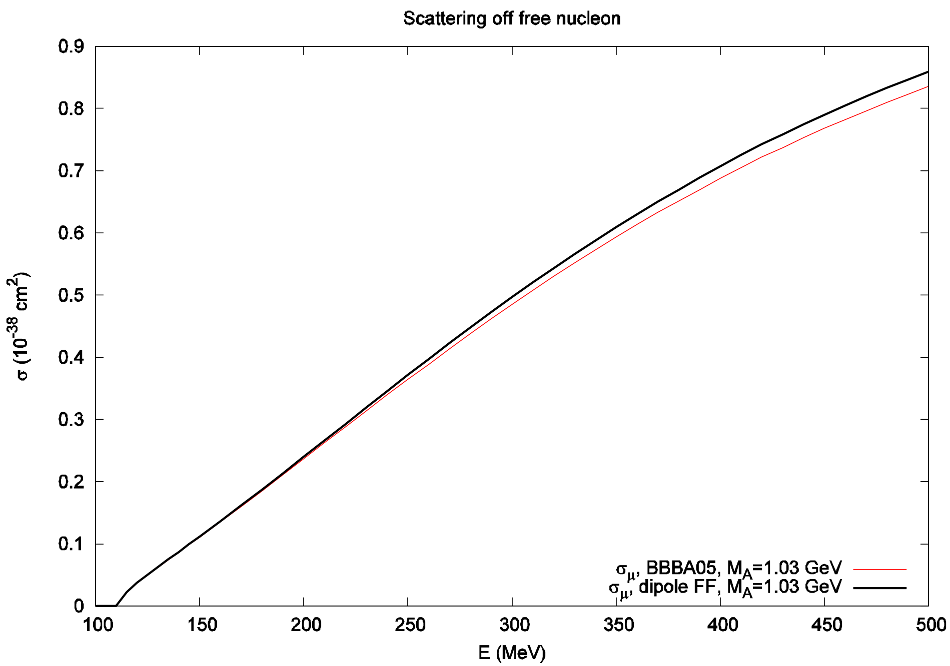


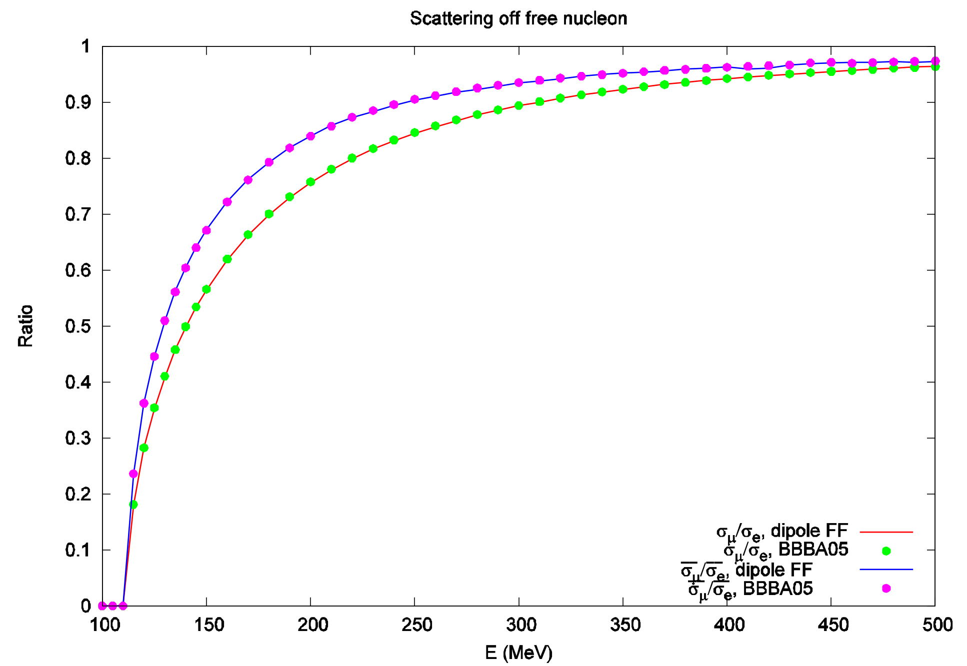


… no change!

Quasi-elastic reaction

Electromagnetic form-factors …





… no change!

Single pion production

3 CC channels for neutrino and 3 CC channels for anti-neutrino reactions:

Characteristic feature is that the dominant contribution comes from resonance excitation (mainly Δ):

$$\nu + p \rightarrow l^- + \Delta^{++} \rightarrow l^- + p + \pi^+$$

$$\nu + n \rightarrow l^- + \Delta^{+} \rightarrow l^- + p + \pi^0$$

$$\nu + n \rightarrow l^- + \Delta^{+} \rightarrow l^- + n + \pi^+$$

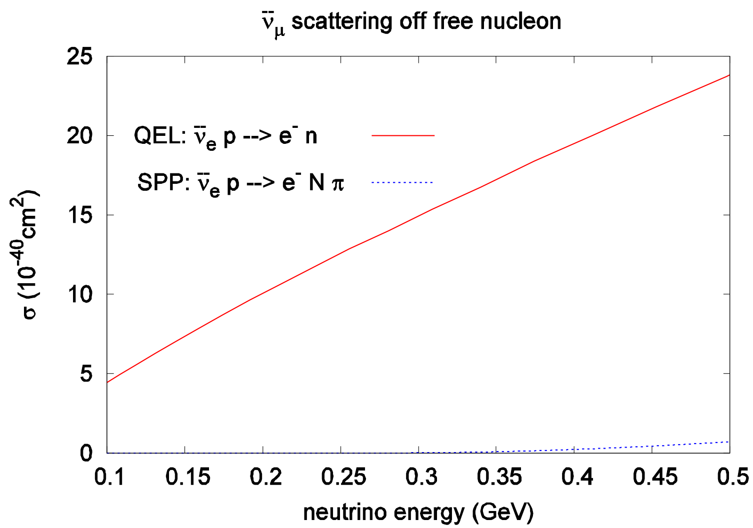
$$\bar{\nu} + p \rightarrow l^+ + \Delta^{0} \rightarrow l^+ + p + \pi^-$$

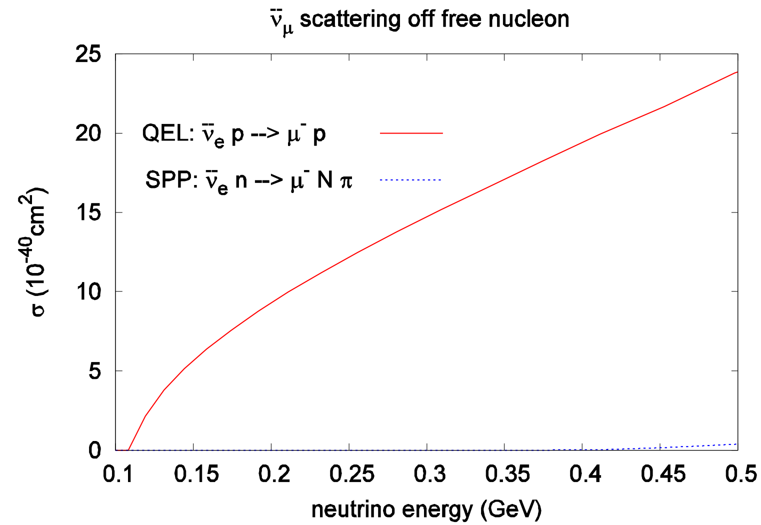
$$\bar{\nu} + p \rightarrow l^+ + \Delta^{0} \rightarrow l^+ + n + \pi^0$$

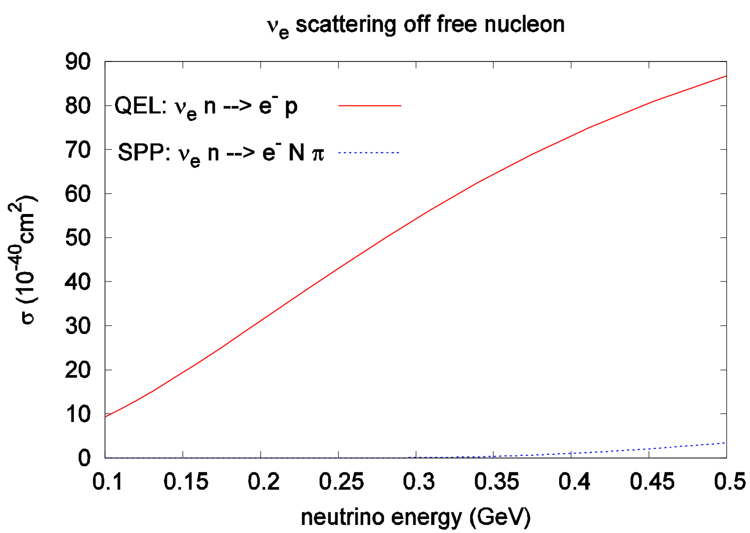
$$\bar{\nu} + n \rightarrow l^+ + \Delta^{-} \rightarrow l^+ + n + \pi^-$$

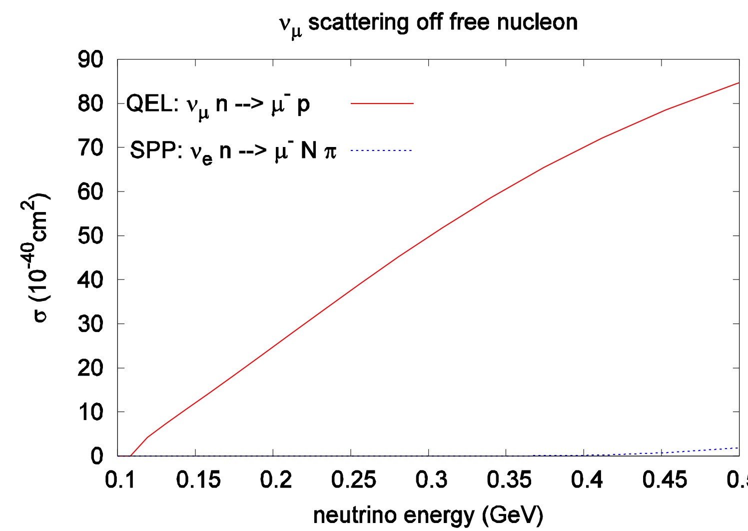
Single pion production

What is a significance of spp channels in few hundred MeV energy region?
What is their impact on total cross sections ratios?





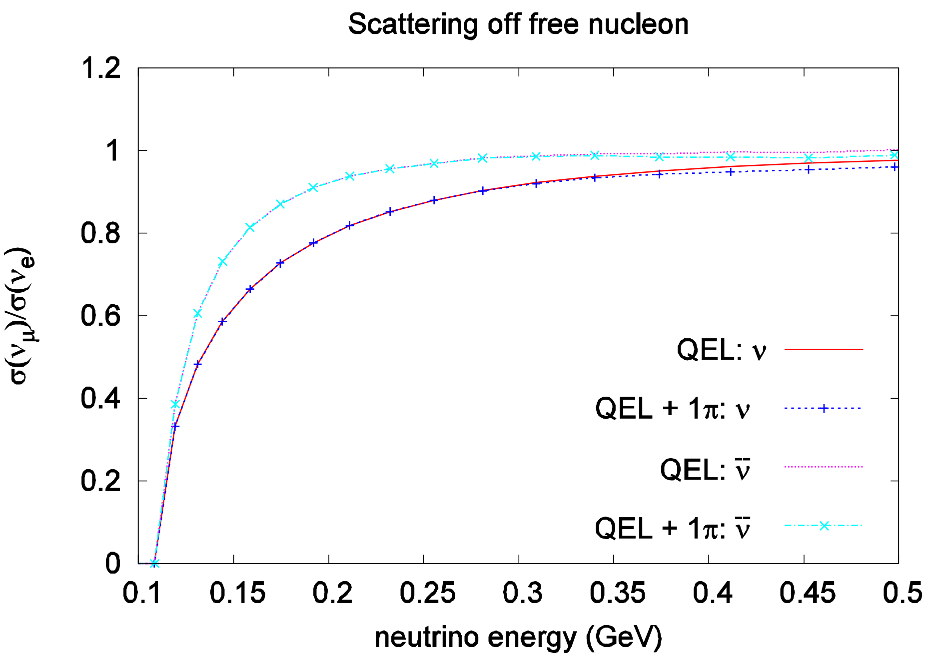


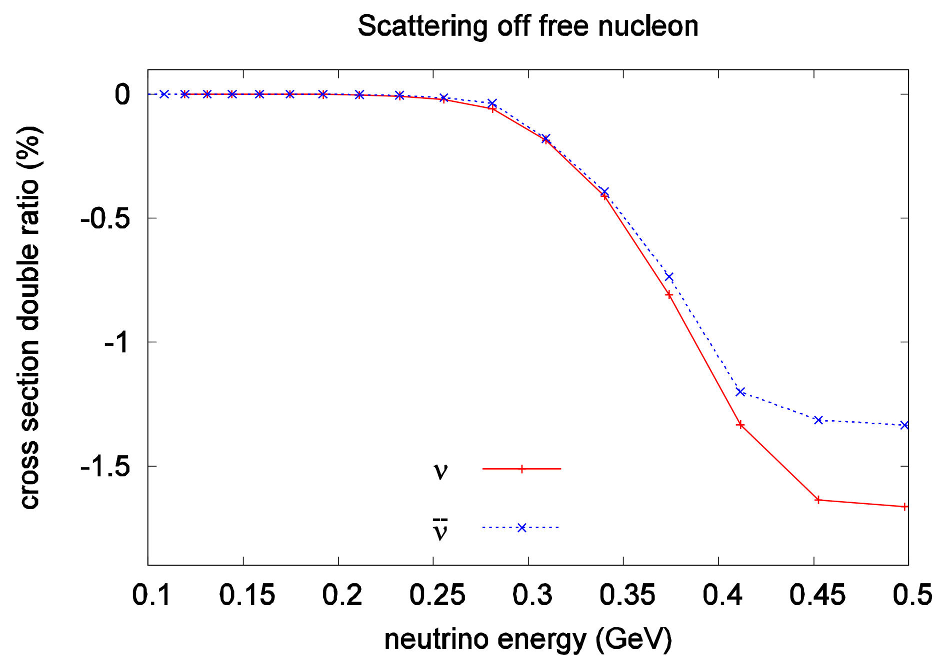


Single pion production

The measure of the impact of spp on cross section ratios

$$\frac{\dfrac{\sigma^{qel+1\pi}(\nu_\mu)}{\sigma^{qel+1\pi}(\nu_e)} - \dfrac{\sigma^{qel}(\nu_\mu)}{\sigma^{qel}(\nu_e)}}{\dfrac{1}{2}\left(\dfrac{\sigma^{qel+1\pi}(\nu_\mu)}{\sigma^{qel+1\pi}(\nu_e)} + \dfrac{\sigma^{qel}(\nu_\mu)}{\sigma^{qel}(\nu_e)}\right)}$$





Relevant at 1% level only for E>350MeV

Jan T. Sobczyk, ISS meeting, RAL, April 2006

Nuclear effects – general remarks

The treatment is energy-dependent:

- low energies: shell model
- intermediate energies: CRPA
- higher energies: impulse approximation (Fermi gas, spectral function)

What does it mean: „low", „intermediate", „higher"?!

Peter Vogel (nucl-th/9901027):

For neutrino energies starting from ≈ 200 MeV
CRPA and FG give rise to very similar total and differential cross-sections.

Giampaolo Co':

Impulse approximation methods make sense for momentum transfer > 400 MeV.

Nuclear effects – general remarks

The methods well justified in GeV region will be used

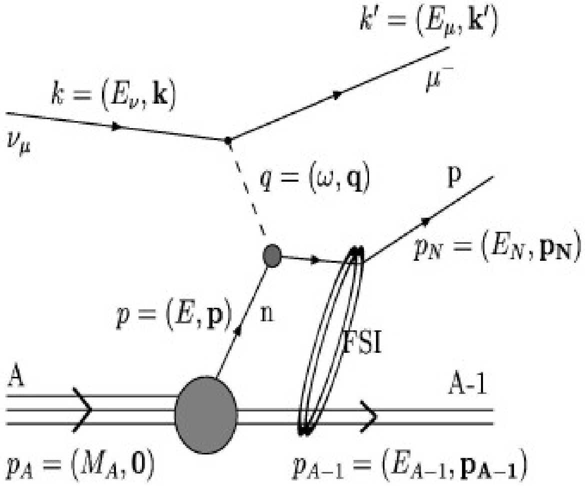
and…

the results which follow should be treated with caution.

Impulse approximation based computations will be presented.

The hope is that ratios are not very much sensitive to weakness of the models.

Impulse approximation



(from Ch. Maieron, XX Max Born Symposium)

- neutrino interacts with an individual (bound) nucleons
- „final state interactions" (FSI) follows (does not change inclusive cross-section)

The simplest realization:
Fermi gas model.

Fermi gas model

Fermi momentum

$$\frac{d\sigma_{\text{S-M}}}{dE_\mu} = \frac{G_F^2 \cos^2\theta_C}{4\pi E_\nu^2}\frac{3N}{4\pi p_F^3}\int d|\mathbf{q}|\, d^3p\, \theta(p_F - |\mathbf{p}|) \times \delta(\omega + E_{\mathbf{p}} - \bar{\epsilon}_B - E_{\mathbf{p}'})\, \theta(|\mathbf{p}+\mathbf{q}| - p_F) \times \frac{|\mathbf{q}|}{E_{\mathbf{p}}E_{\mathbf{p}'}} L_{\mu\nu}\widetilde{H}^{\mu\nu}_{\text{S-M}}. \quad (30)$$

average binding energy

Pauli blocking

off shell matrix element

Fermi gas model is
defined by 2 parameters.
Simple generalization is to take into
account nucleus density profile (LDA).

Spectral function

Realistic distribution of momenta

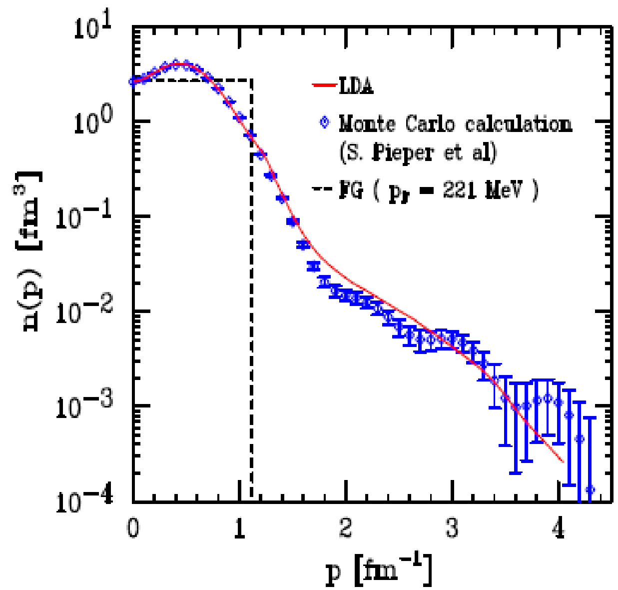
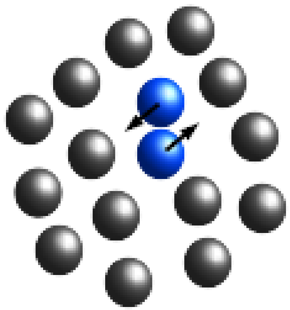


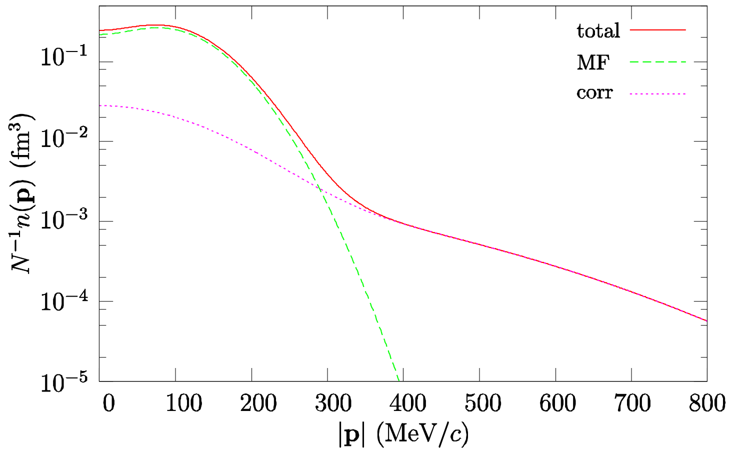
FIG. 3: (Color online) Momentum distribution of nucleons in the oxygen ground state. Solid line: LDA approximation. Dashed line: FG model with Fermi momentum $p_F = 221$ MeV. Diamonds: Monte Carlo calculation carried out by S.C. Pieper [40] using the wave function of Ref. [41].

(from O. Benhar et al. hep-ph/0516116)

Short range correlations (SRC): correlated pairs of nucleons

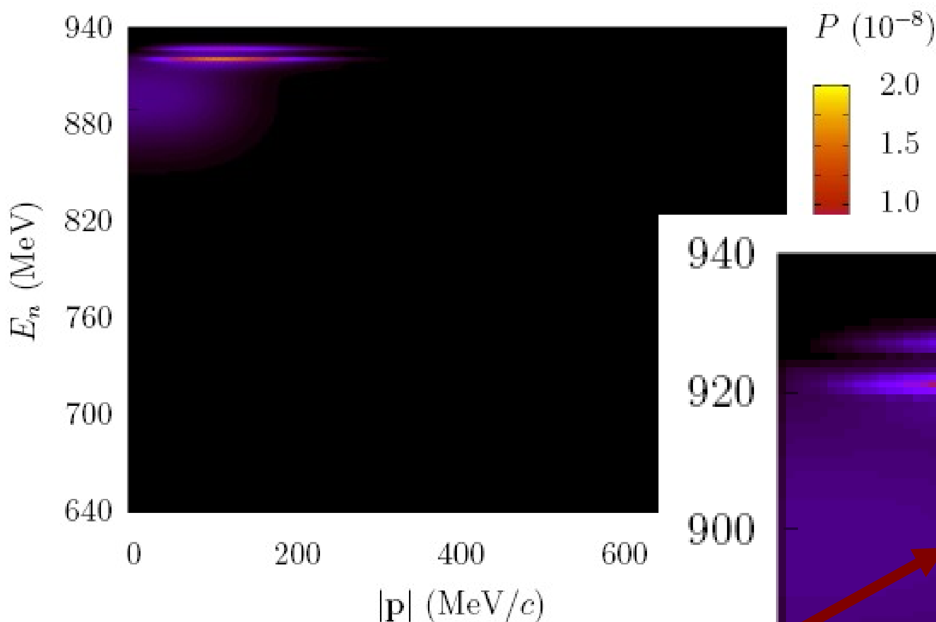


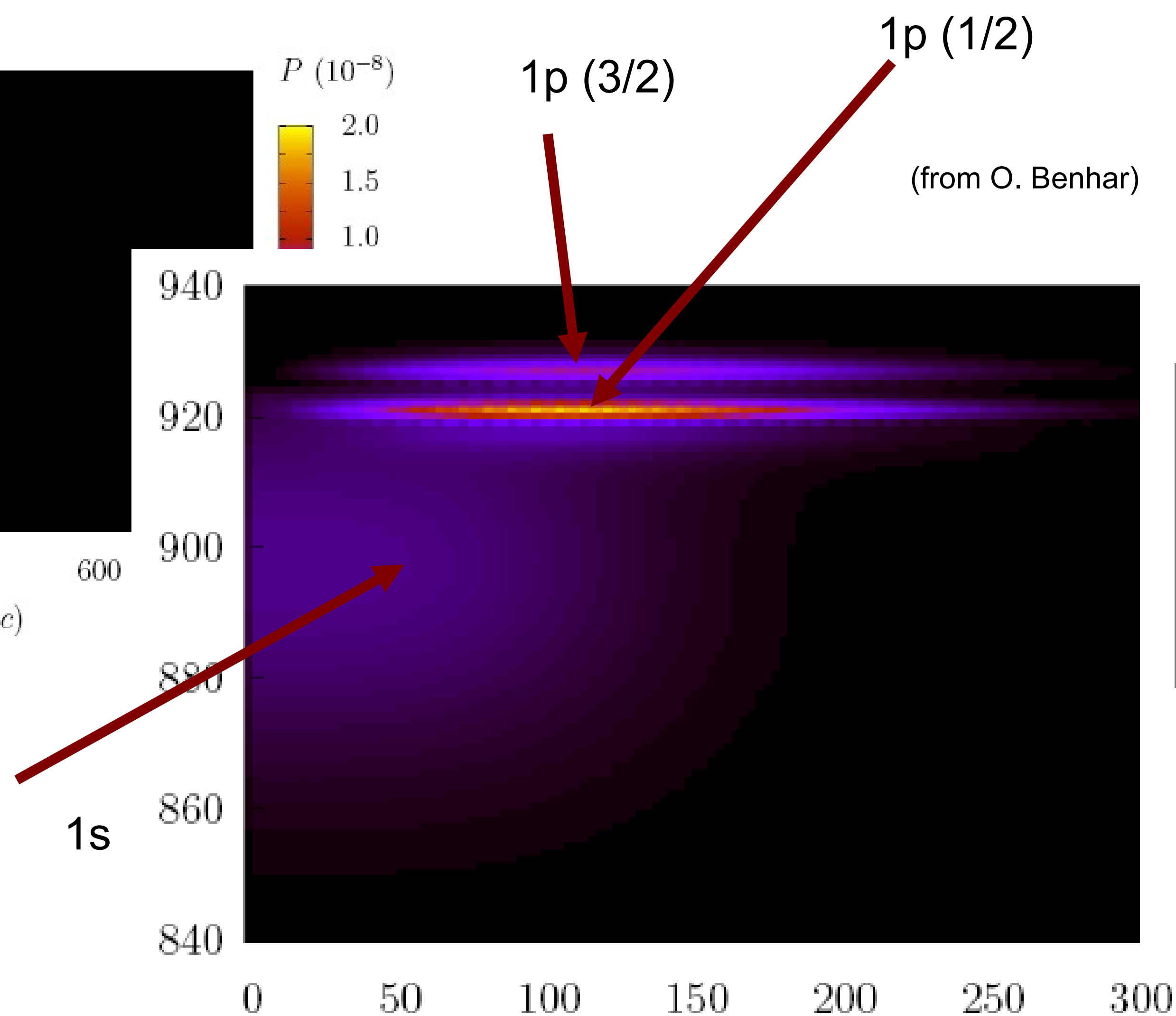
(from A. Ankowski)



Spectral function

Spectral function for oxygen





Spectral function approach is parameter free.
But FSI/Pauli blocking effects must be added.

Effective (momentum dependent) potential

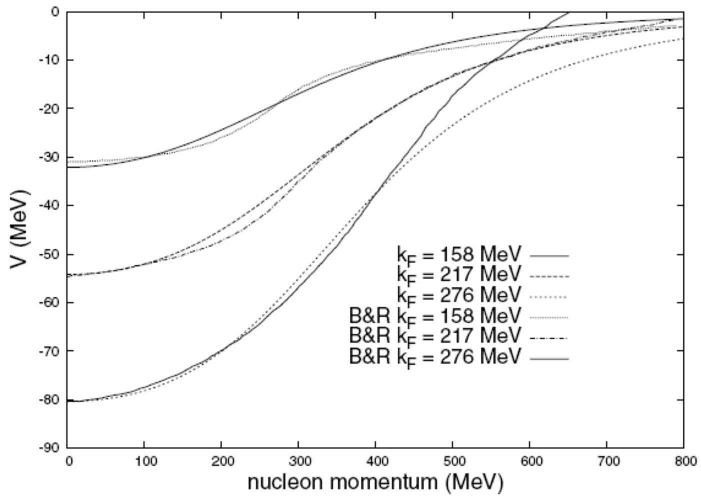
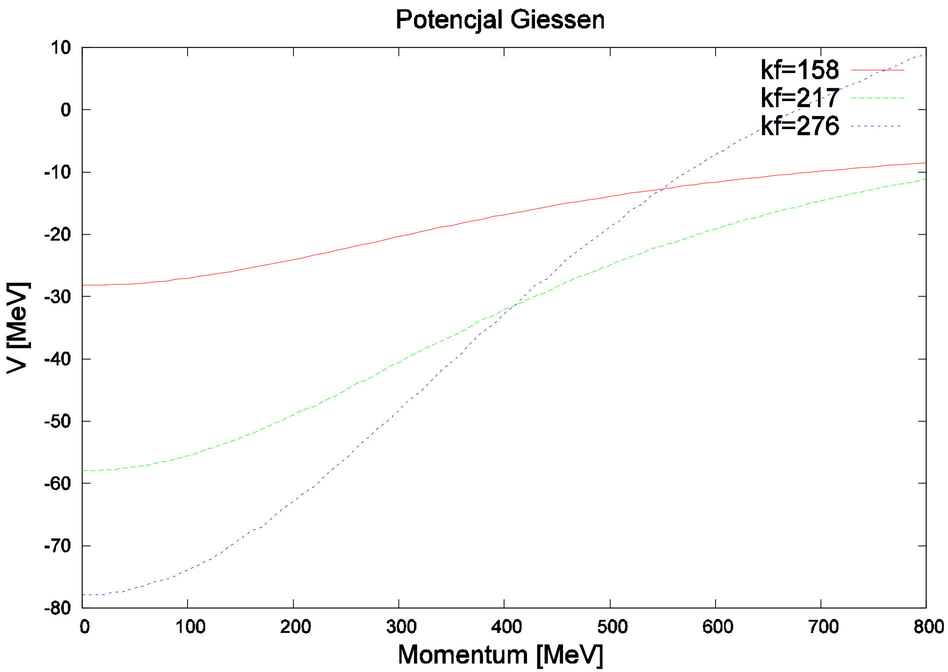


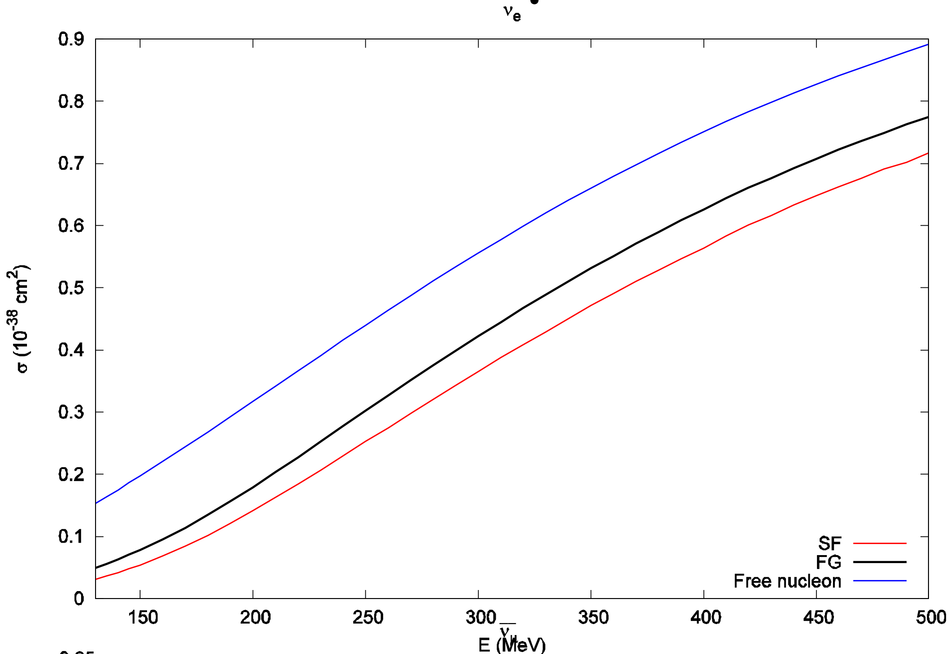
Fig. 2. The Momentum dependent potential $V(k_F, p)$ for 3 values of Fermi momentum (see (8)) compared with original plots, labeled B&R taken from [11]

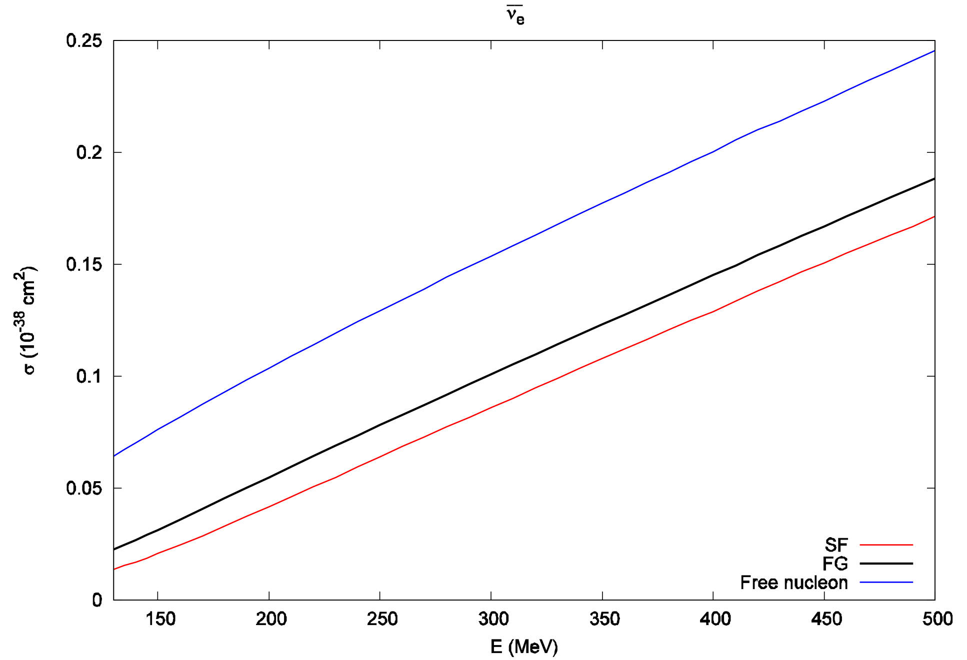
(from Juszczak, Nowak, Sobczyk, Eur. Phys. J. C39 (2005) 195)

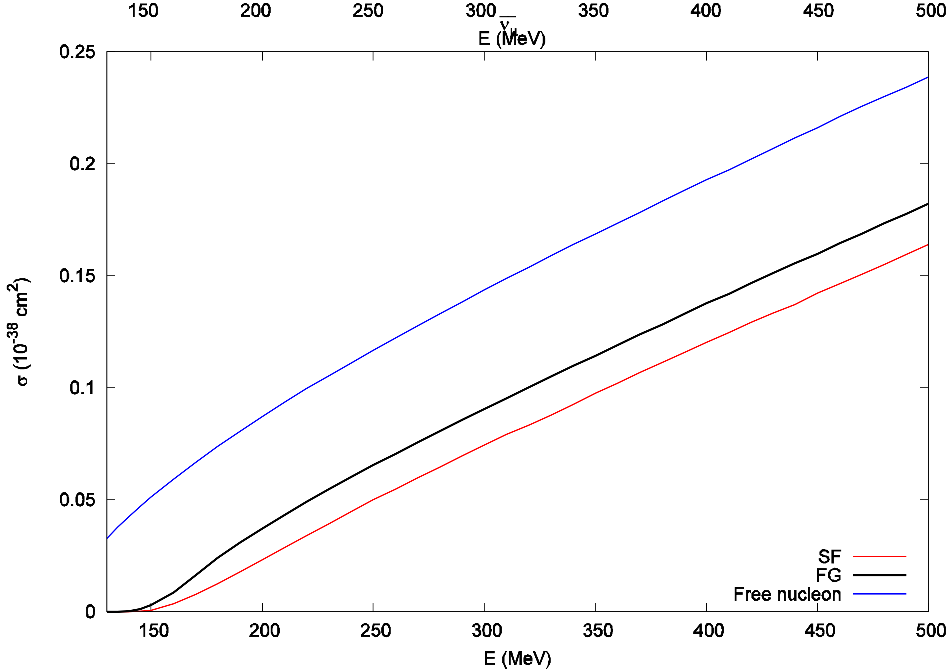


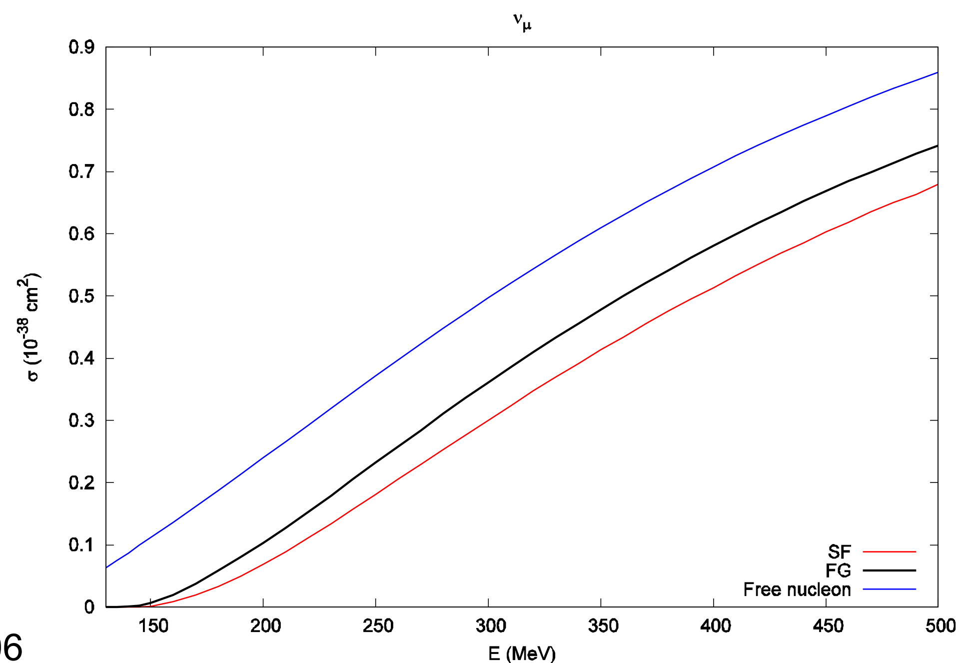
(from Leitner, Alvarez-Ruso, Mosel, nucl-th/0601103)

Spectral function - results

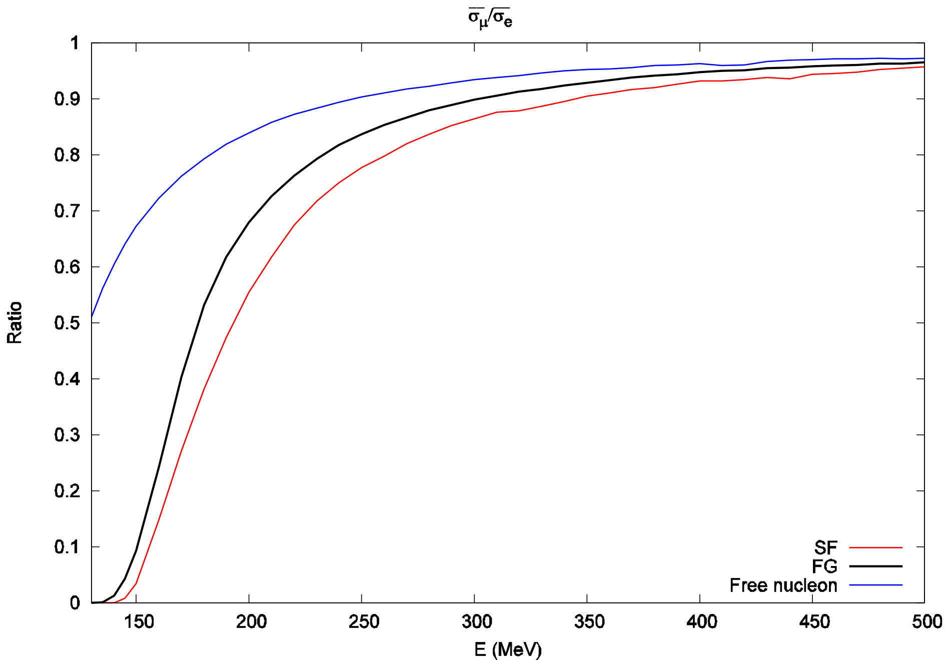




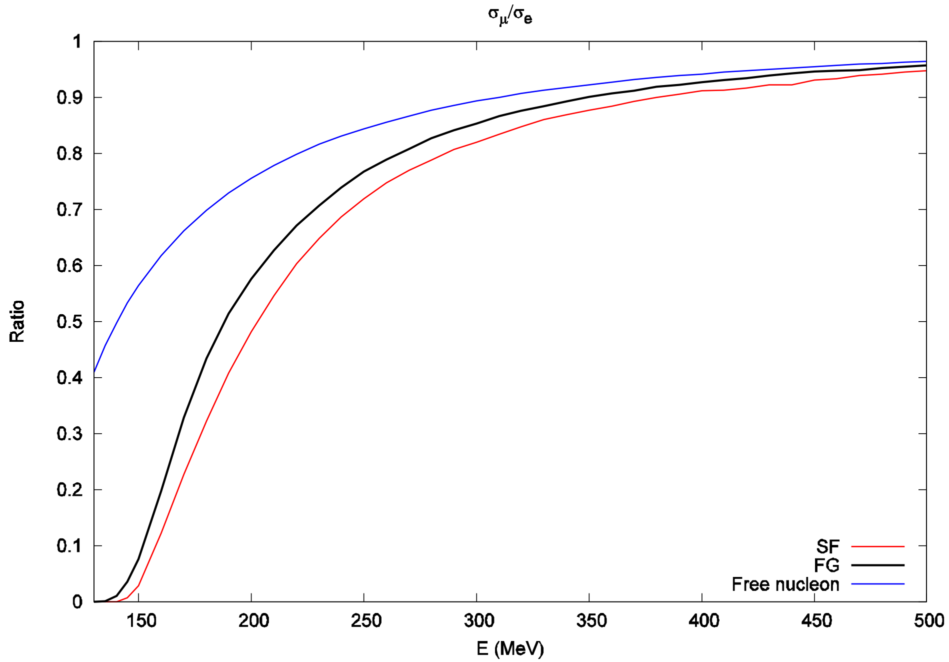




Spectral function - results



$$\frac{\sigma(\nu_\mu)}{\sigma(\nu_e)}, \quad \frac{\sigma(\bar{\nu}_\mu)}{\sigma(\bar{\nu}_e)}$$



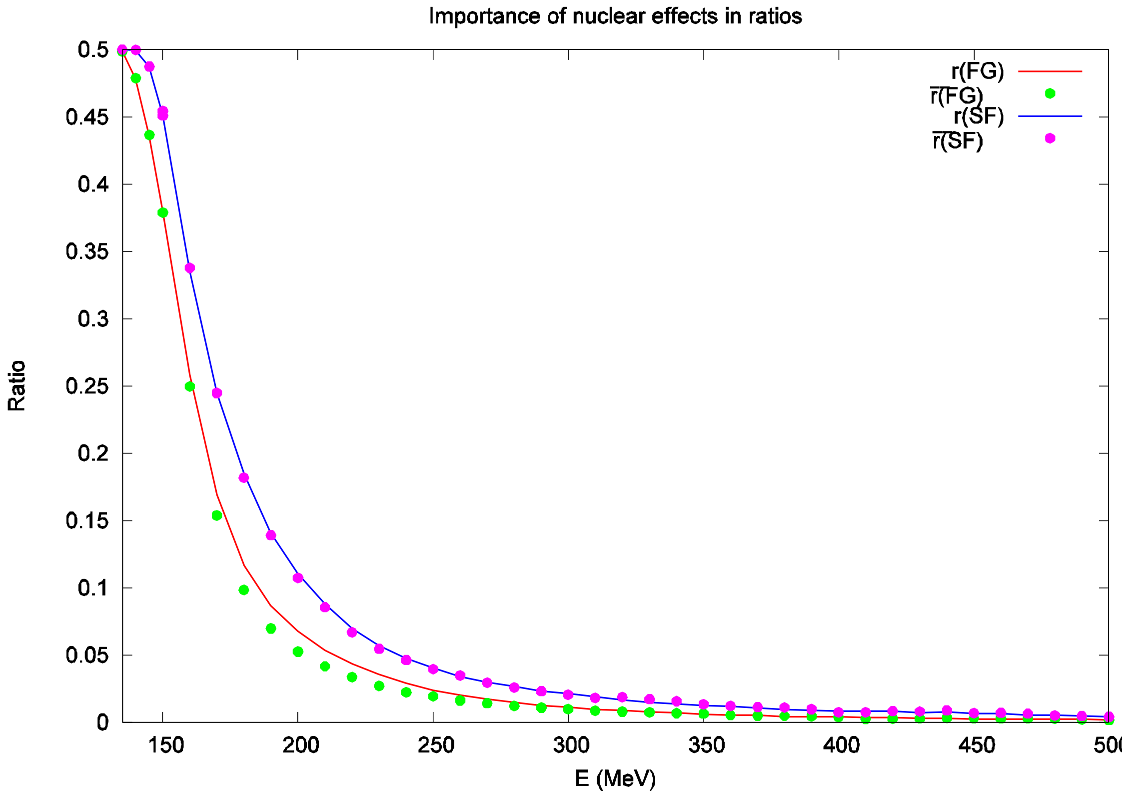
FG – Fermi gas model
SF – spectral function with Pauli blocking

Spectral function - results

„Double" ratios:

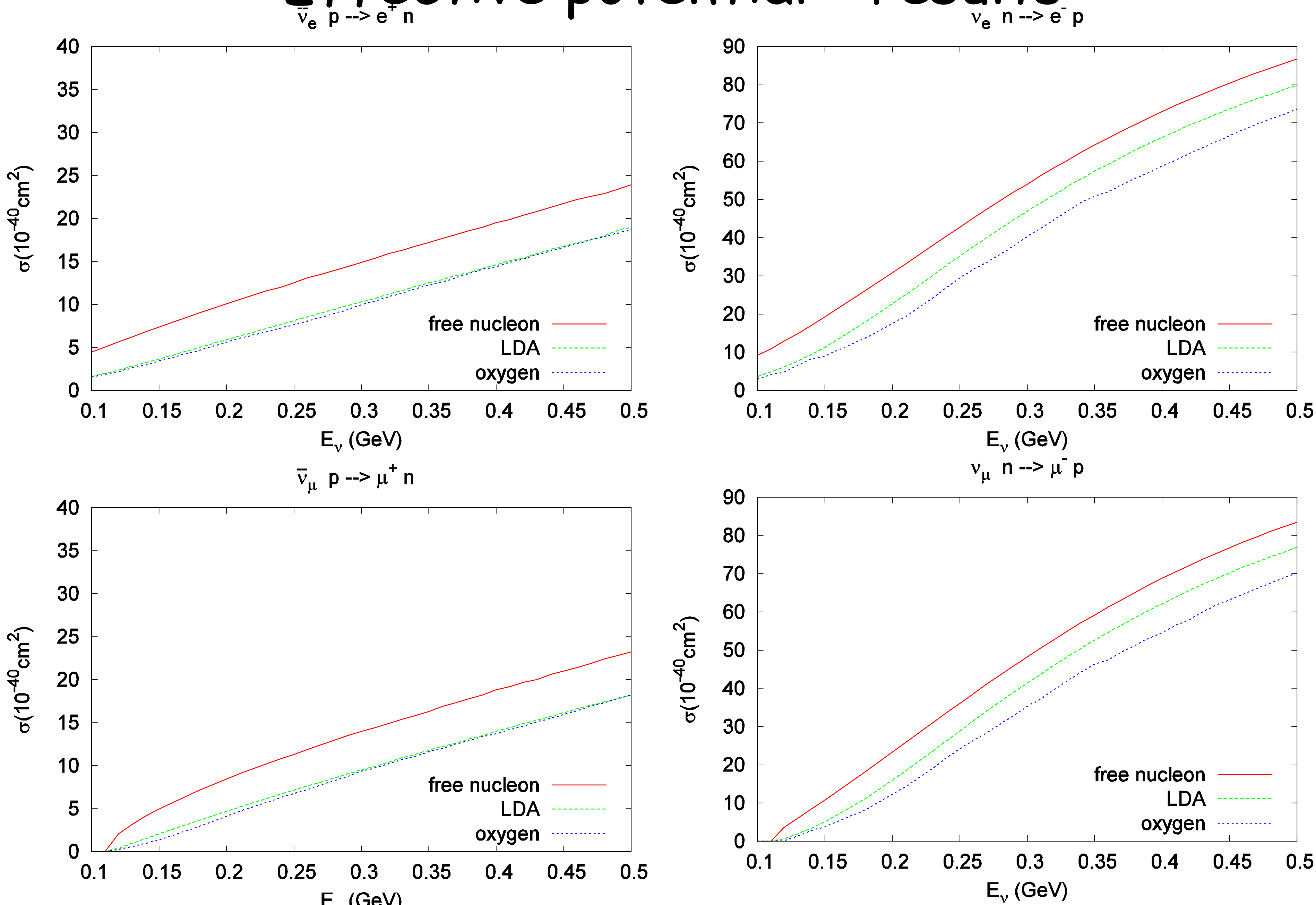
$$\frac{\dfrac{\sigma^{nuclear}(\nu_\mu)}{\sigma^{nuclear}(\nu_e)} - \dfrac{\sigma^{free}(\nu_\mu)}{\sigma^{free}(\nu_e)}}{\dfrac{1}{2}\left(\dfrac{\sigma^{nuclear}(\nu_\mu)}{\sigma^{nuclear}(\nu_e)} + \dfrac{\sigma^{free}(\nu_\mu)}{\sigma^{free}(\nu_e)}\right)}$$

$$\frac{\dfrac{\sigma^{nuclear}(\bar{\nu}_\mu)}{\sigma^{nuclear}(\bar{\nu}_e)} - \dfrac{\sigma^{free}(\bar{\nu}_\mu)}{\sigma^{free}(\bar{\nu}_e)}}{\dfrac{1}{2}\left(\dfrac{\sigma^{nuclear}(\bar{\nu}_\mu)}{\sigma^{nuclear}(\bar{\nu}_e)} + \dfrac{\sigma^{free}(\bar{\nu}_\mu)}{\sigma^{free}(\bar{\nu}_e)}\right)}$$

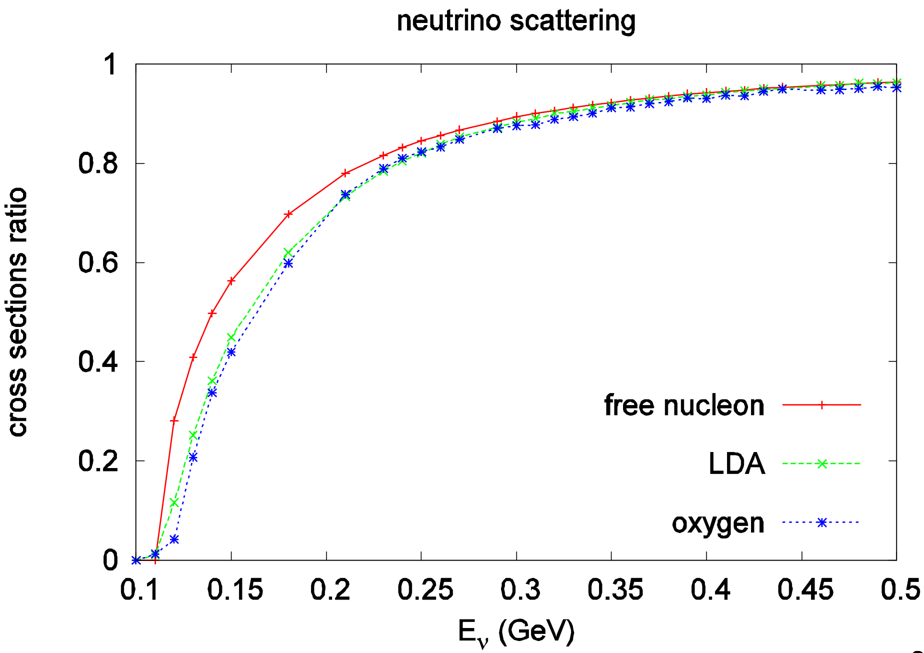


FG - Fermi gas model
SF - spectral function with Pauli blocking

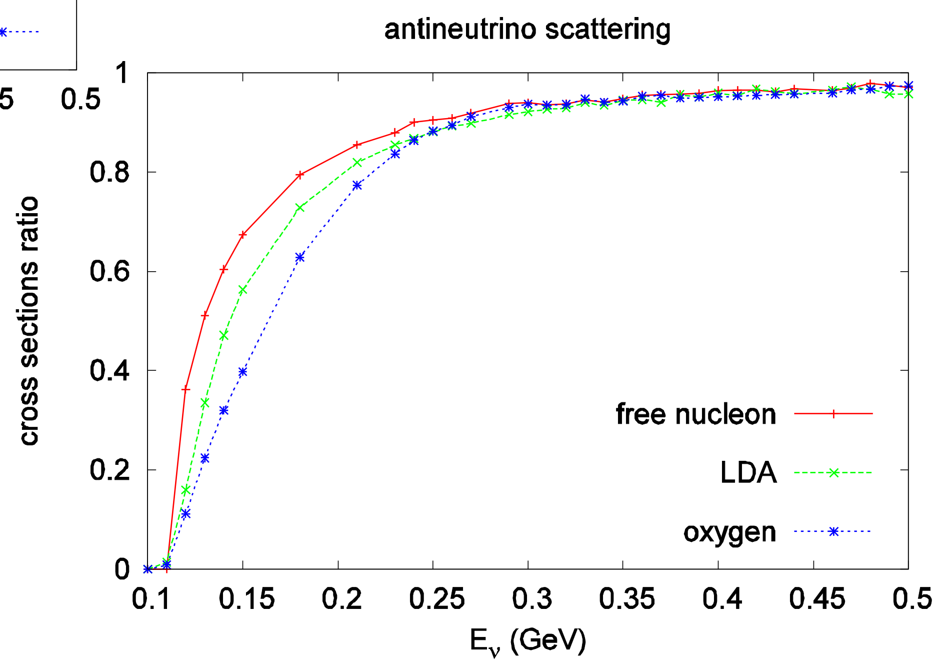
Effective potential - results



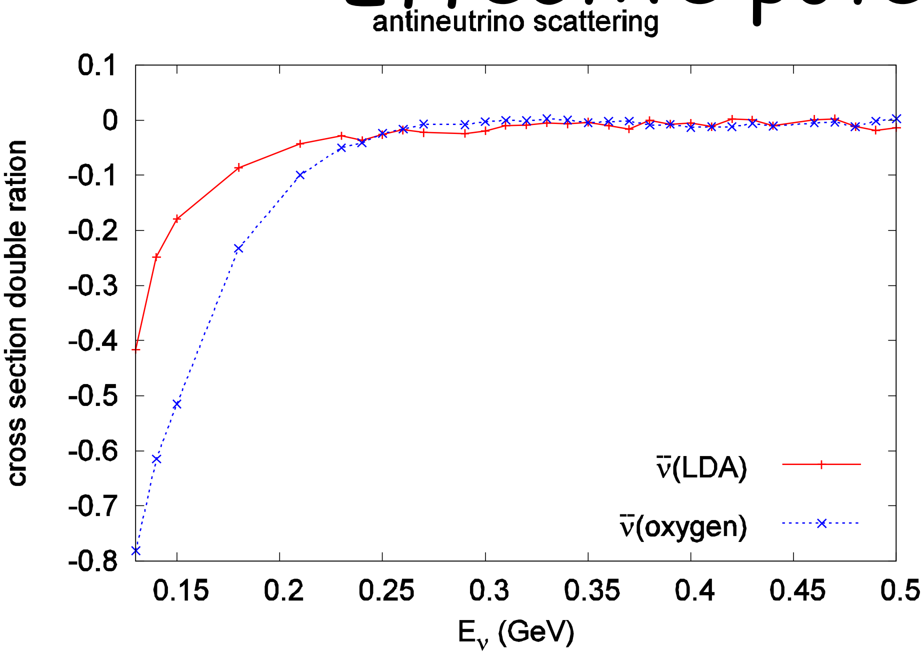
Effective potential - results



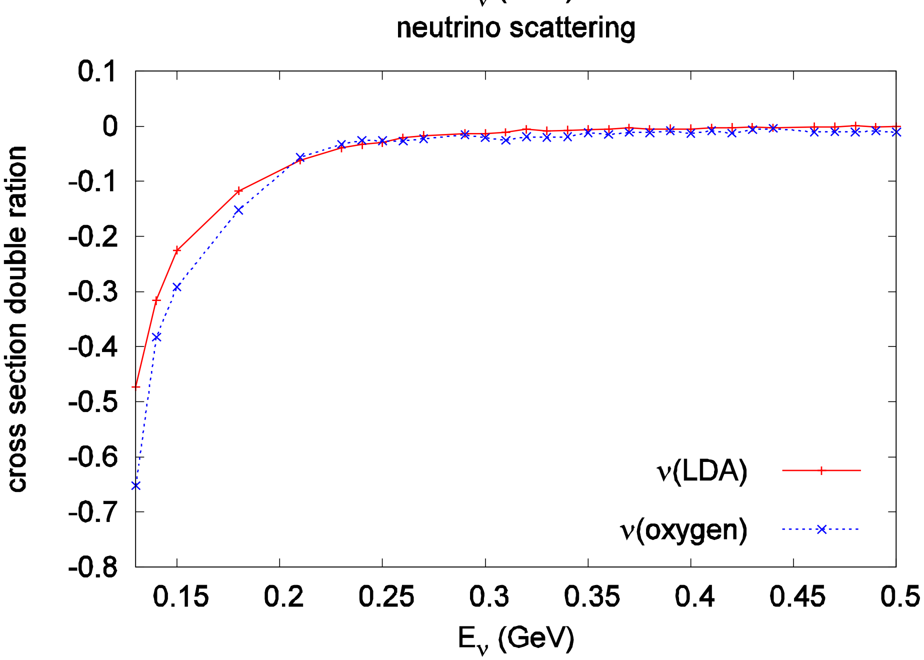
$$\frac{\sigma\ (\nu_\mu)}{\sigma\ (\nu_e)},\quad \frac{\sigma\ (\bar{\nu}_\mu)}{\sigma\ (\bar{\nu}_e)}$$



Effective potential - results



$$\frac{\dfrac{\sigma^{nuclear}(\bar{\nu}_\mu)}{\sigma^{nuclear}(\bar{\nu}_e)} - \dfrac{\sigma^{free}(\bar{\nu}_\mu)}{\sigma^{free}(\bar{\nu}_e)}}{\dfrac{1}{2}\left(\dfrac{\sigma^{nuclear}(\bar{\nu}_\mu)}{\sigma^{nuclear}(\bar{\nu}_e)} + \dfrac{\sigma^{free}(\bar{\nu}_\mu)}{\sigma^{free}(\bar{\nu}_e)}\right)}$$



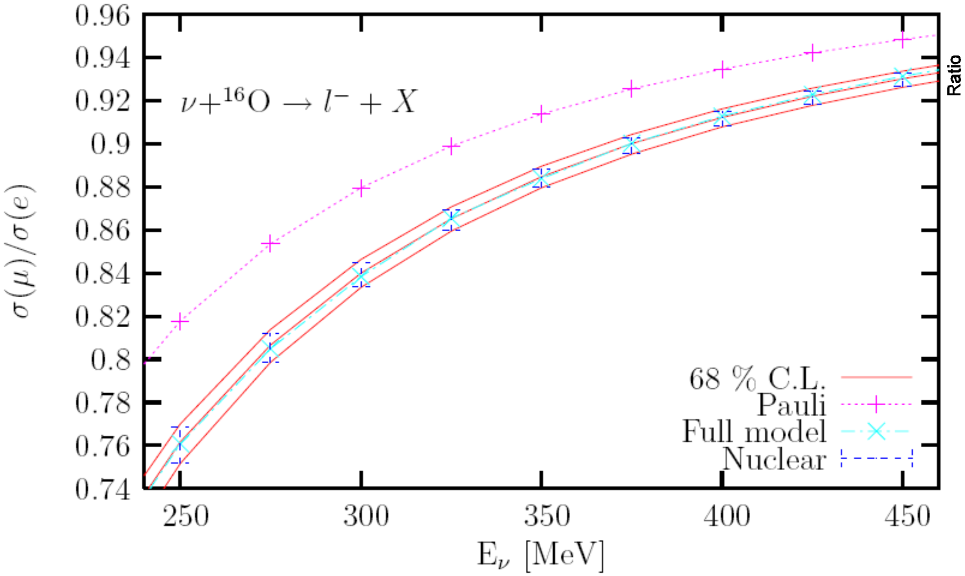
$$\frac{\dfrac{\sigma^{nuclear}(\nu_\mu)}{\sigma^{nuclear}(\nu_e)} - \dfrac{\sigma^{free}(\nu_\mu)}{\sigma^{free}(\nu_e)}}{\dfrac{1}{2}\left(\dfrac{\sigma^{nuclear}(\nu_\mu)}{\sigma^{nuclear}(\nu_e)} + \dfrac{\sigma^{free}(\nu_\mu)}{\sigma^{free}(\nu_e)}\right)}$$

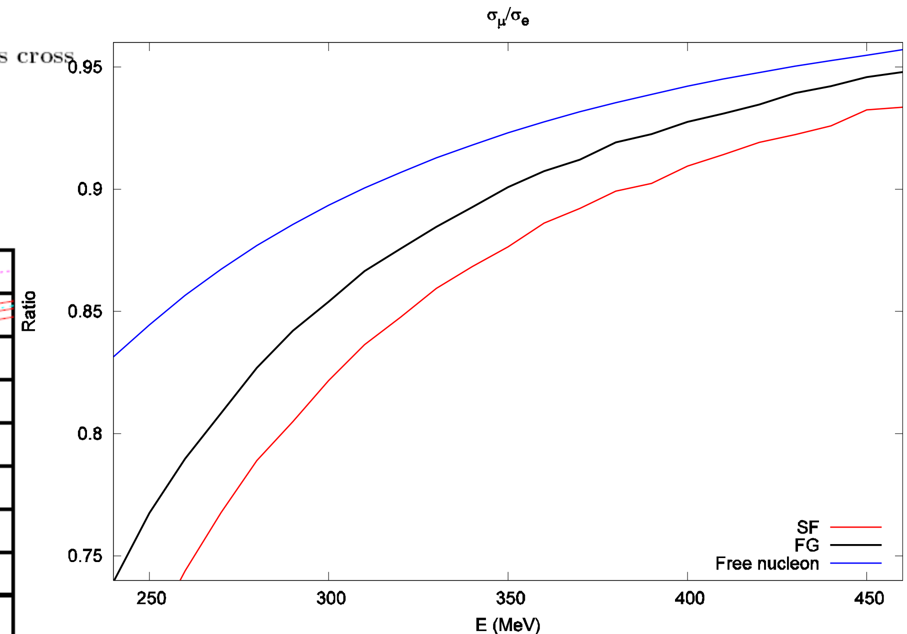
Comparison with Amaro-Nieves group

Theoretical uncertainties on quasielastic charged–current neutrino–nucleus cross sections

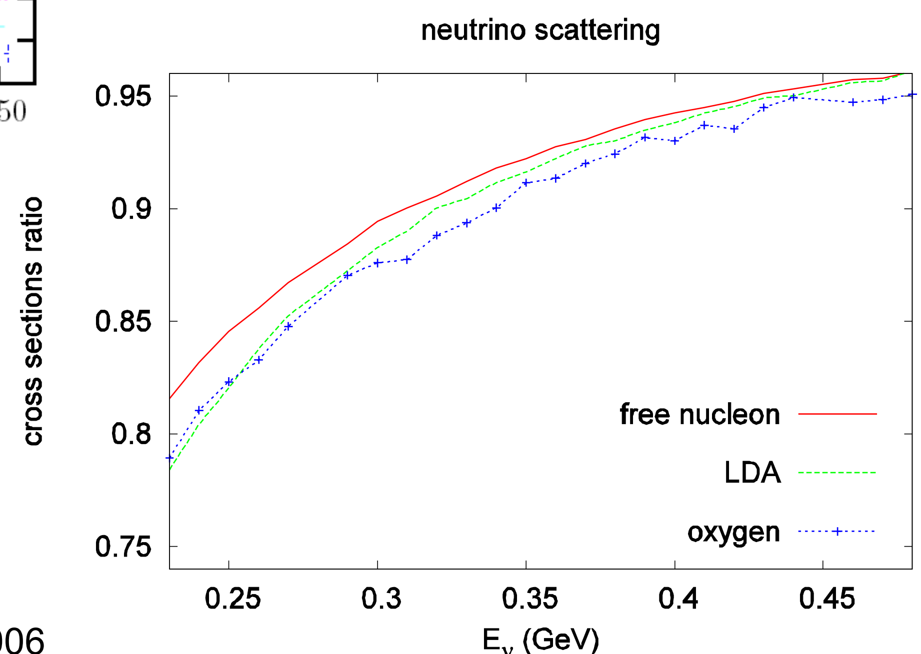
M. Valverde,[1] J. E. Amaro,[1] and J. Nieves[1]

(very recent paper: hep-ph/0604042)





FG – Fermi gas model
SF – spectral function with Pauli blocking



Conclusions (preliminary)

Ratios are not sensitive to uncertainties in free quasi-elastic description.

If 1% precision is required then for energies above 350 MeV pion production must be considered.

For energies below 250 MeV nuclear effects change ratios by more than 5%.

Much more detailed study is necessary, if few % precision is required.